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An analytical and numerical investigation of heat transport in superfluid helium(*)

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ONE PARTICULAR feature of superfluid helium is the propagation of temperature waves as an efficient mechanism of heat transport. Nevertheless, this phenomenon can be attenuated due to the formation of superfluid turbulence [1]. Thus the thermal behaviour seems to be similar to the classical heat diffusion, as in a normal fluid.

Szczególną własnością nadciekłego helu jest zdolność przenoszenia fal temperaturowych stanowiąca wydajny mechanizm transportu ciepła. Zjawisko to może być jednak tłumione dzięki tworzeniu się turbulencji [1]. W ten sposób przebieg zjawisk cieplnych wydaje się podobny do klasycznego procesu przepływu ciepła w zwykłym płynie.

Особенным свойством сверхтекучего гелия является способность переноса температурных волн, составляющая эффективный механизм теплопереноса. Это явление однако может затухать из-за образования турбулентности [1]. Таким образом ход тепловых явлений кажется быть аналогичным классическому процессу теплопереноса в обыкновенной жидкости.

Nomenclature

A_0 equivalent viscosity,
$A(T)$ Gorter–Mellink constant,
C_p specific heat,
$f, \psi, \bar{\bar{\tau}}, \Sigma$ dissipative terms,
K thermal conductivity,
P intrinsic momentum,
Q heat flux,
s entropy,
T temperature,
U internal energy,
V_n normal fluid velocity,
α mutual friction constant,
γ intrinsic velocity,
π pressure,
ϱ density,
ϱ_n normal density,
ϱ_s superfluid density.

THIS PAPER is concerned with the influence of propagation on heat transport efficienc y especially in the presence of a vapour-liquid transition. A numerical simulation is presented, based on the resolution of the energy and internal momentum conservation equations,

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according to the KHALATNIKOV's theory [2]. The dimensionless form of these coupled, nonlinear equations introduces three characteristic times related to the transport, propagation and diffusion of the temperature. The relative importance of the three modes of energy transport is discussed in accordance with the amplitude and duration of the energy to evacuate. The influence of the propagative properties is particularly important in the presence of a liquid-vapour transition.

1. Conservation equations and different processes of transport

1.1. Equations and model

The equations describing the thermal behaviour of helium are the conservation of internal momentum and of energy. Their general shape can be written as follows, in accordance with [3]:

$$\frac{\partial P}{\partial t}+\nabla(\bar{\bar{P}}V+\bar{\bar{P}}+\bar{\bar{\tau}})+P_J\nabla(V+\gamma)_J+S\nabla T-\varrho\nabla\psi-P\times(\nabla\times\mathbf{V}_s)=f, \tag{1.1}$$

$$\frac{\partial U}{\partial t}+\nabla[(U+p)\mathbf{V}+TS\boldsymbol{\gamma}+(\mathbf{V}+\boldsymbol{\gamma})(\boldsymbol{\gamma}\bar{\bar{P}}+\bar{\bar{\tau}})+\bar{\bar{\tau}}\bar{\bar{S}}-\varrho\psi\boldsymbol{\gamma}]=0. \tag{1.2}$$

The meaning of different terms have been discussed in [4]. For the numerical resolution, the following assumptions have been made: i) heat transport without net mass flow ($J = \varrho V = 0$), ii) one-dimensional heat transport. Thus the previous set of equations becomes

$$\frac{\partial T}{\partial t}=\frac{K}{\varrho C_p}\frac{\partial^2 T}{\partial x^2}-V_N\frac{\partial T}{\partial x}-\frac{s}{C_p}\frac{\partial V_N}{\partial x}T+\frac{\alpha}{\varrho C_p}V_N^2+\frac{A_0}{C_p}\left(\frac{\partial V_N}{\partial x}\right)^2, \tag{1.3}$$

$$\frac{\partial V_N}{\partial t}=A_0\frac{\varrho_s}{\varrho_N}\frac{\partial^2 V_N}{\partial x^2}-V_N\left[3\frac{\partial V_N}{\partial x}+A(T)\frac{\varrho^3}{\varrho_s^2}V_N^2\right]-s\frac{\varrho_s}{\varrho_N}\frac{\partial T}{\partial x}. \tag{1.4}$$

These equations are able to describe completely the temperature and normal fluid velocity component evolution in superfluid helium, when a temperature or heat flux perturbation is generated at the origin of the coordinates ($x = 0$ in our case). Note that the V_n space and time evolution is equivalent to the heat flux transport because of the relation $Q = \varrho s T V_n$ meaning that entropy is transported at the V_n velocity by the normal fluid component [5].

