Archives of Mechanics • Archiwum Mechaniki Stosowanej • 29, 2, pp. 223–227, Warszawa 1977

Steady motion of thin profile near interface of two heavy fluids

V. V. GOLOVCHENKO and D. N. GORELOV (NOVOSIBIRSK)

A NON-LINEAR problem of the motion of a thin profile at a constant velocity near the interface of two heavy fluids having different densities has been considered. The problem is reduced to a system of non-linear equations. The solution of the latter determines the shape of the interface, pressure distribution along this interface and hydrodynamic reactions on the profile. Some results of calculations for the air-water media are given.

Rozważono nieliniowe zagadnienie cienkiego profilu poruszającego się ze stałą prędkością w pobliżu powierzchni rozdziału dwóch płynów ciężkich o różnych gęstościach. Problem sprowadzono do układu równań nieliniowych, którego rozwiązanie określa kształt powierzchni rozdziału, rozkład ciśnień wzdłuż tej powierzchni i reakcje hydrodynamiczne występujące na profilu. Podano pewne wyniki obliczeń dotyczące ośrodków wodno-powietrznych.

Рассмотрена нелинейная задача тонкого профиля движущегося с постоянной скоростью вблизи поверхности раздела двух тяжелых жидкостей с разными плотностями. Задача сводится к системе нелинейных уравнений, решение которой определяет форму поверхности раздела, распределение давлений вдоль этой поверхности и гидродинамические реакции выступающие на профиле. Приведены некоторые результаты вычислений, касающиеся водно-воздушных сред.

THE PROBLEM of wing motion near an interface of two fluids has been considered in a great number of papers in which the flow was assumed to be two-dimensional and fluid interface to be free surface, the pressure being constant. Most problems were solved in terms of the linear theory of small disturbances; the non-linear theory was only used for problems of motion of the vortex with a given intensity (e.g., see [1, 2, 3]).

The present paper gives a solution to the non-linear problem of the thin profile steady motion near the interface of two heavy fluids in a more general statement, without assuming that on this interface the pressure is constant. The problem is reduced to the system of non-linear equations. The solution of this system determines the shape of the interface, the pressure distribution the realong and the hydrodynamic reactions on the profile. Some results of calculations for an air-water medium are also given.

1. Let us consider the motion of the profile at constant velocity V_∞ near the interface of two fluids having different densities, ϱ_1 and ϱ_2. The general assumptions are:

(a) Fluids are heavy, perfect and incompressible.

(b) The flow is steady, two-dimensional and potential.

(c) The profile is a plate of negligible thickness.

The axis of the fixed coordinates Ox and Oy is taken as shown in Fig. 1. The perfect-fluid equations in regions D_1 and D_2 (see Fig. 1) are

$$\mathbf{v}_k \nabla \mathbf{v}_k = -\frac{1}{\varrho_k}\nabla p - \mathbf{g}, \tag{1}$$

$$\operatorname{div}\mathbf{v}_k = 0, \quad \operatorname{rot}\mathbf{v}_k = 0, \qquad k = 1, 2, \tag{2}$$

where $\mathbf{v}_k(v_{kx}, v_{ky}), \mathbf{g}(0, -g)$ are the velocity and acceleration of the gravity vectors, respectively; p is the hydrodynamic pressure.

Equations (1) and (2) must be solved under corresponding boundary conditions which will be formulated below.

From Eq. (1) we have Bernouli's equation:

$$p/\varrho_k+\frac{1}{2}(\mathbf{v}_k\cdot\mathbf{v}_k)+gy = C_k, \quad C_k = \text{const}, \quad k = 1,2. \tag{3}$$

Equations (2) are Cauchy-Riemann's conditions for the function

$$\overline{v}_k(z) = v_{kx}-iv_{ky}, \quad z = x+iy. \tag{4}$$

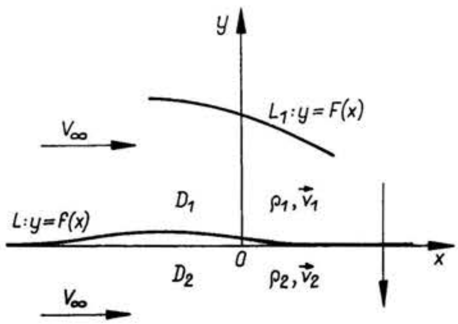

FIG. 1.

Accordingly, we will find the analytical functions

$$\overline{v}(z) = \begin{cases} v_1(z) & \text{at} \quad z\in D_1, \\ v_2(z) & \text{at} \quad z\in D_2 \end{cases} \tag{5}$$

which satisfy the following boundary conditions.

