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A flutter analysis of a system of two airfoils with aerodynamic interference

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A THEORETICAL study is made of the influence of an aerodynamic interaction of two thin profiles in the plane flow of an ideal gas on the critical flutter velocity. Presented are results of calculations of aerodynamic coefficients and the flutter velocity for biplane profiles and profiles laying on the straight line one behind the other.

Praca dotyczy teoretycznego zbadania wpływu aerodynamicznego oddziaływania dwóch cienkich profili umieszczonych w płaskim przepływie gazu doskonałego na prędkość krytyczną flatteru. Przedstawiono wyniki obliczeń współczynników aerodynamicznych oraz prędkości flatteru dla profili w układzie dwupłata i dla profili leżących jeden za drugim na jednej linii prostej.

Работа касается исследования теоретическим путем влияния на критическую скорость флаттера аэродинамического взаимодействия двух тонких профилей помещенных в плоском течении идеального газа. Представлены результаты расчетов аэродинамических коэффициентов и скоростей флаттера для профилей в системе биплана и для профилей лежащих один за другим на одной прямой линии.

1. Introduction

IN THE FLUTTER analysis of airplanes it is usually assumed that the aerodynamic forces act on the lifting surfaces only, that is the wings and the tail surfaces. These forces are usually determined for each lifting surface treated as isolated one, therefore by neglecting the flow perturbation due to the presence of the remaining parts of the airplane. However, the aerodynamic interaction between the wing and the tailplane or between the wings of a biplane should not be disregarded for some planes, in particular if the lifting surfaces are not wide apart. For preliminary assessment of the influence of such an aerodynamic interference on the critical flutter speed, a simple model taking into account the fundamental properties of the interaction between the elements considered and requiring no excessive numerical analysis would be particularly useful. This is important, because flutter analysis is always time-consuming and expensive, even if only a few degrees of freedom are assumed and the simplest model is used for determining non-stationary aerodynamic forces. With this in mind the two-dimensional model is assumed as a starting point, which leads to the determination of the critical flutter speed for two interacting airfoils. The principal aim of such a simplified computation is to obtain a prompt reply to the question as to whether the use of the airfoil configuration considered involves a risk of reducing the critical flutter speed as a result of the interaction between them.

2. Formulation of the problem

Let us consider two undeformable airfoils in homogeneous plane flow of a perfect gas at a speed U at infinity (Fig. 1). The airfoils are numbered in the order in which the middle points of their chords are met, if we move in the direction of flow. In the position of equilibrium the chords of the airfoils lie on two straight lines parallel to the direction of unperturbed flow, and coincide with the axes O_1x_1 and O_2x_2 of two rectangular systems of coordinates, $O_1x_1z_1$ and $O_2x_2z_2$, the origins of which are located at the middle points of the airfoil chords. The relative position of the two systems is determined by two coordinates L and H. A system of linear flexural springs, the rigidities of which are K_1, K_2 and K_z, and torsional springs, the rigidities of which are C_1 and C_2, connects the two airfoils

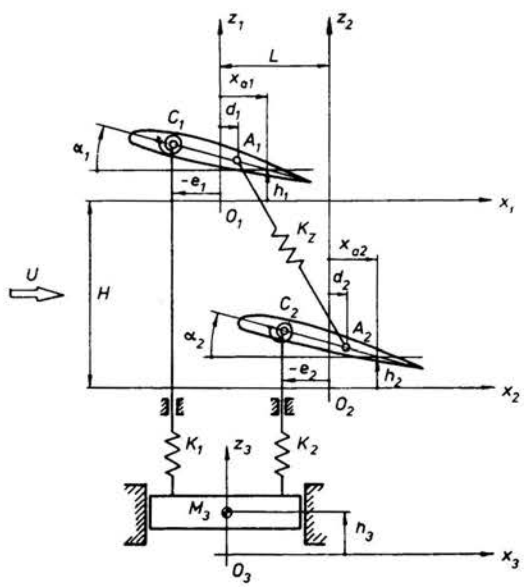


FIG. 1.

with a mass M_3 which is insulated from the flow. In the position of equilibrium its centre of gravity lies at the origin of a rectangular system of coordinates $O_3x_3z_3$, the axes of which are parallel to the corresponding axes of the systems connected with the airfoils. Each airfoil should be considered as constituting a unit segment of a wing of infinite aspect ratio, having a mass M_i, a static moment S_i with reference to a selected reference axis and a moment of inertia J_i about that axis (the index $i = 1, 2$ representing the number of the profile).

The airfoil system just described is freely deplaceable as a rigid body in the direction normal to that of unperturbed flow (the rigid degree of freedom). The flutter analysis of this system will consist in studying the stability of its motion resulting from a deviation from the position of equilibrium. In practice such an analysis depends always on the limited capability of the mathematical apparatus available and reduces to the determination of the limiting value of the speed of unperturbed flow, above which the motion of the airfoils becomes unstable. The objective of the present paper is to study the dependence of that speed on the aerodynamic interaction between the airfoils.

3. The equation of motion

The position of the system of two airfoils considered is described by five generalized coordinates $h_1, \alpha_1, h_2, \alpha_2, h_3$ (Fig. 1), which may be treated as components of a certain displacement vector $\mathbf{u}$. If the motion is stationary, we have $\mathbf{u} = 0$. Our fundamental assumption is one of small perturbations, that is the generalized coordinates and the generalized speeds have their moduli much less than unity. The method for deriving the equations of motion in the principal coordinates selected for the purposes of the present paper is based on previously determined natural frequencies and modes. These quantities represent its mass and rigidity properties in a joint manner. In addition, the flutter equations based on the natural modes have a simple structure which is an essential advantage bearing in mind the necessity of solving them many times.

The method for deriving the equations of motion applied here is a particular case of the method described in Refs. [1] and [2] for a free material system composed of linearly elastic bodies. The starting point is the determination of the vector of displacement $\mathbf{u}$ as a sum of two orthogonal vectors $\mathbf{u}_R$ and $\mathbf{u}_E$

$$\mathbf{u} = \mathbf{u}_R + \mathbf{u}_E.$$

This orthogonality is understood in the sense of a scalar product defined for any two vectors $\mathbf{a}$ and $\mathbf{b}$ in the following manner

$$(\mathbf{a}, \mathbf{b}) = \mathbf{a}^T \mathbf{M} \mathbf{b}, \tag{3.1}$$

where $\mathbf{M}$ is the mass matrix of the system considered and $\mathbf{a}^T$ is the transposed vector. In the considered case of two profiles, the mass matrix is

$$\mathbf{M} = \begin{bmatrix} M_1 & -S_1 & 0 & 0 & 0 \\ -S_1 & J_1 & 0 & 0 & 0 \\ 0 & 0 & M_2 & -S_2 & 0 \\ 0 & 0 & -S_2 & J_2 & 0 \\ 0 & 0 & 0 & 0 & M_3 \end{bmatrix}$$

the elements of which are mass parameters.

