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Waves and vibrations in micropolar elastic medium
I. Steady-state response to moving loads

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THE PRESENT paper is concerned with the steady-state response to moving loads on the free plane boundary of a semi-infinite micropolar elastic medium. It is assumed that a line load moves with a constant velocity over the plane boundary of the semi-space and moves for an infinitely long time so that a steady state prevails in the neighbourhood of the loading. The dynamic deformation is characterized by two asymmetric tensors — the deformation tensor and curvature twist tensor. Similarly, the state of stress in the body is characterized by two asymmetric tensors — the force-stress tensor and the couple-stress tensor.

Niniejsza praca jest związana z otrzymaniem rozwiązania w formie stanu ustalonego (steady-state response) przy danych na swobodnym płaskim brzegu półnieskończonego mikropolarnego ośrodka ruchomych obciążeniach. Przyjęto, że obciążenie liniowe porusza się ze stałą prędkością na płaskim brzegu półprzestrzeni falowej i trwa nieskończenie długo tak, że stan ustalony przeważa w otoczeniu obciążenia. Deformacja dynamiczna scharakteryzowana jest przez dwa tensory osiowo-symetryczne — tensor odkształcenia i skośny tensor krzywizny. Podobnie stan naprężenia w ciele określony jest przez dwa antysymetryczne tensory — tensor naprężenia i tensor naprężenia momentowego.

Настоящая работа связана с получением решения в форме установившегося состояния (steady-state response) при заданных, на свободной плоской границе полубесконечной микрополярной среды, подвижных нагрузках. Принимается, что линейная нагрузка движется с постоянной скоростью на плоской границе волнового полупространства и продолжается бесконечно долго, так что установившееся состояние преобладает в окрестности нагрузки. Динамическая деформация характеризуется двумя осесимметричными тензорами — тензором деформации и косым тензором кривизны. Аналогично напряженное состояние в теле определяется двумя антисимметричными тензорами — тензором напряжения и тензором моментных напряжений.

1. Introduction

IN THE CLASSICAL theory of elasticity the notion of stress is that of balancing internal action and reaction between two parts of a body separated by means of hypothetical plane. It is assumed that the action across an infinitesimal surface element within the solid is equivalent to a force only. It is expected that the elementary forces across a hypothetical plane within the solid should be statically equivalent to a force and a couple. If, following Voigt, we assume that across any infinitesimal element in a solid the action of one part of the material upon the other part is equipollent to a force and a couple, then, in addition to the force-stress vector acting on the surface we must also have a couple-stress vector. These two vectors together are now equipollent to the action of the exterior upon the interior into which the body is divided by hypothetical plane. In the like manner, one might have body-couples similar to body forces as pointed out by Maxwell. If we accept these possibilities, then we must define a couple-stress tensor, μ_{ji}, in addition to the force-stress tensor σ_{ji}. Couple-stresses and body couples are useful concepts in the case of materials with molecules of internal structure and in the dislocation theory of metals.

Recently, some problems of propagation of waves and vibrations in micropolar elastic solid medium have been investigated by NOWACKI [1–5] and CHADWICK [9]. ERINGEN [6, 7] has given a detailed exposition of the theory of micropolar elasticity consisting largely of results obtained by himself and his co-workers. In the present paper the authors consider the steady-state response to moving loads on the free boundary of a semi-infinite micropolar elastic solid medium. In subsequent papers the authors undertake a programme of investigating some basic problems of waves and vibrations in micropolar elastic solids.

2. General theory and boundary conditions

Introducing a set of orthogonal Cartesian coordinate axes $0x_1x_2x_3$, the origin being a point on the plane boundary of the micropolar elastic semi-space, let x_1, x_2, x_3 be the Cartesian coordinates fixed in the medium which occupies the half space $x_2 \geqslant 0$ as shown in the adjoining diagram. Let us assume that a line load moves with a constant speed U over the half space and a plane strain state prevails. Let us assume further that the load has been applied and is moving for such an infinitely long time that a steady state prevails in the neighbourhood of the loading, as seen by an observer moving with the load. Under the action of external loadings the body will be deformed; in general, the field of displace-

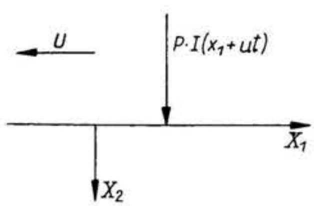

FIG. 1. A moving load over a micropolar elastic half-space.

ment u_i and the rotation field ω_i depend on the space coordinates x_1, x_2, x_3 and time t. The strain state is determined by two-non-symmetrical tensors — the deformation tensor γ_{ji} and curvature twist tensor $\varkappa_{ji}$. These tensors are defined as follows [6, 7, 8]:

(2.1) $$\gamma_{ji} = u_{i,j} - \varepsilon_{kji}\omega_k, \qquad \varkappa_{ji} = \omega_{i,j}.$$