1.2. Dimensional analysis

The numerical treatment of Eqs. (1.3) and (1.4) is easier if dimensionless parameters are introduced:

$$\bar{x}=x/\lambda,\quad \bar{t}=t/\tau,\quad \bar{V}_n=V_n/v_0,\quad \bar{T}=(T-T_0)/\delta T.$$

These new variables lead to the following set of equations, equivalent to Eqs. (1.3) and (1.4)

$$\frac{\partial \bar{V}_N}{\partial \bar{t}}=\left(\frac{A_0\tau\varrho_s}{\varrho_N\lambda^2}\right)\frac{\partial^2\bar{V}_N}{\partial\bar{x}^2}-\left[3\left(\frac{V_0\tau}{\lambda}\right)\frac{\partial\bar{V}_N}{\partial\bar{x}}+\left(\frac{A\varrho^3\tau}{\varrho_s^2}V_0^2\right)\bar{V}_N^2\right]\bar{V}_N-\left(\frac{S_0\varrho_s\tau(\delta T)}{V_0\varrho_N\lambda}\right)\frac{\partial\bar{T}}{\partial\bar{x}}, \tag{1.5}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \left(\frac{K\tau}{\varrho C_p \lambda^2}\right)\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} - \left(\frac{V_0 \tau}{\lambda}\right)\bar{V}_N \frac{\partial \bar{T}}{\partial \bar{x}} - \left(\frac{\tau S_0 V_0}{\lambda C_p}\right)\bar{T}\frac{\partial \bar{V}_N}{\partial \bar{x}} + \left(\frac{A_\varrho^3 \varrho_N V_0^4 \tau}{\varrho_s^3 C_p(\delta T)}\right)\bar{V}_N^2 + \left(\frac{A_0 \tau \bar{V}_0^2}{C_p \lambda^2 (\delta T)}\right)\left(\frac{\partial \bar{V}_N}{\partial \bar{x}}\right)^2. \tag{1.6}$$

Taking into account the very small values of K (heat conductivity) and A_0 (equivalent viscosity), the associated terms can be neglected, what has been numerically verified [4]. Thus Eqs. (1.5) and (1.6) can be written in the following form

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \frac{\tau}{\tau_t}\bar{V}_n \frac{\partial \bar{T}}{\partial \bar{x}} = -\frac{\tau}{\tau_p}\frac{\partial \bar{V}_N}{\partial \bar{x}}\bar{T} + \frac{\tau^*}{\tau_d}\bar{V}_N^4, \tag{1.7}$$

$$\frac{\partial \bar{V}_N}{\partial \bar{t}} + 3\frac{\tau}{\tau_t}\bar{V}_N \frac{\partial \bar{V}_N}{\partial \bar{x}} = -\frac{\tau}{\tau_p}\frac{\partial \bar{T}}{\partial \bar{x}} - \frac{\tau}{\tau_d}\bar{V}_n^3 \tag{1.8}$$

with

$$\tau_t = \lambda / V_0,$$

$$\tau_p = \lambda C_p / S_0 V_0 = \lambda \varrho_N V_0 / S_0 \varrho_s \delta T,$$

$$\tau_d = \varrho_s^3 C_p \delta T / A \varrho^3 \varrho_N V_0^4 = \varrho_s^2 / A \varrho^3 V_0^2.$$

This writing shows the existence of three distinct but coupled heat transport mechanisms that could be called: i) transport of T at V_n ("convection"), ii) propagation of T, iii) diffusion of T, according to VINEN [6] at GENTILE [7]. This mathematical presentation is formal and allows for a numerical separation of the different modes; if the duration τ of the heat flux disturbance is smaller than τ_d (and τ_t), the main process of transport will be propagative. On the contrary, τ being larger than τ_d, the main process will be diffusive.

As it has been demonstrated in [4], since τ_d is strongly dependent of V_0, it is numerically equivalent to maintain τ constant and change the value of the perturbation Q. In particular, τ_d varies with Q^{-2}. At τ constant and according to the value of Q, τ_d can be smaller or greater than τ. The diffusive character can be more or less pronounced, depending on the value of the heat flux at $x = 0$ on the heater.