The vector $\mathbf{u}_R$ is part of the displacement $\mathbf{u}$ representing the rigid displacement of the system and belongs to the one-dimensional space R, the basic vector of which is $\mathbf{\Psi} = [1\ 0\ 1\ 0\ 1]$, and whose components constitute the linear part of the increase in the generalized coordinates corresponding to a unit increase in the coordinate describing the rigid displacement. The vector $\mathbf{u}_E$ is that part of the displacement $\mathbf{u}$ which corresponds to the elastic strain of the system and belongs to a space E, orthogonal to R, with a basis formed by the vectors $\mathbf{\Phi}_i (i = 1, 2, 3, 4)$ of natural modes of vibration.

Any displacement of the moving system $\mathbf{u}(t)$ under the action of external aerodynamic forces belongs to the space $E+R$ and can be represented as a linear combination of the vectors of basis of the component sub-spaces:

$$\mathbf{u}(t) = \sum_{i=1}^{4} \mathbf{\Phi}_i q_i(t) + \mathbf{\Psi} q_5(t),$$

where $q_i(t)$ $(i = 1, 2, \ldots, 5)$ are functions of t alone and constitute new generalized coordinates. The set of equations for the latter has the form

$$m_i(\ddot{q}_i + \omega_i^2 q_i) = Q_i \qquad (i = 1, 2, 3, 4), \tag{3.2}$$

$$(\mathbf{\Psi}, \mathbf{\Psi})\ddot{q}_5 = Q_5, \tag{3.3}$$

where m_i denotes the generalized mass corresponding to the particular mode of natural vibration with a frequency ω_i

$$m_i = (\mathbf{\Phi}_i, \mathbf{\Phi}_i),$$

and Q_i $(i = 1, 2, \ldots, 5)$ are generalized aerodynamic forces corresponding to the natural modes and the rigid mode.

The separation of the Eqs. (3.2) and (3.3) and their coupling by aerodynamic forces alone results from a decomposition of the vector $\mathbf{u}$ into a sum of two vectors, which are orthogonal in the sense of the scalar product (3.1) and the transition to the principal coordinates.

4. The aerodynamic forces

To determine the vector $\mathbf{Q}$ of generalized aerodynamic forces, the components Q_i of $(i = 1, 2, \ldots, 5)$ which occur in the right-hand members of the Eqs. (3.2) and (3.3), it is necessary to know the pressure distribution over each airfoil during the motion of the system. In what follows harmonic motion will only be considered. The pressure distribution over profiles will be found under the usual simplifying assumptions, which enable linearization of the fundamental equations and boundary conditions. These assumptions were used for the computation of the pressure distribution over a single isolated airfoil [3]. Their discussion for a system of two profiles is contained in Ref. [4]. Linearization of the boundary condition at the surfaces of the airfoils, expressing their impermeability, means that each airfoil is replaced by a straight line segment. Linearization of the equations describing the flow enables us to introduce the acceleration potential which satisfies the Laplace equation and is proportional to the pressure on the airfoils. As a result, the problem of determining the pressure distribution over the oscillating airfoils reduces to an external Neumann problem for the acceleration potential with a boundary condition for the surface of the two airfoils only (a vortex wake does not occur in an explicit manner). Now, starting out from an elementary solution having the form of the potential of a double layer and making use of the fundamental equation for harmonic functions, we obtain a set of two integral equations for the amplitude of the reduced pressure difference $\gamma_i(x_i)$, which is related with the amplitude of the pressure difference between the upper and lower side of the i-th airfoil $\Delta p_i(x_i)$ by the equation

$$\gamma_i(x_i) = -\frac{\Delta p_i(x_i)}{\varrho U^2},$$

where ϱ is the density of the medium. The set of equations mentioned is as follows

$$\frac{1}{2\pi}\int_{-1}^{1}\mathbf{k}(x,\xi)\boldsymbol{\gamma}(\xi) = \mathbf{a}(x), \tag{4.1}$$

where $\mathbf{k}(x,\xi)$ is a second-order matrix, the elements of which are the following kernel functions

$$k_{lm}(x,\xi) = \begin{cases} \dfrac{1}{(x-\xi)^2}, & \text{for } l = m \\[2ex] \dfrac{\left(\dfrac{(-1)^m L - b_l x}{b_m} + \xi\right)^2 - \left(\dfrac{H}{b_m}\right)^2}{\left[\left(\dfrac{(-1)^m L - b_l x}{b_m} + \xi\right)^2 + \left(\dfrac{H}{b_m}\right)^2\right]^2}, & \text{for } l \neq m \\ l, m = 1, 2; \end{cases}$$

$\boldsymbol{\gamma}(\xi)$ is a two-dimensional vector, the components of which are the amplitudes of the reduced pressure difference on the first and second airfoil, respectively, $\mathbf{a}(x)$ — a two dimensional vector, the components of which are the amplitudes of the normal acceleration components on the first and second airfoil, respectively and b_1, b_2 — half-chords of the first and second airfoil, respectively.

If $l = m$, there occur in the set of Eq. (4.1) integrals, which are convergent neither in the ordinary sense nor in the sense of the Cauchy principal value. The method for making those integrals regular is strictly determined by the order of operations of integration in the fundamental equation for the harmonic functions and the passage to the limit, thus obtaining the set of Eq. (4.1). It can be shown that this procedure is that of treating divergent integrals in the sense of the Hadamard finite value.