Similarly, the state of stress is defined by two non-symmetrical tensors — the force stress tensor σ_{ji} and the couple-stress tensor μ_{ji}. The linear relations between the stress and strain states are expressed as

(2.2) $$\begin{aligned} \sigma_{ji} &= (\mu+\alpha)\gamma_{ji} + (\mu-\alpha)\gamma_{ij} + \lambda\gamma_{kk}\delta_{ij}, \\ \mu_{ji} &= (\gamma+\varepsilon)\varkappa_{ji} + (\gamma-\varepsilon)\varkappa_{ij} + \beta\gamma_{kk}\delta_{ij}, \end{aligned}$$

where λ, μ, β, ε, γ, α are the material constants, and ε_{kji} is the alternate tensor.

Substituting the relations (2.2) into the equations of motion

(2.3) $$\sigma_{ji,j} + X_i = \varrho\ddot{u}_i, \qquad \varepsilon_{ijk}\delta_{jk} + \mu_{ji,j} + Y_i = J\ddot{\omega}_i$$

and expressing γ_{ji} and $\varkappa_{ji}$ in terms of the displacements u_i and rotations ω_i as determined from Eq. (2.1), Eqs. (2.3) can be written in the vector form in the following way:

$$(2.4)\qquad \begin{aligned}&(\mu+\alpha)\nabla^2\mathbf{u}+(\lambda+\mu-\alpha)\,\mathrm{grad\,div}\,\mathbf{u}+2\alpha\,\mathrm{rot}\boldsymbol{\omega}+\mathbf{X} = \varrho\ddot{\mathbf{u}},\\ &(\gamma+\varepsilon)\nabla^2\boldsymbol{\omega}+(\beta+\gamma-\varepsilon)\,\mathrm{grad\,div}\boldsymbol{\omega}-4\alpha\boldsymbol{\omega}+2\alpha\,\mathrm{rot}\,\mathbf{u}+\mathbf{Y} = J\ddot{\boldsymbol{\omega}},\end{aligned}$$

where $\mathbf{X}$—vector of body forces, $\mathbf{Y}$—vector of body couples, ϱ—density, J—rotational inertia.

Equations (2.4) are coupled equations and can be decoupled by assuming $\alpha = 0$ and thus, one obtains

$$(2.5)\qquad \begin{aligned}&\mu\nabla^2 u+(\lambda+\mu)\,\mathrm{grad\,div}\,\mathbf{u}+\mathbf{x} = \varrho\ddot{\mathbf{u}},\\ &(\gamma+\varepsilon)\nabla^2\boldsymbol{\omega}+(\gamma+\beta-\varepsilon)\,\mathrm{grad\,div}\boldsymbol{\omega}+\mathbf{y} = J\ddot{\boldsymbol{\omega}}.\end{aligned}$$

The first equation of (2.5) is the equation of motion in the classical theory of elasticity and the second represents the motion of a hypothetical medium in which rotations only are possible.

In the present plane-strain problem the external loadings, the displacement vector $\mathbf{u}$ and the rotation vector $\boldsymbol{\omega}$ depend only on the coordinates x_1, x_2 and time t; the body couples and body forces are neglected. In this case the system of Eqs. (2.4) splits into two independent systems of equations:

$$(2.6)\qquad \begin{aligned}&(\mu+\alpha)\nabla^2 u_1+(\lambda+\mu-\alpha)e_{,1}+2\alpha\omega_{3,2} = \varrho\ddot{u}_1,\\ &(\mu+\alpha)\nabla^2 u_2+(\lambda+\mu-\alpha)e_{,2}-2\alpha\omega_{3,1} = \varrho\ddot{u}_2,\\ &(\gamma+\varepsilon)\nabla^2\omega_3-4\alpha\omega_3+2\alpha(u_{2,1}-u_{1,2}) = J\ddot{\omega}_2,\end{aligned}$$

and

$$(2.7)\qquad \begin{aligned}&(\gamma+\varepsilon)\nabla^2\omega_1+(\gamma+\beta-\varepsilon)\varkappa_{,1}-4\alpha\omega_1+2\alpha u_{3,2} = J\ddot{\omega}_1,\\ &(\gamma+\varepsilon)\nabla^2\omega_2+(\gamma+\beta-\varepsilon)\varkappa_{,2}-4\alpha\omega_2-2\alpha u_{3,1} = J\ddot{\omega}_2,\\ &(\mu+\alpha)\nabla^2 u_3+2\alpha(\omega_{2,1}-\omega_{1,2}) = \varrho\ddot{u}_3,\end{aligned}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2},\qquad e = u_{1,1}+u_{2,2},\qquad \varkappa = \omega_{1,1}+\omega_{2,2}$$