The normal velocities at the profile and the interface are zero

$$\mathrm{Im}\{\overline{v}(\zeta_1)[1+iF'(x)]\} = 0 \quad \text{at} \quad \zeta_1\in L_1, \tag{6}$$

$$\mathrm{Im}\{\overline{v}(\zeta_2)[1+if'(x)]\} = 0 \quad \text{at} \quad \zeta_2\in L. \tag{7}$$

The pressure is continuous across the interface

$$\lim_{z_1\to\zeta} p(z_1) = \lim_{z_2\to\zeta} p(z_2) \quad \text{at} \quad \zeta\in L, \quad z_k\in D_k, \quad k = 1,2. \tag{8}$$

The conditions at infinity are

$$\lim_{|z|\to\infty} \overline{v}(z) = V_\infty, \quad \lim_{x\to-\infty} f(x) = 0. \tag{9}$$

Kutta-Joukovski's condition at the trailing edge point is

$$\lim_{\zeta_1\to z_*} |\overline{v}(\zeta_1)| < \infty \quad \text{at} \quad \zeta_1\in L_1. \tag{10}$$

This problem contains 3 unknown functions: two components v_x, v_y of the complex-conjugate velocity $\overline{v} = v_x-iv_y$ and the function determining the interface.

2. The analytical function $\bar{v}(z)$ may be represented by the Cauchy's integral

$$\bar{v}(z) = V_\infty + \frac{1}{2\pi i}\int_{L_1} \frac{\gamma_1(\zeta_1)d\zeta_1}{\zeta_1 - z} + \frac{1}{2\pi i}\int_{L} \frac{\gamma(\zeta)d\zeta}{\zeta - z}, \tag{11}$$

where $\gamma_1(\zeta_1)$, $\gamma(\zeta)$ are new unknown functions.

Substituting Eq. (11) into Eqs. (6) and (7) we have two equations with respect to the functions γ_1, γ and f. To obtain the third equation let us consider condition (8). Taking into account Bernoulli's integrals (3) this condition can be written in the form

$$[2gf(x) - V_\infty^2](\varrho_1 - \varrho_2) = \varrho_2 \lim_{z_2\to\zeta} \bar{v}(z_2)v(z_2) - \varrho_1 \lim_{z_1\to\zeta} \bar{v}(z_1)v(z_1) \tag{12}$$

$$\text{at} \quad \zeta \in L, \quad z_k \in D_k, \quad k = 1, 2.$$

According to Condition (7) and Plemeli-Sokhotski's formulas [4] we can write

$$\begin{aligned} \lim_{z_1\to\zeta} \bar{v}(z_1)v(z_1) &= \left[V_0(\zeta) - \frac{1}{2}\gamma(\zeta)\right]^2, \\ \lim_{z_2\to\zeta} \bar{v}(z_2)v(z_2) &= \left[V_0(\zeta) + \frac{1}{2}\gamma(\zeta)\right]^2, \end{aligned} \tag{13}$$

where

$$V_0(\zeta) = \mathrm{Re}\left\{\bar{v}_0(\zeta)\frac{1 + if'(x)}{\sqrt{1 + (f'(x))^2}}\right\} \tag{14}$$

and the function $\bar{v}_0(\zeta)$ is determined by Formula (11) at $z \in L$ (the second integral is considered here in the Cauchy sense). Substituting Eqs. (13) into Eq. (12) we obtain the following third equation for the unknown functions:

$$f(x) = \frac{1}{2g}\left[V_\infty^2 - V_0^2(\zeta) - \frac{1}{\varrho_*}\gamma(\zeta)V_0(\zeta) - \frac{1}{4}\gamma^2(\zeta)\right], \tag{15}$$

where

$$\varrho_* = \frac{\varrho_2 - \varrho_1}{\varrho_2 + \varrho_1}. \tag{16}$$

3. The system of equations (6), (7) and (15) is non-linear. It was solved numerically using the method of successive approximation. As a result, the functions γ_1, γ and f are determined for the given parameters ϱ_*, b/h (b is the profile chord, h is the distance from the middle of the chord to the undisturbed interface) and the Froude number $F_r = V_\infty^2/(gb)$. Knowing these functions, one can determine the total hydrodynamic force on the profile $Y = 1/2\varrho_1 V_\infty^2 bC_{y\alpha}\alpha$, where α is the angle of attack.