The set of integral Eq. (4.1) does not represent a complete mathematical formulation of the problem, because prescription of the normal acceleration component on the airfoils does not determine in a complete manner the pressure field, which depends also on the normal velocity component. This physical fact has its mathematical reflection in the solution of the set of integral equations being not unique. It is determined with an accuracy to within the solution of the homogeneous set that in for $\mathbf{a}(x) \equiv 0$. There are various methods for ensuring the uniqueness of solution, by solving the homogeneous set, for instance, and requiring the equality of the normal velocity component to prescribed values at any two points on the airfoils. Another method is by transforming both members of Eq. (4.1) by means of an operator transforming the acceleration potentional into that of velocity. Then, we obtain a set of integral equations of strong singularity, of the Possio type, constituting a complete mathematical formulation of the problem. There are, however, some difficulties in the numerical solution for high values of the frequency coefficient k, which is defined thus

$$k = \frac{\omega b}{U},$$

where ω is the circular frequency of oscillation, b denotes half-chord of one of the airfoils selected as a reference length. The duration of the computation increases considerably, for the same accuracy, which is due to the necessity of computing integrals of rapidly oscillating functions. These difficulties can be avoided by transforming both members of Eqs. (4.1) by means of an operator defining the integral acceleration potential $\Psi(x, z)$ (or, strictly speaking, its amplitude of harmonic vibration) [5]:

$$\Psi(x, z) = \int_{-\infty}^{x} \psi(\xi, z)\, d\xi, \tag{4.2}$$

where $\psi(\xi, z)$ is the amplitude of the acceleration potential. The transformation of the left-hand member of the set of Eq. (4.1) is reduced to some simple mathematical operations, the transformation of the vector $\mathbf{a}(x)$ requiring a brief discussion, however. The integration path in Eq. (4.2) is a ray from minus infinity to a point on the airfoil, but the components of the vector $\mathbf{a}(x)$ (acceleration normal to the airfoil) are directly known on the segment occupied by the airfoils only. For the remaining part of the integration path these components are determined by the sought for pressure distribution on the airfoils. The integral (4.2) should, therefore, be resolved into a sum of two integrals, one of which extends over the segment of the airfoil up to the leading edge and the other over the ray in front of the airfoil. The latter integrand will be found by expressing the normal acceleration component in terms of the unknown reduced pressure difference, making use of the operator converting the acceleration potential into the velocity potential. On performing all the operations described we obtain, for the reduced pressure difference (the vector $\boldsymbol{\gamma}$), the following set of integral equations with a strong singularity

$$\frac{1}{2\pi} \int_{-1}^{1} \mathbf{K}(x, \xi)\boldsymbol{\gamma}(\xi)\, d\xi = \mathbf{d}(x) + \frac{1}{2\pi} \int_{-\infty}^{-1} \mathbf{C}(x) \int_{-1}^{1} \mathbf{K}(x, \xi)\boldsymbol{\gamma}(\xi)\, d\xi, \tag{4.3}$$

where $\mathbf{K}(x,\xi)$ is a second-order matrix composed of the following kernel functions

$$K_{lm}(x, \xi) = \begin{cases} \dfrac{1}{\xi - x} & \text{for } l = m, \\[2ex] \dfrac{\dfrac{(-1)^m L - b_l x}{b_m} + \xi}{\left(\dfrac{(-1)^m L - b_l x}{b_m} + \xi\right)^2 + \left(\dfrac{H}{b_m}\right)^2} & \text{for } l \neq m, \\[2ex] l, m = 1, 2; \end{cases}$$

$\mathbf{d}(x)$ is a two-dimensional vector, the components of which are given by the equation

$$d_l(x) = w_l(x) + ik_l \int_{-1}^{x} w_l(\xi)\, d\xi \qquad (l = 1, 2),$$

where $w_l(x)$ denotes the amplitude of the normal velocity components on the l-th airfoil and k_l — the frequency coefficient as referred to the half-chord of that airfoil, the symbol i denoting the imaginary unit; $\mathbf{C}(x)$ is a diagonal matrix of the second order, the elements of which are the functions

$$c_{lm}(x) = \begin{cases} ik_l e^{ik_l(x+1)}, & l = m, \\ 0, & l \neq m, \end{cases}$$
$$l, m = 1, 2.$$

The latter term of Eqs. (4.3) is a constant, independent of the coordinate x, constituting a functional of the sought-for solution $\boldsymbol{\gamma}(x)$.

The interaction between the profiles is expressed, in the set of Eq. (4.3), by non-singular components of the kernel K_{lm} (for $l \neq m$). These kernels tend rapidly to zero, if the airfoils are moved apart for $H \neq 0$. This is obvious because for $H = 0$ the second profile lies always in the vortex wake extending from the first airfoil to infinity.

For airfoils in tandem configuration a closed-form solution can be obtained [6], which cannot be generalized, however, to the case of Eq. (4.3) due to the geometry of the boundary of the region and the form of the kernels. Under such conditions, one of the simplified methods must be used for actual solution. In the present work the method of least squares will be used, which seems to be the most effective of the existing approximate methods of functional analysis as regards the set of integral equations obtained.

The Eq. (4.3) can be written in the operational form

$$\mathbf{D}\boldsymbol{\gamma}(x) = \mathbf{d}(x),$$

where $\mathbf{D}$ is the operator described by the matrices $\mathbf{K}$ and $\mathbf{C}$. Each component $\gamma_j(x)$ $(j = 1, 2)$ of the vector $\boldsymbol{\gamma}(x)$ is approximated by the series

$$\gamma_j(x) \approx \gamma_j^{(n)}(x) = 2\sqrt{\frac{1-x}{1+x}} \sum_{l=0}^{n} a_l^{(j)} P_l^{\left(\frac{1}{2}, -\frac{1}{2}\right)}(x) \qquad (j = 1, 2), \tag{4.4}$$

where $P_l^{\left(\frac{1}{2}, -\frac{1}{2}\right)}(x)$ are Jacobi polynomals and $a_l^{(j)}$ denote numerical coefficients, which in the method of least squares are determined from the condition that the square of the norm

$$||\mathbf{D}\boldsymbol{\gamma}^{(n)}(x) - \mathbf{d}(x)||^2 = \int_{-1}^{1} \sqrt{\frac{1+x}{1-x}} |\mathbf{D}\boldsymbol{\gamma}^{(n)} - \mathbf{d}(x)|^2 dx$$

should be minimum. This leads to the following set of linear algebraic equations

$$\sum_{m=1}^{N} g_{lm} a_m = p_l \qquad (l = 1, 2, \ldots, N),$$

where a_m are the sought-for expansion coefficients (4.4) on both airfoils, the total number being N, and the coefficients g_{lm} and p_l of the set of equations are given by the formulae (the bar denoting the conjugate complex number)

$$g_{lm} = \int_{-1}^{1} \sqrt{\frac{1+x}{1-x}} F_m^{(1)}(x) \overline{F_l^{(1)}(x)} dx + \int_{-1}^{1} \sqrt{\frac{1+x}{1-x}} F_m^{(2)}(x) \overline{F_l^{(2)}(x)} dx,$$

$$p_l = \int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\, d_1(x) \overline{F_l^{(1)}(x)}\, dx + \int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\, d_2(x) \overline{F_l^{(2)}(x)}\, dx.$$