and the stresses σ_{ji} and couple-stresses μ_{ji} corresponding to the fields $(u_1, u_2, 0)$ and $(0, 0, \omega_3)$; and $(0, 0, u_3)$ and $(\omega_1, \omega_2, 0)$ related with the systems of Eqs. (2.6) and (2.7) respectively, are given in the following form:

$$(2.8)\qquad \begin{aligned}&\sigma_{11} = 2\mu u_{1,1}+\lambda e,\quad \sigma_{22} = 2\mu u_{2,2}+\lambda e,\quad \sigma_{33} = \lambda e,\\ &\sigma_{12} = \mu(u_{2,1}+u_{1,2})+\alpha(u_{2,1}-u_{1,2})-2\alpha\omega_3,\\ &\sigma_{21} = \mu(u_{2,1}+u_{1,2})-\alpha(u_{2,1}-u_{1,2})+2\alpha\omega_3,\\ &\mu_{13} = (\gamma+\varepsilon)\omega_{3,1},\quad \mu_{31} = (\gamma-\varepsilon)\omega_{3,1},\quad \mu_{23} = (\gamma+\xi\varepsilon)\omega_{3,2},\\ &\mu_{32} = (\gamma-\varepsilon)\omega_{3,2}\end{aligned}$$

and

$$(2.9)\qquad \begin{aligned}&\sigma_{13} = (\mu+\alpha)u_{3,1}+2\alpha\omega_2,\quad \sigma_{31} = (\mu-\alpha)u_{3,1}-2\alpha\omega_2,\\ &\sigma_{23} = (\mu+\alpha)u_{3,2}-2\alpha\omega_1,\quad \sigma_{32} = (\mu-\alpha)u_{3,2}+2\alpha\omega_1,\\ &\mu_{11} = 2\gamma\omega_{1,1}+\beta\varkappa,\quad \mu_{22} = 2\gamma\omega_{2,2}+\beta\varkappa,\quad \mu_{33} = \beta\varkappa,\\ &\mu_{12} = \gamma(\omega_{2,1}+\omega_{1,2})+\varepsilon(\omega_{2,1}-\omega_{1,2}),\quad \mu_{21} = \gamma(\omega_{2,1}+\omega_{1,2})-\varepsilon(\omega_{2,1}-\omega_{1,2}).\end{aligned}$$

Now, we have already stated in our problem that the load moves in the negative direction of the x_1-axis with uniform speed U.

Therefore, an observer moving with a load at the same speed would see the load as stationary. Thus, if a Galilean transformation

$$x_1' = x_1 + Ut, \quad x_2' = x_2, \quad t' = t \tag{2.10}$$

is introduced, then, the boundary conditions are independent of t' since x_1 and t enter the boundary conditions only in the combination $x_1 + Ut$. We may assume that the loadings of the boundary can be divided into two groups. The first produces the displacement $\mathbf{u} = (u_1, u_2, 0)$ and the rotation $\boldsymbol{\omega} = (0, 0, \omega_3)$ involved in the system of Eqs. (2.6), and the second causes the displacement $\mathbf{u} = (0, 0, u_3)$ and the rotation $\boldsymbol{\omega} = (\omega_1, \omega_2, 0)$ involved in the system of Eq. (2.7). In the first case, there exists only a stress normal to the semi-space. The boundary conditions for a concentrated load moving over the plane boundary of the semi-space in moving coordinates are

$$\sigma_{22} = -P\delta(x_1'), \quad \sigma_{21} = 0, \quad \mu_{23} = 0 \tag{2.11}$$

where $\delta(x_1')$ is the Dirac-delta function.

In the second case, only the moment stress on the semi-space surface exists and the vector of this moment stress is directed along the positive x_2-axis. The boundary conditions for concentrated loading in moving coordinates are

$$\mu_{22} = -l\delta(x_1'), \quad \mu_{21} = 0, \quad \sigma_{23} = 0. \tag{2.12}$$

We are to seek solutions of Eqs. (2.6) and (2.7) subject to the boundary conditions (2.11) and (2.12), respectively.

3. General solution of the equations (2.6)

In the present plane strain problem the elastic displacements $u_1, u_2, (u_3 = 0)$ are derivable from the displacement potentials $\phi(x_1, x_2, t)$ and $\psi(x_1, x_2, t)$, so that

$$u_1 = \phi_{,1} - \psi_{,2}, \quad u_2 = \phi_{,2} + \psi_{,1} \tag{3.1}$$

and the only component of rotation $\omega_3 (\omega_1 = \omega_2 = 0)$ is a function of x_1, x_2 and t. Substituting Eqs. (3.1) into Eqs. (2.6), we find that the functions ϕ, ψ and ω_3 satisfy the following differential equations:

$$\begin{gathered}
\left(\nabla^2 - \frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}\right)\phi = 0,\\
\left(\nabla^2 - \frac{1}{c_2^2}\frac{\partial^2}{\partial t^2}\right)\psi - p\omega_3 = 0,\\
\left(\nabla^2 - v_0^2 - \frac{1}{c_4^2}\frac{\partial^2}{\partial t^2}\right)\omega_3 + s\nabla^2\psi = 0,
\end{gathered} \tag{3.2}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\varrho}, \quad c_2^2 = \frac{\mu + \alpha}{\varrho}, \quad c_4^2 = \frac{\gamma + \varepsilon}{J}, \quad s = \frac{2\alpha}{\gamma + \varepsilon},$$

$$p = \frac{2\alpha}{\mu + \alpha}, \quad v_0^2 = \frac{4\alpha}{\gamma + \varepsilon}, \quad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}.$$