To take an example, the calculations are suggested for a plate moving in air near a water surface. The results are given in Figs. 2–4.

The Froude number effect on the lifting force coefficient $C_{y\alpha}$ at $\alpha = 0.1$ for different distances from the undisturbed interface is represented in Figs. 3, 4. The sharp change of the force coefficient is observed for narrow regions in the vicinity of the Froude numbers $F_r = 1$ and $F_r = 1.5$. This effect is associated with the interface shape instability for these

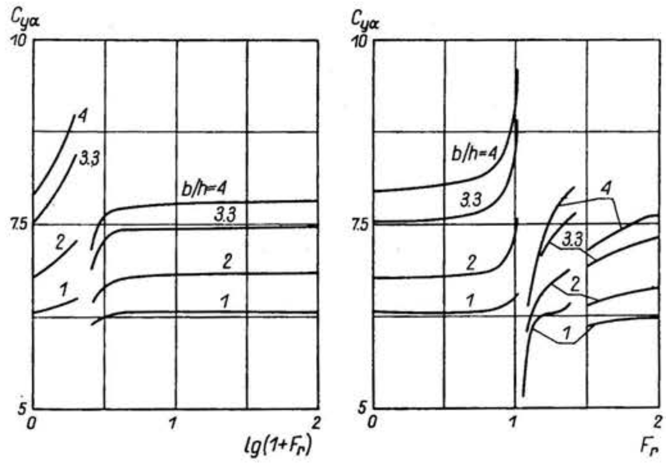

Fig. 2.

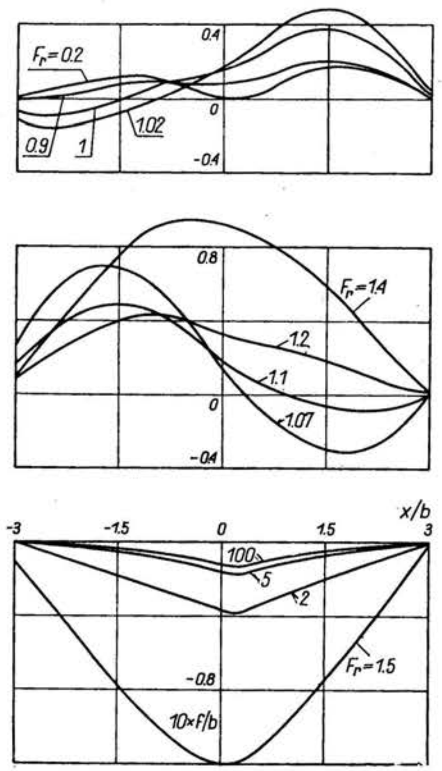

Fig. 3.

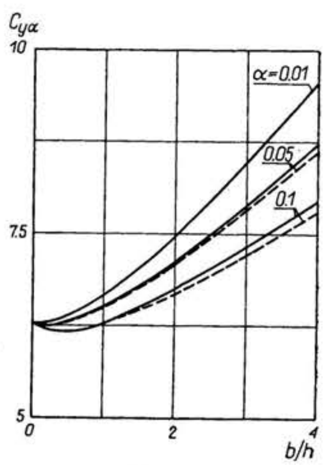

Fig. 4.

Froude numbers (see Fig. 3). At the further increase of the Froude number the coefficient $C_{y\alpha}$ slightly depends on F_r. Moreover, at $F_r > 10$ which are of practical interest the coefficient $C_{y\alpha}$ is close to the corresponding value at $F_r = 0$ (this value corresponds to the profile motion near a solid plane). Figure 4 which shows the dependence of the coefficient $C_{y\alpha}$ on the b/h parameter for different values of the angle of attack α, illustrates it rather well. The solid line corresponds to $F_r = 0$, and the dashed line to $F_r = 100$. The given results show sufficient non-linear dependence of the profile lifting force on the angle of attack.

References

1. Н. Е. Кочин, *О волновом сопротивлении погруженных в жидкость тел*, Собрание сочинений, т. II. АН СССР, 1949.
2. А. И. Некрасов, *О точечном вихре под поверхностью тяжёлой идеальной жидкости в плоско-параллельном потоке*, Собрание сочинений, т. II. АН СССР, 1962.
3. А. М. Тер-Крикоров, *Точное решение задачи о движений вихря под поверхностью жидкости*, Изв. АН СССР. Сер. матем., **22**, 2, 1958.
4. L. M. Milne-Thomson, *Theoretical hydrodynamics*, IV ed, London 1960.

Received November 24, 1975.