In these formulae $d_1(x)$ and $d_2(x)$ are the components of the vector $\mathbf{d}(x)$ occurring in Eq. (4.4). The functions $F_m^{(1)}(x)$ and $F_m^{(2)}(x)$ are integrals of various types on which largely depends the time of computation and the error of the aerodynamic coefficients obtained.

Most of those integrals can be readily evaluated which is an essential advantage of the method for deriving the set of integral equations by means of an integral operator of the acceleration potential. The remaining integrals, with regular integrands, have been computed by means of the Gauss-Jacobi quadrature.

To assess the overall error, with which the aerodynamic coefficients have been obtained, an ODRA 1204 computer was used to perform the analysis by the method described for a single isolated airfoil with two degrees of freedom. In this case a closed-form solution is known, which is limited to two terms in the expansion (4.4) for translational oscillation of the airfoil, and to three terms for rotational oscillation. The approximate solution obtained can be compared to the exact solution. The test computation was performed for two, three, four and five terms of the expansion (4.4). In each of those cases the frequency coefficient took the values $k = 0.01, 0.1, 1, 10, 100$. In all the computations for $k < 10$ (except the aerodynamic coefficients corresponding to the two-term expansion for rotational oscillation) an agreement of ten significant digits was found for the real and imaginary parts of the aerodynamic coefficients. With increasing frequency coefficient and increasing number of expansion terms the relative error increased, reaching at most 0.001%. All the integrals, except those of the method of least squares, have been determined by analytic methods, the result obtained shows that the error of the method is insignificant and makes us suppose that its main source is the procedure of rounding off the scalar products of the quadratures. This conclusion is confirmed by the analogous test described in Ref. [7], in which the method of least squares was used to solve the Possio equation. The form of the kernel of that equation suggests its decomposition into a singular part and regular part, which means numerical solution of more integrals than in the case of integral acceleration potential. As a result, the agreement for the coefficient of the moment of rotating motion of a single isolated profile and a frequency coefficient $k = 1$, was of two or three significant digits only. It remained unchanged independent of the number of expansion terms of the reduced pressure difference. In the present case, that is in the case of two airfoils, the value of the result was already established for four terms of the expansion (4.4) for each of the airfoils.

The lift L and the moment M about the reference axis x_a (Fig. 1) acting on the profile are expressed in terms of the reduced pressure difference $\gamma(x)$ as follows

$$L = \varrho U^2 b e^{i\omega t} \int_{-1}^{1} \gamma(x)\, dx,$$

$$M = -\varrho U^2 b^2 e^{i\omega t} \int_{-1}^{1} (x - x_a)\gamma(x)\, dx,$$

where b is the half-chord of the airfoil.

For purely formal reasons it is more convenient to use the following expressions of the aerodynamic coefficients of the lift A_1 and moment A_2 [8]

$$A_1 = \frac{Le^{-i\omega t}}{\pi\varrho U^2 b} = \frac{1}{\pi}\int\limits_{-1}^{1}\gamma(x)\,dx,$$

$$A_2 = -\frac{Me^{-i\omega t}}{\pi\varrho U^2 b^2} = \frac{1}{\pi}\int\limits_{-1}^{1}(x-x_a)\gamma(x)\,dx.$$

Bearing in mind the orthogonality property of the Jacobi polynomials, these coefficients depend only on the first two terms of the expansion (4.4) for the reduced pressure difference on the airfoil.

The problem considered, that is the problem of determining the pressure distribution over the airfoils has been reduced to a set of linear equations with linear boundary conditions. It follows that the system of forces acting on airfoils moving in any harmonic manner can be obtained by superposition of forces for simple component motions. Any harmonic motion of two airfoils can be described by superposing four simple motions, during which only one generalized coordinate varies, the remaining ones being zero. In the present case they will be translational and rotational oscillations of each profile. Each of those motions produces forces and moments acting on the oscillating airfoil (their values being different from those obtained for the isolated airfoil due to the presence of the other airfoil, which is fixed) and on the fixed airfoil.

The latter result from the aerodynamic interference. Thus, each elementary motion corresponds to a set of four aerodynamic coefficients. The set of forces acting on the airfoils during any harmonic motion, in which all the generalized coordinates vary, that is h_1, α_1, h_2, α_2 can now be expressed in the form of the product

$$(4.5)\qquad \begin{Bmatrix} L_1 b_1 \\ -M_1 \\ \delta L_2 b_2 \\ -\delta M_2 \end{Bmatrix} = \pi\varrho U^2 b_1^2 e^{i\omega t}\left[\begin{array}{cc|cc} A_{11}^{11} & A_{12}^{11} & A_{11}^{12} & A_{12}^{12} \\ A_{21}^{11} & A_{22}^{11} & A_{21}^{12} & A_{22}^{12} \\ \hline A_{11}^{21} & A_{12}^{21} & A_{11}^{22} & A_{12}^{22} \\ A_{21}^{21} & A_{22}^{21} & A_{21}^{22} & A_{22}^{22} \end{array}\right]\begin{Bmatrix} h_1 \\ \alpha_1 \\ h_2 \\ \alpha_2 \end{Bmatrix},$$

where $\delta = (b_2/b_1)^2$.

The matrix of aerodynamic coefficients is composed of four blocks of four elements each. The diagonal blocks represent the action of each airfoil on itself, the remaining two being an effect of the aerodynamic interference. If the latter is disregarded, the matrix will be composed of two diagonal blocks only.

The first subscript indicates whether the coefficient that of lift or moment, the other determining the oscillation type (translational or rotational). Either of the two subscripts may become 1 or 2. The superscripts indicate the airfoil to which the coefficient pertains and the airfoil performing the oscillation indicated by the second subscript. In other words the superscripts determine the object of the action of an airfoil.