The second and third equations in (3.2) are coupled partial differential equations. On eliminating one of the unknown functions ψ and ω_3, we obtain separately the same partial differential equation

$$\left[\left(\nabla^2-v_0^2-\frac{1}{c_4^2}\frac{\partial^2}{\partial t^2}\right)\left(\nabla^2-\frac{1}{c_2^2}\frac{\partial^2}{\partial t^2}\right)+v_1^2\nabla^2\right](\psi,\omega_3)=0, \tag{3.3}$$

where

$$v_1^2 = ps = 4\alpha^2/(\gamma+\varepsilon)(\mu+\alpha).$$

In view of Eqs. (2.10), the differential equations satisfied by ϕ and ψ, ω_3 are

$$\left(1-\frac{U^2}{c_1^2}\right)\frac{\partial^2\phi}{\partial x_1'^2}+\frac{\partial^2\phi}{\partial x_2'^2}=0,$$

$$\left[\left\{\left(1-\frac{U^2}{c_4^2}\right)\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_2'^2}-v_0^2\right\}\left\{\left(1-\frac{U^2}{c_2^2}\right)\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_2'^2}\right\}+v_1^2\left(\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_2'^2}\right)\right](\psi,\omega_3)=0. \tag{3.4}$$

Let us now introduce the Mach numbers

$$\mathsf{M}_i=\frac{U}{c_i},\quad i=1,2,4 \tag{3.5}$$

and the parameters

$$\beta_i^2=1-\frac{U^2}{c_i^2}=1-\mathsf{M}_i^2,\quad (i=1,2,4)\quad \text{if}\quad \mathsf{M}_i<1, \tag{3.6}$$

and

$$\beta_i'^2=\frac{U^2}{c_i^2}-1=\mathsf{M}_i^2-1,\quad (i=1,2,4)\quad \text{if}\quad \mathsf{M}_i>1.$$

In the case when $\mathsf{M}_i > 1$ (supersonic), we obtain the following partial differential equations:

$$\beta_1'^2\frac{\partial^2\phi}{\partial x_1'^2}-\frac{\partial^2\phi}{\partial x_2'^2}=0,$$

$$\left[\left(\beta_4'^2\frac{\partial^2}{\partial x_1'^2}-\frac{\partial^2}{\partial x_2'^2}+v_0^2\right)\left(\beta_2'^2\frac{\partial^2}{\partial x_1'^2}-\frac{\partial^2}{\partial x_2'^2}\right)+v_1^2\left(\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_2'^2}\right)\right](\psi,\omega_3)=0. \tag{3.7}$$

Assuming $c_1 > c_2 > c_4$ and c_4 to be very small, we consider the cases in which $\mathsf{M}_1 < \mathsf{M}_2 < 1$, $\mathsf{M}_4 > 1$ (subsonic case) and $\mathsf{M}_1 < 1$, $\mathsf{M}_2 > 1$, $\mathsf{M}_4 > 1$ (transonic case). The partial differential equations corresponding to these cases are, respectively,

$$\beta_2^1\frac{\partial^2\phi}{\partial x_1'^2}+\frac{\partial^2\phi}{\partial x_1'^2}=0, \tag{3.8}$$

$$\left[\left(\beta_4'^2\frac{\partial^2}{\partial x_1'^2}-\frac{\partial^2}{\partial x_2'^2}+v_0^2\right)\left(\beta_2'^2\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_2'^2}\right)-v_1^2\left(\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_2'^2}\right)\right](\psi,\omega_3)=0;$$

$$\beta_1^2\frac{\partial^2\phi}{\partial x_1'^2}+\frac{\partial^2\phi}{\partial x_2'^2}=0, \tag{3.9}$$

$$\left[\left(\beta_4'^2\frac{\partial^2}{\partial x_1'^2}-\frac{\partial^2}{\partial x_1'^2}+v_0^2\right)\left(\beta_2'^2\frac{\partial^2}{\partial x_1'^2}-\frac{\partial^2}{\partial x_2'^2}\right)+v_1^2\left(\frac{\partial^2}{\partial x_1'^2}+\frac{\partial^2}{\partial x_2'^2}\right)\right](\psi,\omega_3)=0.$$