2. Numerical solution

The physical conclusions have been demonstrated through a numerical solution of the coupled equations (1.5) and (1.6). The numerical algorithm has been explained in [4]. As to the boundary conditions, a heat flux perturbation Q, of duration τ, has been applied at $x = 0$. Q is converted into a condition on T and $V_n \cdot T_{(x=0)}$ is determined from an energy balance on a small slice of fluid (between the first two nodes) and $V_{n(x=0)}$ from the relation $Q/\varrho s T_{(x=0)}$. At the free extremity, $x = L$, the temperature is maintained constant and equal to the initial temperature T_0, and the velocity gradient $(dV_n/dx)_{(x=L)}$ is equal to zero.

Three values of Q have been assumed, for a duration of $\tau = 3$ mS, corresponding to Figs. 1a, 1b, and 1c.

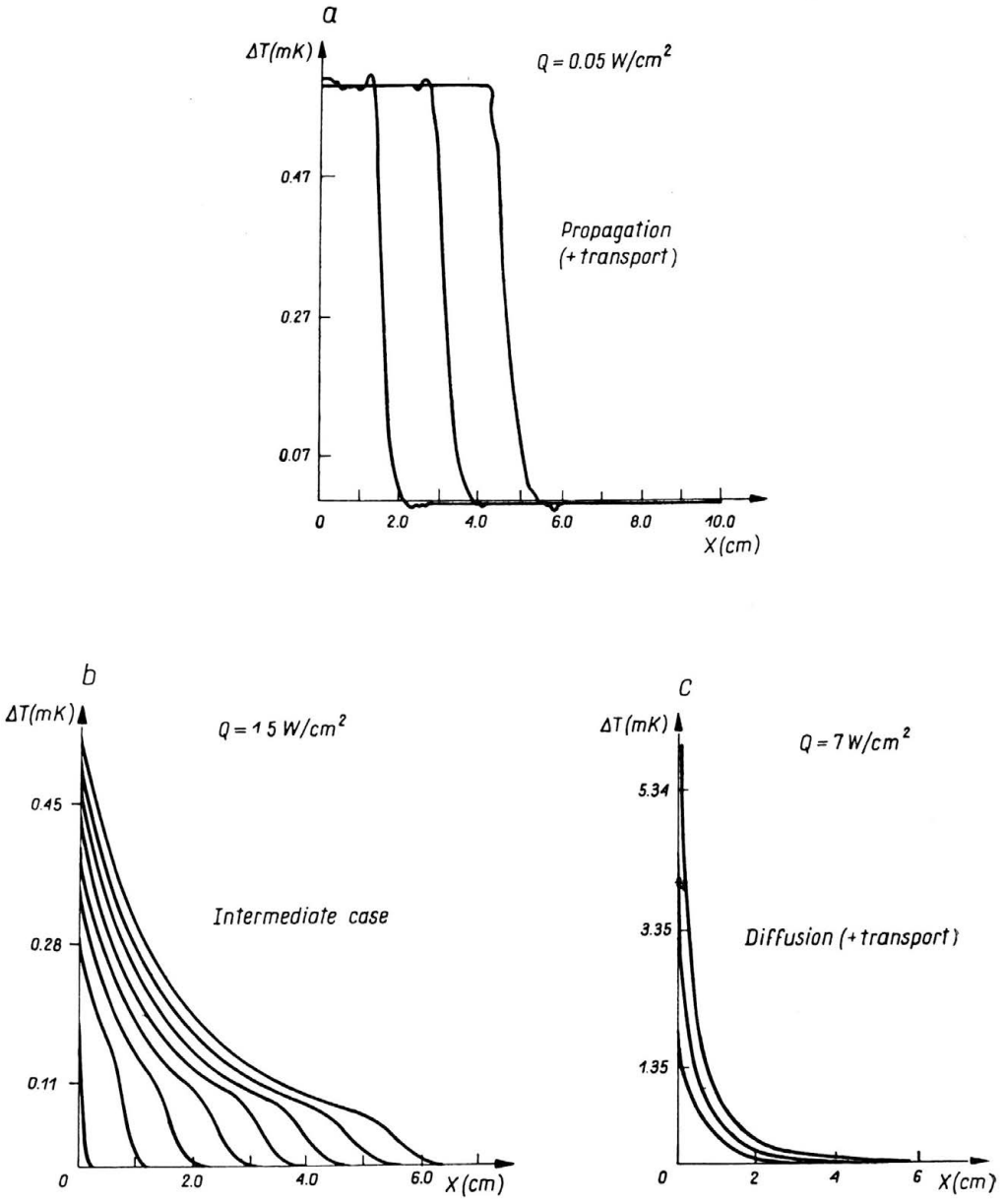

FIG. 1.

i) $Q = 0.05$ W/cm²,
ii) $Q = 1.5$ W/cm²,
iii) $Q = 7$ W/cm².