For a biplane configuration the influence of the aerodynamic interference decreases rapidly with increasing distance between the airfoils. The examples of computation results

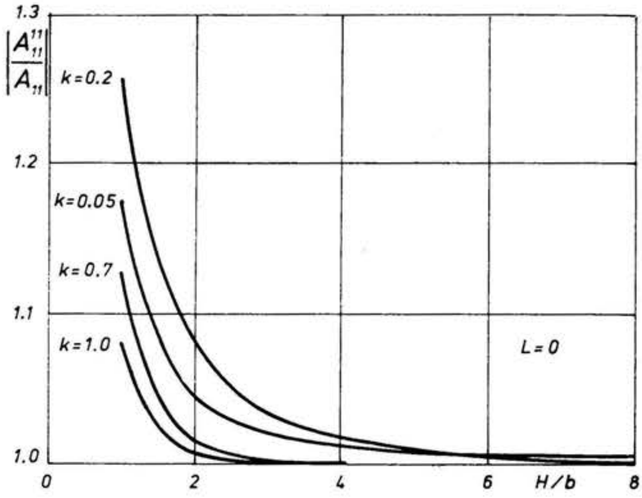


FIG. 2.

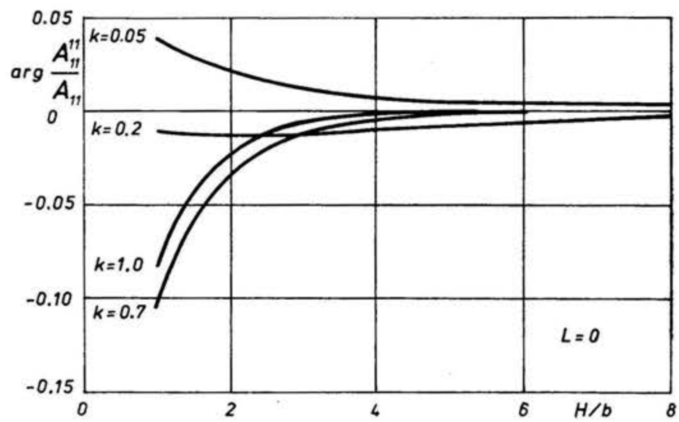


FIG. 3.

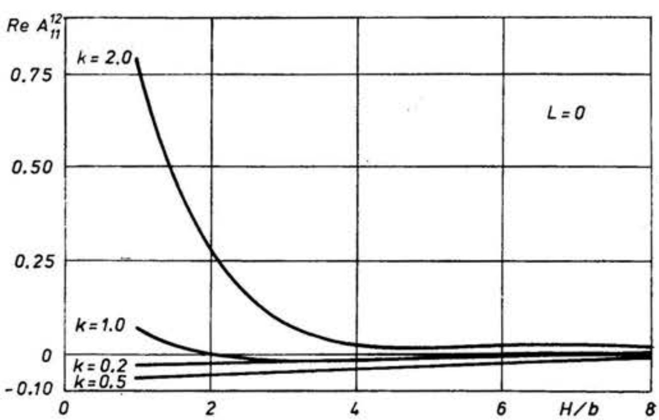


FIG. 4.

presented here concern a system of two identical airfoil of a chord 2b and a distance $L = 0$ between the middle points of the chords in the direction of flow (Fig. 1). Various distance H between the airfoils and various values of the frequency coefficient k were assumed. The influence on the oscillation of an airfoil of the presence of the other airfoil decreases very rapidly with increasing distance between the airfoils, which is illustrated in Figs. 2 and 3, representing the modulus and the argument of the ratio of the lift coefficient of an airfoil due to its translational oscillation in the presence of the other airfoil, which is fixed, to the relevant standard coefficient (that is the lift coefficient for a single isolated airfoil). The character of the variation is independent of the frequency coefficient. The ratios of the remaining aerodynamic coefficients, representing the action of each airfoil on itself and the relevant standard coefficient behave in a similar manner. The coefficients of mutual action of the two airfoils decrease also rapidly with increasing distance between them. This is illustrated in Figs. 4 and 5, which represent the real and the imaginary part of the lift coefficient for one airfoil due to translational oscillation of the other airfoil. The remaining aerodynamic coefficient of mutual action behave in the same manner. The influence of the aerodynamic interference on the forces acting on two airfoils in tandem configuration

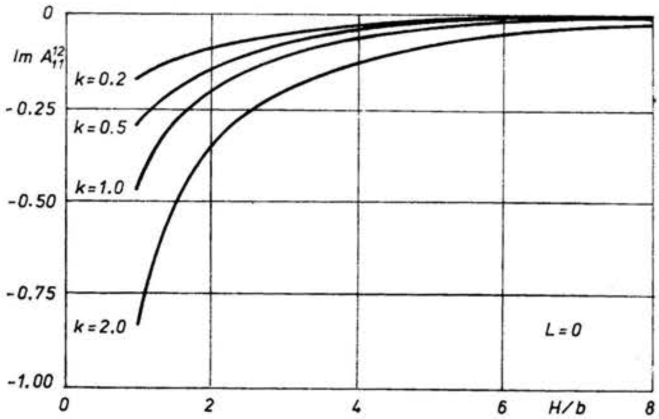


FIG. 5.

is different. This configuration corresponds to $H = 0$ (Fig. 1). The aerodynamic coefficients resulting from the action of each airfoil on itself vary very little with increasing distance between the airfoils. The effect of the aerodynamic interference is described by the coefficients of mutual action. Examples of results are shown for lift coefficients (Figs. 6, 7) for a frequency coefficient $k = 1$. The aerodynamic interference has here a distinct directional character. The coefficients resulting from the action of the first airfoil on the second airfoil have an oscillatory character, the corresponding wavelength being determined by the speed of flow and the frequency of oscillation. On the other hand, the aerodynamic coefficients representing the action of the second airfoil on the first decrease in a monotonic manner to zero with increasing distance between the airfoil. The aerodynamic coefficients representing the action of the first airfoil on the second airfoil are of the same order of magnitude as those of the action of each airfoil on itself.