Solution of the problem in different cases

Case I. Supersonic case: $M_i > 1$ $(i = 1, 2, 4)$. In this case the first equation of (3.7) admits the elementary solution

$$\phi(x_1', x_2') = Ae^{i\lambda x_1'}e^{-i\beta_1'\lambda x_2'} \tag{3.10}$$

and the solutions of the second equations of (3.7) will be sought for in the form

$$\big(\psi(x_2', x_2'), \omega_3(x_1', x_2')\big) = \big(\overline{\psi}(x_2'), \overline{\omega}_3(x_2')\big)e^{i\lambda x_1'}. \tag{3.11}$$

These solutions are as follows:

$$\begin{aligned}\psi(x_2') &= B(\lambda)e^{-i\lambda_1 x_2'} + C(\lambda)e^{-i\lambda_2 x_2'},\\ \omega_3(x_2') &= B_1(\lambda)e^{-i\lambda_1 x_2'} + C_1(\lambda)e^{-i\lambda_2 x_2'},\end{aligned} \tag{3.12}$$

where

$$\lambda_{1,2} = \frac{1}{\sqrt{2}}\left[\{\lambda^2(\beta_4'^2+\beta_2'^2) - v_0^2 + v_1^2\}\right.$$
$$\left.\pm\sqrt{\{\lambda^2(\beta_4'^2+\beta_2'^2) - v_0^2 + v_1^2\}^2 - 4\{\lambda^2\beta_2'^2(\lambda^2\beta_4'^2 - v_0^2) - \lambda^2 v_1^2\}}\right]^{1/2}.$$

The other possible solutions of the form $\exp[i\lambda_2 x_2']$ are rejected on the basis of the radiation condition at infinity.

Since λ is an arbitrary constant and the system is linear, we may let the constant A and B depend on λ and integrate over λ. Hence, we may write the general solution in the form

$$\begin{aligned}\phi(x_1', x_2') &= \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\lambda)e^{i\lambda(x_1' - \beta_1' x_2')}d\lambda,\\ \psi(x_1', x_2') &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{B(\lambda)e^{-i\lambda_1 x_2'} + C(\lambda)e^{-i\lambda_2 x_2'}\}e^{i\lambda x_1'}d\lambda,\\ \omega_3(x_1', x_2') &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{B_1(\lambda)e^{-i\lambda_1 x_2'} + C_1(\lambda)e^{-i\lambda_2 x_2'}\}e^{i\lambda x_1'}d\lambda.\end{aligned} \tag{3.13}$$

The quantities B_1, B and C_1, C are related by means of the second or third equation of (3.2).

Equating the coefficient of $e^{-i\lambda_1 x_2'}$ and $e^{-i\lambda_2 x_2'}$, we obtain from the second equation of (3.2) the relations

$$B_1(\lambda) = \varkappa_1 B(\lambda), \quad C_1(\lambda) = \varkappa_2 C(\lambda), \tag{3.14}$$

where

$$\varkappa_1 = \frac{1}{p}(\beta_2'^2\lambda^2 - \lambda_1^2), \quad \varkappa_2 = \frac{1}{p}(\beta_2'^2\lambda^2 - \lambda_2^2).$$

In view of Eqs. (2.8), (2.10), (3.1), (3.5) and (3.13), we obtain from the boundary conditions (2.11)

$$(M_2^2-2)\lambda^2 A(\lambda)-2\{\lambda_1 B(\lambda)+\lambda_2 C(\lambda)\}\lambda = i\lambda P(\lambda),$$

(3.15)
$$2\lambda^2\beta_1' A(\lambda)+\left[\lambda^2\left(\alpha\frac{M_2^2}{\mu}+M_2^2-2\right)-\left\{(p-2)\frac{\alpha}{\mu}+p\right\}\varkappa_1\right]B(\lambda)$$
$$+\left[\lambda^2\left(\alpha\frac{M_2^2}{\mu}+M_2^2-2\right)-\left\{(p-2)\frac{\alpha}{\mu}+p\right\}\varkappa_2\right]C(\lambda) = 0,$$
$$\varkappa_1\lambda_1 B(\lambda)+\varkappa_2\lambda_2 C(\lambda) = 0,$$

where $i\lambda P(\lambda)$ is the Fourier transform of $\frac{P}{\mu}\delta(x_1')$, the representation of $\delta(x_1')$ being $\lim\limits_{\epsilon\to 0}\frac{1}{2\pi\epsilon}\int\limits_{-\infty}^{\infty}\frac{\sin\lambda\epsilon}{\lambda}e^{i\lambda x_1'}d\lambda$. Hence,