According to the wave concept in the theory of heat, developed by V. A. BUBNOV [8] and based on the analysis of temperature field isotherms, calculation of the "diffusion velocity" g_T has been done for the intermediate case which corresponds to the velocity

of an isothermal line ΔT = const, progressing through the fluid, where $\Delta T = T(x, t) - T_0$. g_T is defined as

$$g_T = -(d\Delta T/dt)_x/(d\Delta T/dx)_t.$$

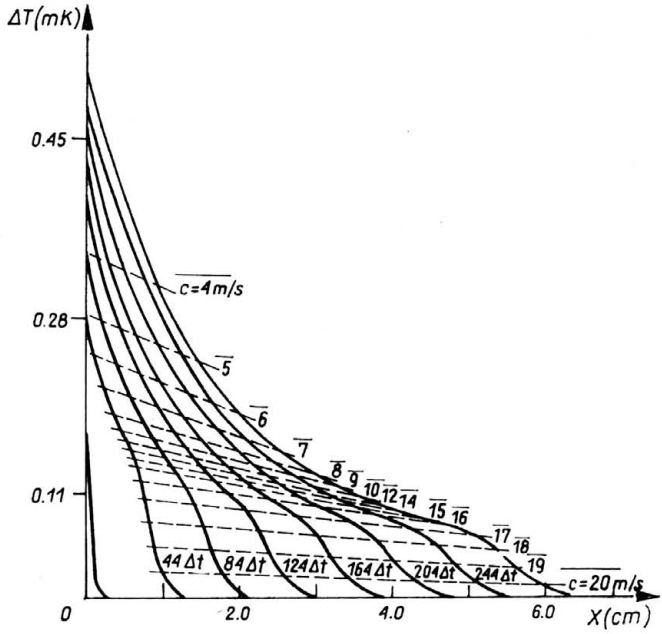

FIG. 2.

Figure 2 gives the values of g for different ΔT. The dashed lines correspond in fact to different velocities, every line indicates the points of temperature which move at constant speed. It is remarkable that in the intermediate case the points of the temperature profile move at velocities between small values (< 4 m/S) up to the second sound velocity (20 m/S at the equilibrium temperature 1.8 K). A temperature front propagates at low values of ΔT, and even some points of very low temperature move at velocities greater than the second sound one. The coupling between the different modes can certainly explain this point [9].

3. Simulation of film boiling of helium II

Numerical simulation was extended to the case of a liquid-vapour transition. Before describing the numerical algorithm and discussing the obtained results, it seems necessary to review briefly the main conclusions of film boiling of helium II that one can find in the literature.

3.1. A review of He II film boiling

As mentioned by LABUNTSOV [10], despite the increased interest in heat transfer abilities to liquid helium, analytical studies of heat exchange with He II in presence of a phase

change lag behind experimental studies. Only one analytical investigation can be cited [11] which postulates the existence of a special heat flux Q_b removed from the boundary of the vapour-liquid interface in the volume of helium. From this heat flux, the existence of which has been postulated also in [12] and correlated to experimental data of IREY and al. [13, 14, 15], LABUNTSOV and al. [16, 17, 18] have made some analytical attempts to explain the process of film boiling in He II. Their analysis is based on the non-equilibrium effects taking place on the interface and, through the kinetic theory equations, they could give an interpretation of this heat flux Q_b. In particular, it appears that in certain conditions the vapour film boiling of He II remains invariable in dimension in time, and vapour does not accumulate in the film. The film thickness is automatically established for a particular value of heat flux, given by the relation between dimensionless fluxes of mass, of heat and momentum, directly deduced from the kinetic theory. If the heat flux is different from this value, either vaporisation or condensation takes place [10]. Moreover, this analysis shows that a cooling effect occurs with a supply of heat from the vapour to the interface. This point has been observed experimentally by one of the authors [19]. Our purpose is not to discuss in detail these results but to show that it is possible to find them by solving numerically Eqs. (1.5) and (1.6) modified by the presence of phase-change.