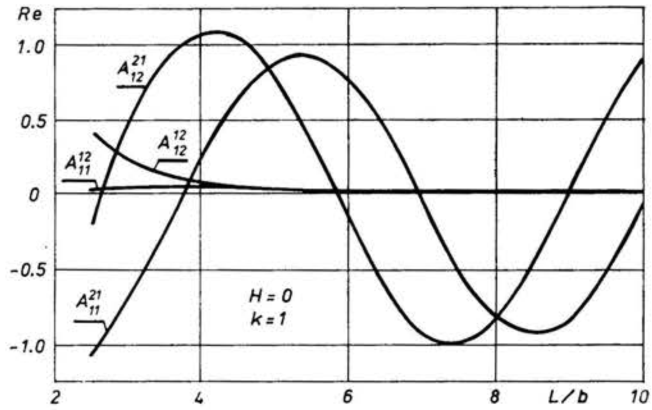


FIG. 6.

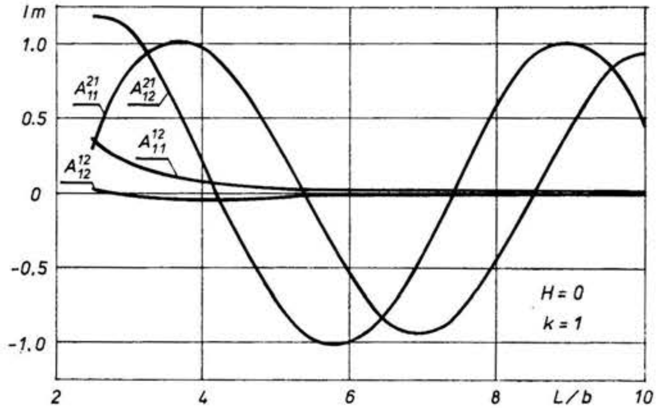


FIG. 7.

The knowledge of the aerodynamic coefficients enables us to determine the vector of generalized aerodynamic forces **Q**, which constitutes the right-hand member of the Eqs. (3.2) and (3.3). This vector has the form

$$\mathbf{Q} = \varrho\omega^2 \mathbf{A}\mathbf{q}e^{i\omega t}, \tag{4.6}$$

where ϱ denotes, as before, the density of the medium, ω is the angular frequency of oscillation and $\mathbf{q}$ is the vector of principal coordinates. The matrix **A** is the aerodynamic matrix, the elements of which are expressed by the formula

$$a_{lm} = \frac{\pi b_1^4}{k^2} \boldsymbol{\varphi}_l^T \mathbf{A}_c \boldsymbol{\varphi}_m \qquad (l, m = 1, 2, \ldots, 5), \tag{4.7}$$

where k is the frequency coefficient referred to the half-chord of the first airfoil, $\boldsymbol{\varphi}_l$ denotes a vector, the components of which are equal to the generalized coordinates of the airfoils corresponding to the l-th mode (the rigid mode being included) and $\mathbf{A}_c$ is the matrix of aerodynamic coefficients appearing in Eq. (4.5). The aerodynamic matrix **A** is a function of the frequency coefficient, therefore also the frequency ω, the square of which enters Eq. (4.6) in an explicit manner. Such an expression is due to the requirements of the analysis of flutter.

5. The flutter analysis

The starting point for the flutter analysis of the system of two airfoils considered are Eqs. (3.2) and (3.3). However, bearing in mind the complicated manner in which the aerodynamic forces depend on the history of the motion, the classical methods of stability analysis cannot be applied to those equations. The present flutter analysis consists in finding such values of the parameters (the flow velocity, in particular), for which harmonic motion as described by the equations considered is possible. By analogy to the methods of stability analysis consisting in assuming solution in the form $e^{\lambda t}$ (where $\lambda = \mu + i\omega$) and studying the sign of the real part of the parameter λ, the harmonic motion is treated here as a motion which separates damped vibration from undamped vibration. The relevant value of the speed of flow U is the critical flutter speed. Such a method has commonly been used since the beginning of the history of flutter analysis and is as yet the only one enabling some inferences to be drawn on the stability of the set of Eqs. (3.2) and (3.3), which is always a complicated integro-differential set of equations.

The set of Eqs. (3.2) and (3.3) completed by a term of damping forces, which are assumed to be proportional to the elastic forces but in phase with the velocities, can be represented in the form of the classical flutter equation

$$(\mathbf{M}+\varrho\mathbf{A})^{-1}\mathbf{K}\mathbf{q} = \lambda\mathbf{q},$$

that is in the form of an eigenvalue problem of an asymmetric singular (due to the rigid degree of freedom) matrix with complex elements. In this equation $\mathbf{M}$ is a diagonal matrix, the elements of which are the generalized masses corresponding to the natural modes of vibration (the last element, which corresponds to the rigid mode is equal to the total mass of the system), $\mathbf{K}$ is a matrix, the elements of which are equal to the squares of the natural frequencies (the last element being zero) and $\mathbf{A}$ is an aerodynamic matrix, the elements of which are given by Eq. (4.7). The eigenvalue

$$\lambda = \frac{\omega^2}{1+ig}$$

depends on the parameter g, which is interpreted as an overall damping coefficient.

The determination of the critical flutter speed consists in finding such values of the speed U and the frequency ω that, for a prescribed coefficient g, the value of $\omega^2/(1+ig)$ may be an eigenvalue and, in addition, the speed U may take the lowest value among those corresponding to particular eigenvalues. This is not feasible in a direct manner, the aerodynamic matrix $\mathbf{A}$ being a function of the frequency coefficient known only from numerical solution of a set of integral equations. As a consequence, the computation was repeated many times for different values of the frequency coefficient until the given value was obtained for the damping coefficient. The form of the harmonic motion at the critical speed can be found from the computet eigenvector $\mathbf{q}$.

To illustrate the influence of the aerodynamic interference on the critical flutter speed, examples of computation results will be presented for a system of airfoils with parameters having values as for a typical aircraft. The airfoils are of equal chords and equal mass parameters. The rigidity properties of the l-th airfoil are characterized by the frequencies $f_h^{(l)}$ and $f_\alpha^{(l)}$

$$f_h^{(l)} = \frac{1}{2\pi}\sqrt{\frac{K_l}{M_l}} \qquad (l = 1, 2),$$

$$f_\alpha^{(l)} = \frac{1}{2\pi}\sqrt{\frac{C_l}{J_l}} \qquad (l = 1, 2),$$

where K_l and C_l are the flexural and torsional rigidity of the l-th airfoil. If the static moments of both airfoils about the torsion axis were zero, these frequencies would be those of flexural and torsional vibration, respectively, of the l-th profile, under conditions of fixed mass M_3.