(3.16)
$$A(\lambda) = \left(\varDelta_1-\frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}\varDelta_2\right)\frac{iP(\lambda)}{\varDelta},\qquad B(\lambda) = \frac{-2iP(\lambda)}{\varDelta}\lambda^2\beta_1',$$
$$C(\lambda) = -\frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}B(\lambda),$$

where

$$\varDelta_i = \lambda^2\left(\alpha\frac{M_2^2}{\mu}+M_2^2-2\right)-\left\{(p-2)\frac{\alpha}{\mu}+p\right\}\varkappa_i\quad (i = 1, 2),$$
$$\varDelta = \lambda\left(\varDelta_1-\frac{\varkappa_1\lambda_1}{\varkappa_1\lambda_2}\varDelta_2\right)(M_2^2-2)+4\left(1-\frac{\varkappa_1}{\varkappa_2}\right)\lambda^2\lambda_1\beta_1'.$$

Hence, in view of Eqs. (3.13), (3.14), (3.16) and (3.1), we obtain the displacements u_1, u_2 and rotation ω_3

$$u_1(x_1', x_2') = -\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\left\{\left(\varDelta_1+\frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}\varDelta_2\right)e^{-i\lambda\beta_1'x_2'}+2\lambda\lambda_1\beta_1'\left(e^{-i\lambda_1x_2'}\right.\right.$$
$$\left.\left.-\frac{\varkappa_1}{\varkappa_2}e^{-i\lambda_2x_2'}\right)\right\}\frac{\lambda P(\lambda)}{\varDelta}e^{i\lambda x_1'}d\lambda,$$

(3.17)
$$u_2(x_1', x_2') = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\left\{\left(\varDelta_1-\frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}\varDelta_2\right)e^{-i\lambda\beta_1'x_2'}+2\lambda\beta_1'\left(e^{-i\lambda_1x_2'}\right.\right.$$
$$\left.\left.-\frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}e^{-i\lambda_2x_2}\right)\right\}\frac{P(\lambda)}{\varDelta}e^{i\lambda x_1'}d\lambda,$$

$$\omega_3(x_1'x_2') = -\frac{1}{\pi}\int\limits_{-\infty}^{\infty}\varkappa_1\left(e^{-i\lambda_1x_2'}-\frac{\lambda_1}{\lambda_2}e^{-i\lambda_2x_2'}\right)\frac{i\lambda^2\beta_1'P(\lambda)}{\varDelta}e^{i\lambda x_1'}d\lambda.$$

The displacements u_1, u_2 and the rotation ω_3 being known, stresses and couple-stresses can be determined from the formulae (2.8).

For $\alpha \to 0$ what corresponds to the classical theory of elasticity, we obtain [10, 13]

$$
\begin{aligned}
u_1 &= \frac{P}{\mu\Delta_0}\left[(2-M_2^2)I(x_1'-\beta_1'x_2')+2\beta_1'\beta_2 I(x_1'-\beta_2'x_2')\right],\\
u_2 &= \frac{P}{\mu\Delta_0}\left[-\beta_1'(2-M_2^2)I(x_1'-\beta_1'x_2')+2\beta_1' I(x_1'-\beta_2'x_2')\right], \qquad (3.18)\\
\omega_3 &= 0,
\end{aligned}
$$

where

$$
\Delta_0 = (2-M_2^2)^2+4\beta_1'\beta_2', \qquad \frac{P}{\mu}I(x_1') = \frac{1}{2\pi}\int_{-\infty}^{\infty} P(\lambda)e^{i\lambda x_1'}\,d\lambda.
$$

Case II. Subsonic case, $M_1 < 1$, $M_2 < 1$, $M_4 > 1$. In this case the solutions of the Eqs. (3.8) may be taken in the form

$$
\begin{aligned}
\phi(x_1', x_2') &= \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\lambda)e^{-\beta_1|\lambda|x_2'}e^{i\lambda x_1'}\,d\lambda,\\
\psi(x_1', x_2') &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(B(\lambda)e^{-\lambda x_2'}+C(\lambda)e^{-\lambda_2 x_2'}\right)e^{i\lambda x_1'}\,d\lambda, \qquad (3.19)\\
\omega_3(x_1', x_2') &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(B_1(\lambda)e^{-\lambda_1 x_2'}+C_1(\lambda)e^{-\lambda_2 x_2'}\right)e^{i\lambda x_1'}\,d\lambda,
\end{aligned}
$$

where

$$
\lambda_{1,2} = \frac{1}{\sqrt{2}}\Big[\{(\beta_2^2-\beta_4'^2)\lambda^2+v_0^2-v_1^2\} \pm \sqrt{\{(\beta_2^2-\beta_4'^2)\lambda^2+v_0^2-v_1^2\}^2-4\{\beta_2^2\lambda^2(v_0^2-\beta_4'^2\lambda^2)-v_1^2\lambda^2\}}\Big]^{1/2}.
$$