3.2. Numerical model

It has been accurately described in a recent paper [20]. We only recall here the main assumptions and invite the reader to [20] for numerical details.

ASSUMPTIONS. i) one-dimensional model without movement of liquid, ii) conductive heat transfer through the vapour layer (the Rayleigh number corresponding to our numerical examples being smaller than the critical Rayleigh number, it is conceivable to neglect any convective effects) iii) coupling of heat conduction equation in the vapour with Eqs. (1.3) and (1.4) through typical boundary conditions on the interface [20].

If $T(0, t)$ reaches the saturation temperature, T_{sat} is imposed on the liquid-vapour interface and the necessary amount of heat quantity is calculated to assure this temperature condition. This heat flux — called critical heat flux — corresponds to the heat flux Q_b of the kinetic theory. The process of calculation is iterative [20]. It must be noted that the start of film boiling corresponds to a change of the boundary conditions from the Neuman type (power Q imposed at $x = 0$ if $T(0, t) < T_{sat}$) to the Dirichlet type (temperature $T(0, t)$ imposed equal to T_{sat} and critical heat flux calculated).

An energy balance corresponding to a comparison between the imposed heat flux Q and the calculated critical flux Q_b allows to conclude to either vaporisation or condensation, or equilibrium of the thickness of the film. The difference between this approach and the kinetic approach is the introduction of heat vaporisation (latent heat) in our model and the lack of this notion in the kinetic model.

3.3. Results and discussion

Figures 3a, 3b and 3c show the time evolution of the gas film thickness for the following cases:

3a — main process: propagation (no diffusion),

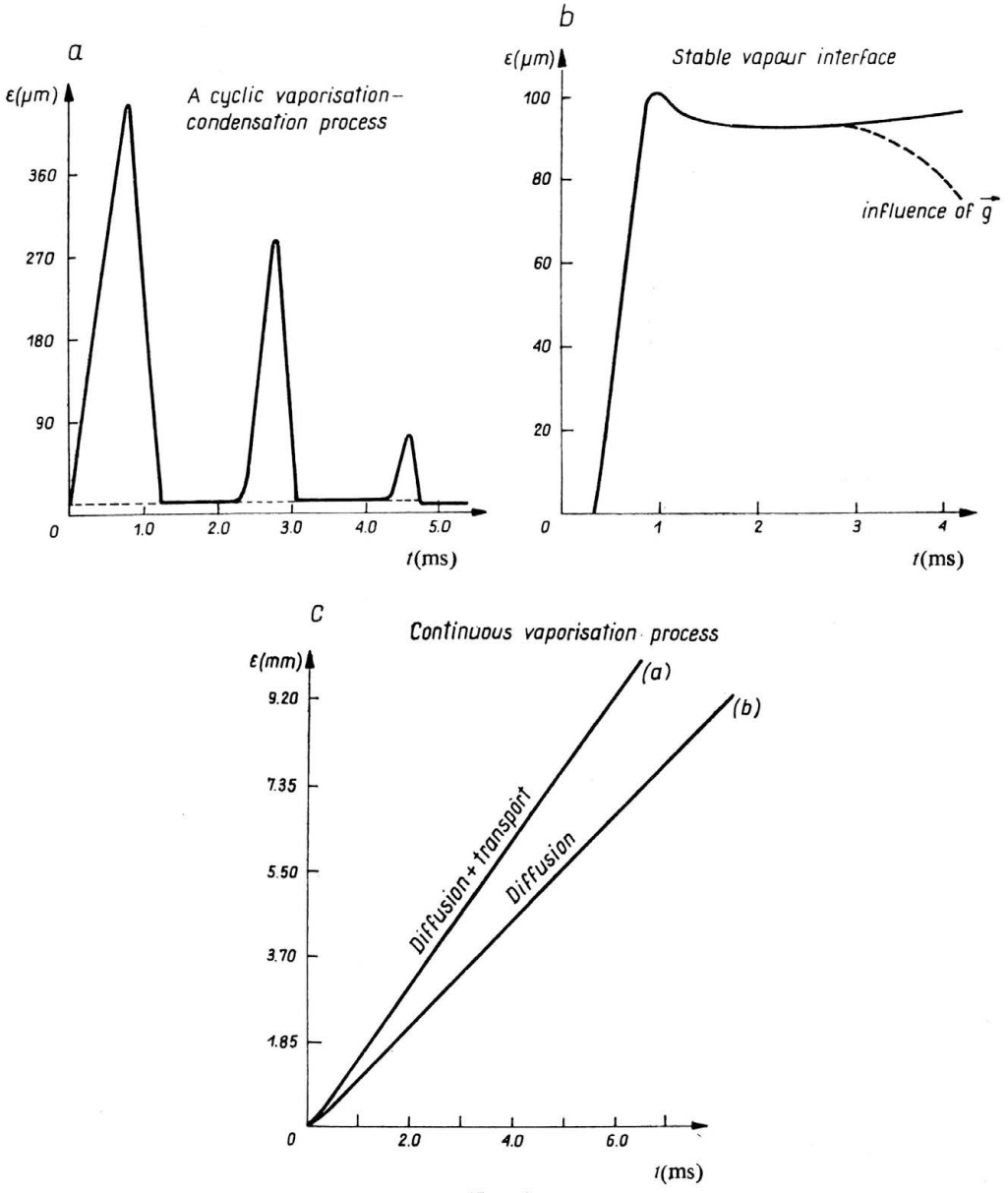

FIG. 3.

3b — coupling between the three modes,

3c — main process: diffusion or diffusion + transport.

It is obvious that the behaviour of the interface and consequently, the mechanisms of phase change, is quite different in presence or absence of propagating temperature waves. Particularly interesting to note is:

i) the possible existence of film thickness oscillations (leading to vaporisation and recondensation processes) which are attenuated after a few milliseconds;

ii) the possible existence of a stable interface or quasi-stable because a small disturbance of Q or (equivalent of pressure) leads to either a vaporisation or recondensation process (Fig. 4);

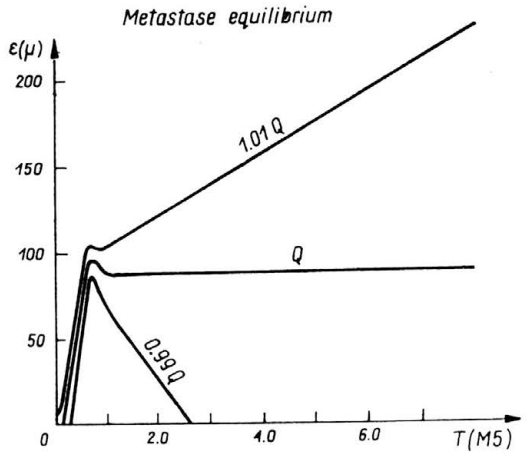

FIG. 4.

iii) a continuous process of vaporisation if the diffusion mechanism is the main transport process.

Our model is simplified because only thermal considerations at the gas-liquid interface have been taken into account. In particular, the gravity effect has been neglected.

The problem will not be so simple if, experimentally for example, we use a surface immersed in helium and heated from below. The influence of gravity will contribute to the disappearance of the stable interface, as indicated by the dashed line in Fig. 3b. A simple calculation can be made to compare thermal and gravity effects in a geometry where the liquid would be above the vapour. The liquid will fall and lead to dynamic instabilities on the interface in an approximate time $\tau_g = \sqrt{\varepsilon/g}$, where ε is the thickness of the film. For the case of Fig. 3b, $\varepsilon \approx 100$ μ leads to a falling-time of 3 mS. In the case of Fig. 3c, gravity effects will not influence evaporation before a few tenth of milliseconds ($\tau_g = 30$ mS for $\varepsilon = 9$ mm).

Under these previous assumptions, the numerical results are easily interpreted through the temperature profiles and different heat fluxes time evolution [20].

4. Conclusion

In this work, the thermohydrodynamic equations of superfluid helium have been presented in dimensionless forms. This allows to separate easily the different heat transport modes by this fluid. Through a numerical simulation, the influence of each mode has been demonstrated in the case of a single liquid and liquid-vapour transition (film). In particular, it seems that the results cited in the literature must be connected to the propagative character of this fluid. Moreover, numerical simulation makes it possible

to do certain assumptions and allows to follow the dynamics of very fast phenomena which, it is not evident, can be assured experimentally. Nevertheless, the confirmation of the main conclusions now imply future experimental measurements.

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