In the biplane configuration a reduction in the critical flutter speed has been found as a result of the aerodynamic interference (Fig. 8). The values of the ratio of the critical flutter speed V_s to the speed V_{0s} of a system of two isolated airfoils are measured along

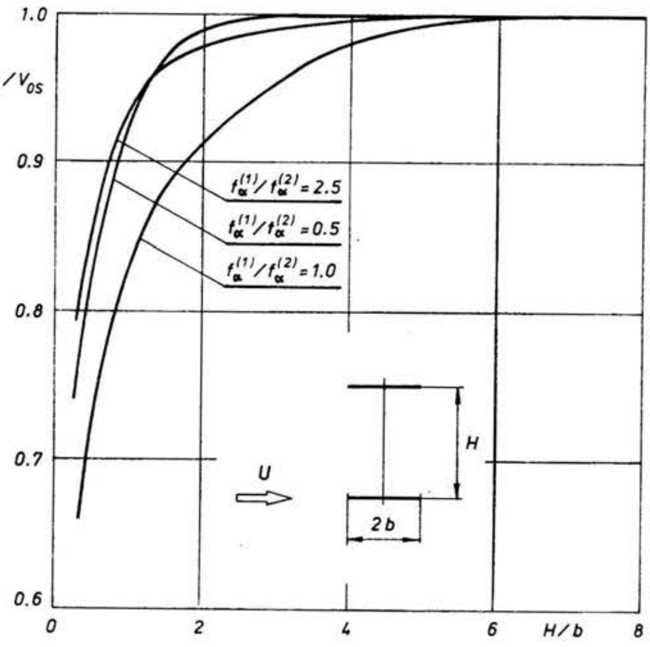
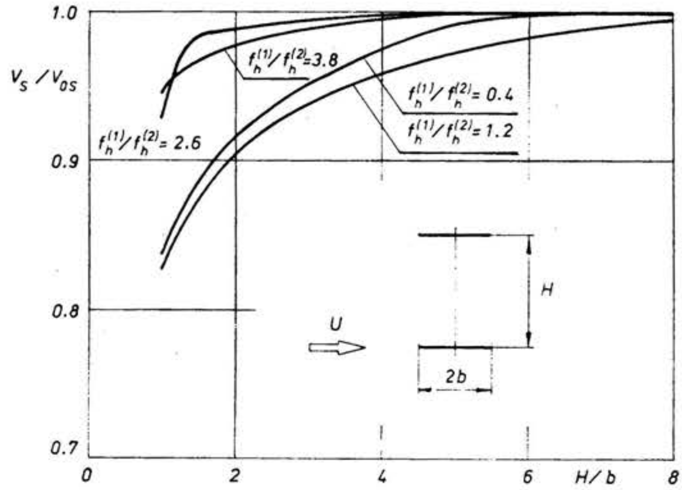


FIG. 8.

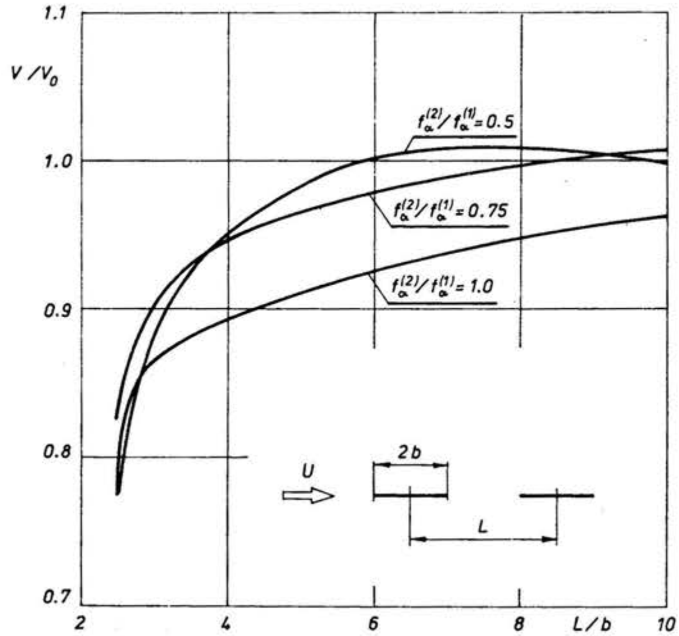
the axis of ordinates taking into account the interference. Each particular curve corresponds to a different value of the ratio of frequencies of torsional vibration, for $f_h^{(1)}/f_h^{(2)} = 1.4$. The influence of the aerodynamic interference in airfoil configurations of various ratios of flexural rigidities is analogous (Fig. 9).

The results of computation of the flutter speed for a system of airfoils in tandem configuration concern two variants: with a fixed mass M_3 (without the rigid degree of freedom) and with the rigid degree of freedom, the value of the mass M_3 being zero. The first variant corresponds to the case of symmetric vibration of the real structure. In the



1.0
0.9
0.8
0.7
V_S/V_{0S}
$f_h^{(1)}/f_h^{(2)}=3.8$
$f_h^{(1)}/f_h^{(2)}=0.4$
$f_h^{(1)}/f_h^{(2)}=1.2$
$f_h^{(1)}/f_h^{(2)}=2.6$
H
U
2b
0
2
4
6
H/b
8

FIG. 9.



1.1
1.0
0.9
0.8
0.7
V/V_0
$f_\alpha^{(2)}/f_\alpha^{(1)}=0.5$
$f_\alpha^{(2)}/f_\alpha^{(1)}=0.75$
$f_\alpha^{(2)}/f_\alpha^{(1)}=1.0$
2b
U
L
2
4
6
8
L/b
10

FIG. 10.

second variant the possibility of free displacement of the system of airfoils normal to the direction of unperturbed flow represents, for the plane model, the free rotatory motion of the airplane. For a fixed system of airfoils of the same flexural rigidity the aerodynamic interference results, over the range of distances tested, in a reduction in flutter speed with a clear distinction of the near zone of action (Figs. 10 and 11).