In view of the coupling of the second and third equations of (3.2), we have

$$
B_1(\lambda) = \bar{\varkappa}_1 B(\lambda), \qquad C_1(\lambda) = \bar{\varkappa}_2 C(\lambda), \qquad (320)
$$

where

$$
\bar{\varkappa}_1 = \frac{1}{p}(\lambda_1^2-\lambda^2\beta_2^2), \qquad \bar{\varkappa}_2 = \frac{1}{p}(\lambda_2^2-\lambda^2\beta_2^2).
$$

In view of the boundary conditions (2.11), we obtain

$$
\begin{aligned}
A(\lambda) &= \left(\Delta_1'-\frac{\bar{\varkappa}_1\lambda_1}{\bar{\varkappa}_2\lambda_2}\Delta_2'\right)\frac{P(\lambda)}{i\lambda\Delta'}, \qquad B(\lambda) = -2|\lambda|\beta_1\frac{P(\lambda)}{\Delta'}, \qquad (3.21)\\
C(\lambda) &= -\frac{\bar{\varkappa}_1\lambda_1}{\bar{\varkappa}_2\lambda_2}B(\lambda),
\end{aligned}
$$

where
$$\Delta_i' = \left[\left\{(p-2)\frac{\alpha}{\mu}+p\right\}\bar{\varkappa}_i - \lambda^2\left(\alpha\frac{M_2^2}{\mu}+M_2^2-2\right)\right] \quad (i = 1, 2),$$
$$\Delta' = (2-M_2^2)\left(\Delta_1' - \frac{\bar{\varkappa}_1\lambda_1}{\bar{\varkappa}_2\lambda_2}\Delta_2'\right) - 4\lambda_1|\lambda|B_1\left(1-\frac{\bar{\varkappa}_1}{\bar{\varkappa}_2}\right).$$

In view of Eqs. (3.19)–(3.21) and (3.1), we obtain the displacements u_1, u_2 and the rotation ω_3 as

$$u_1(x_1', x_2') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left(\Delta_1' - \frac{\bar{\varkappa}_1\lambda_1}{\bar{\varkappa}_2\lambda_2}\Delta_2'\right)e^{-\beta_1|\lambda|x_2'} - 2\lambda_1\left(e^{-\lambda_1 x_2'} - \frac{\bar{\varkappa}_1}{\bar{\varkappa}_2}e^{-\lambda_2 x_2'}\right)|\lambda|\beta_1\right\} \times \frac{P(\lambda)}{\Delta'}e^{i\lambda x_1'}d\lambda,$$

(3.22)
$$u_2(x_1'x_2') = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left(\Delta_1' - \frac{\bar{\varkappa}_1\lambda_1}{\bar{\varkappa}_2\lambda_2}\Delta_2'\right)\frac{e^{-\beta_1|\lambda|x_2'}}{i\lambda} + 2\left(e^{-\lambda_1 x_2'} - \frac{\bar{\varkappa}_1\lambda_1}{\bar{\varkappa}_2\lambda_2}e^{-\lambda_2 x_2'}\right)i\lambda\right\}\frac{\beta_1|\lambda|P(\lambda)}{\Delta'}e^{i\lambda x_2'}d\lambda,$$
$$\omega_3 = -\frac{1}{\pi}\int_{-\infty}^{\infty}\bar{\varkappa}_1\left(e^{-\lambda_1 x_2'} - \frac{\lambda_1}{\lambda_2}e^{-\lambda_2 x_2'}\right)\frac{P(\lambda)|\lambda|\beta_1}{\Delta'}e^{i\lambda x_1'}d\lambda.$$

Making use of the formulae (2.8) we can determine the stresses and couple-stresses in the semi-infinite micropolar medium.

In the particular case, $\alpha \to 0$, we obtain the displacement components in elastokinetics [11] in which no rotation occurs

(3.23)
$$u_1 = \frac{PK_1}{\mu}\left(\frac{1}{2}-\frac{1}{\pi}\tan^{-1}\frac{\beta_1 x_2'}{x_1'}\right) - \frac{P}{\mu}\beta_2 K_2\left(\frac{1}{2}-\frac{1}{\pi}\tan^{-1}\frac{\beta_2 x_2'}{x_1'}\right),$$
$$u_2 = \frac{P}{2\pi\mu}\{K_2\log(x_1'^2+\beta_2^2 x_2'^2) - K_1\beta_1\log(x_1'^2+\beta_1^2 x_2'^2)\},$$

where
$$K_1 = \frac{2-M_2^2}{(2-M_2^2)^2-4\beta_1\beta_2}, \quad K_2 = \frac{2\beta_1}{(2-M_2^2)^2-4\beta_1\beta_2}.$$

Case III. Transonic case: $M_1 < 1$, $M_2 > 1$, $M_4 > 1$. In this case the solution of Eqs. (3.9) are