The axis of ordinates is that of the ratio of the flutter speed V, taking into consideration the aerodynamic interference, to the flutter speed V_0 for a system of two isolated profiles. The near zone of action increases with increasing flexibility of mounting of the other airfoil (Fig. 10), because then its deflections dominate in the flutter mode, which makes constant the level of the total aerodynamic forces of that action despite the fact that the aerodynamic coefficients of action on the first profile decrease with increasing distance.

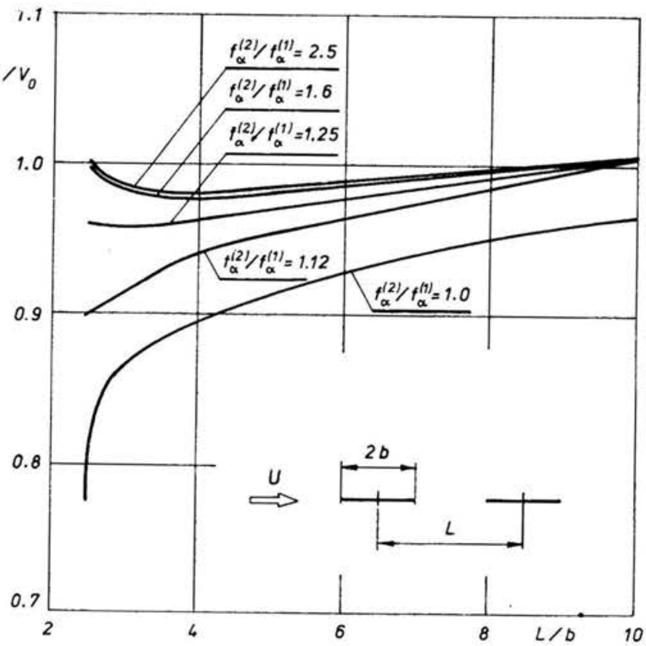


FIG. 11.

If the mounting of the first airfoil is more flexible (Fig. 11), its deflections dominate in the flutter mode. For this reason Fig. 11 mainly illustrates the influence of the action of the second airfoil on the first and, for higher values of the ratio $f_\alpha^{(2)}/f_\alpha^{(1)}$, the influence of the presence of the second profile, which can practically be treated as fixed. The aerodynamic interference reduces, here also, the critical flutter speed over the range of distances for which computation was performed. In addition, the form of the curves shows that the influence of the presence of the other airfoil is insignificant. If the system of airfoils is made free, there are some changes in its flutter properties, manifested chiefly in an increased ability to the influences of the aerodynamic interference. This is illustrated by Fig. 12, which represents the ratio of the flutter speed V_s of a free system to the flutter speed of the same system of isolated airfoils.

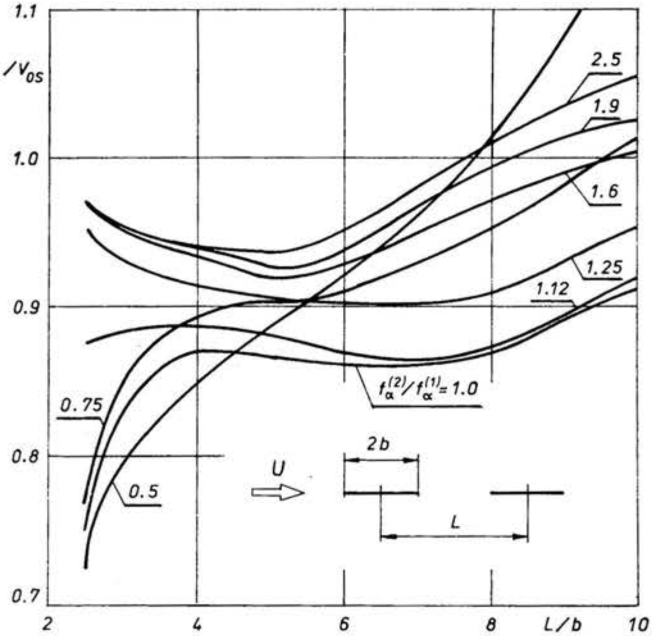


FIG. 12.

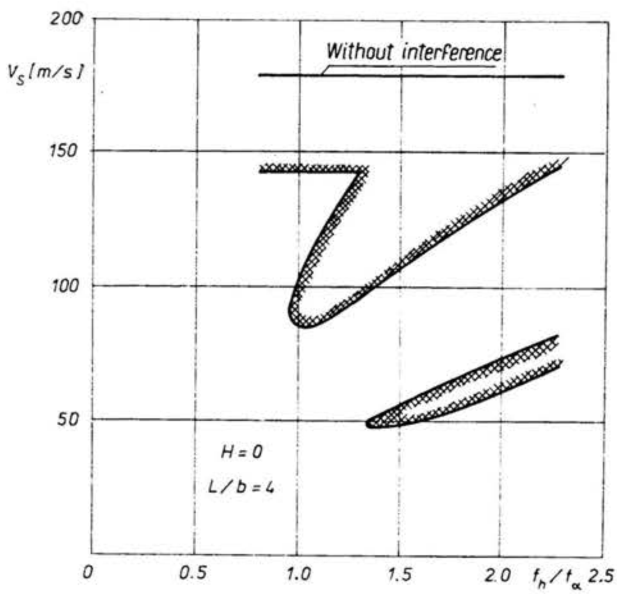


FIG. 13.

By contrast with the biplane configuration, which has a characteristic feature of monotonic decrease in the critical flutter speed with decreasing distance between the airfoils, there is no such feature for profiles arranged one behind the other. The influence of the aerodynamic interference may be different for different configurations of airfoils, depending on the values of geometrical, mass and rigidity parameters. In some cases even qualitative changes are observed, consisting in the occurrence of flutter where it did not occur if the aerodynamic interference was rejected. This effect is shown in Fig. 13 representing the dependence of the critical speed for a free system of airfoils on the ratio of the frequency of flexural vibration to that of torsional vibration. The system being free, there is a single flexural frequency, marked in the diagram by f_h. The frequencies of torsional vibration of the two airfoils are equal and denoted by f_α. The curves shown in the figure bound the stability regions (the regions of occurrence of flutter are those on the shaded side). It is seen that flutter occurs for low speeds due to the aerodynamic interference between the profiles. Additional computation showed that this flutter vanishes very rapidly with increasing distance H (Fig. 1) between the profiles normal to the direction flow. Such a flutter was observed during wind tunnel tests of an airplane model with variable sweepback [9].

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