(3.24)
$$\phi(x_1', x_2') = \frac{1}{2\pi}\int_{-\infty}^{\infty}A(\lambda)e^{i\lambda x_1'}e^{-\beta_1|\lambda|x_2'}d\lambda,$$
$$\psi(x_1', x_2') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\{B(\lambda)e^{-i\lambda_1 x_2'} + C(\lambda)e^{-i\lambda_2 x_2'}\}e^{i\lambda x_1'}d\lambda,$$
$$\omega_3(x_1', x_2') = \frac{1}{2\pi}\int_{-\infty}^{\infty}\{B_1(\lambda)e^{-i\lambda_1 x_2'} + C_1(\lambda)e^{-i\lambda_2 x_2'}\}e^{i\lambda x_1'}d\lambda,$$

where λ_1 and λ_2 are given by Eqs. (3.12). The relations between $B_1(\lambda)$, $B(\lambda)$ and $C_1(\lambda)$, $C(\lambda)$ are given by Eqs. (3.14).

In view of the boundary conditions (2.11), we obtain

$$A(\lambda) = -i\frac{P(\lambda)}{\lambda\Delta}\left(\Delta_1-\frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}\Delta_2\right), \quad B(\lambda) = 2|\lambda|\beta_1 P(\lambda)/\Delta, \tag{3.25}$$

$$C(\lambda) = -\frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}B(\lambda),$$

where

$$\Delta = (2-M_2^2)\left(\Delta_1-\frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}\Delta_2\right)+4i|\lambda|\beta_1\lambda_1\left(1-\frac{\varkappa_1}{\varkappa_2}\right),$$

Δ_1 and Δ_2 are given by Eqs. (3.16). Hence, we obtain the displacements u_1, u_2 and rotation ω_3 as

$$u_1 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\left(\Delta_1-\frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}\Delta_2\right)e^{-\beta_1|\lambda|x_2'}+2\left(e^{-\lambda_1 x_2'} - \frac{\varkappa_1}{\varkappa_2}e^{-\lambda_2 x_2'}\right)\lambda_1\beta_1|\lambda|\right]\frac{P(\lambda)}{\Delta}e^{i\lambda x_1'}d\lambda,$$

$$u_2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[i\beta_1|\lambda|\left\{\left(\Delta_1-\frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}\Delta_2\right)\frac{e^{-\beta_1|\lambda|x_2'}}{\lambda}+2i\lambda\left(e^{-i\lambda_1 x_2'} - \frac{\varkappa_1\lambda_1}{\varkappa_2\lambda_2}e^{-i\lambda_2 x_2'}\right)\right\}\right]\frac{P(\lambda)}{\Delta}e^{i\lambda x_1'}d\lambda, \tag{3.26}$$

$$\omega_3 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(e^{-i\lambda_1 x_2'}-\frac{\lambda_1}{\lambda_2}e^{-i\lambda_2 x_2'}\right)\varkappa_1\frac{2|\lambda|\beta_1 P(\lambda)}{\Delta}e^{i\lambda x_1'}d\lambda.$$

The stresses and couple-stresses can be determined from the formulae (2.8).

In the particular case, $\alpha \to 0$, the displacement components and rotations are found to be [12, 13]

$$u_1 = \frac{P}{\mu}K_4\left\{\left(\frac{1}{2}-\frac{1}{\pi}\tan^{-1}\frac{\beta_1 x_2'}{x_1'}\right)+\frac{2\beta_1\beta_2'}{\pi(2-M_2^2)}\log|x_1'-\beta_2' x_2'|\right\} + \frac{P}{\mu}K_3\left\{\frac{\beta_1}{2\pi}\log(x_1'^2+\beta_1^2 x_2'^2)-\frac{2\beta_1\beta_2'}{2-M_2^2}I(x_1'-\beta_2' x_2')\right\},$$

$$u_2 = -\frac{P}{\mu\pi}K_4\left\{\frac{\beta_1}{2}\log(x_1'^2+\beta_1^2 x_2'^2)-\frac{2\beta_1}{2-M_2^2}\log|x_1'-\beta_2' x_2'|\right\} + \frac{P}{\mu}K_3\left\{\left(\frac{1}{2}-\frac{1}{\pi}\tan^{-1}\frac{\beta_1 x_2'}{x_1'}\right)\beta_1-\frac{2\beta_1}{2-M_2^2}I(x_1'-\beta_2' x_2')\right\}, \tag{3.27}$$

$$\omega_3 = 0,$$

where

$$K_3 = \frac{-4\beta_1\beta_2'(2-M_2^2)}{(2-M_2^2)^4+16\beta_1^2\beta_2'^2}, \quad K_4 = \frac{(2-M_2^2)^3}{(2-M_2^2)^4+16\beta_1^2\beta_2'^2}.$$

4. General solution of the equations (2.7)

Let us now consider the displacement and rotation fields described by the vectors $\mathbf{u} = (0, 0, u_3)$ and $\boldsymbol{\omega} = (\omega_1, \omega_2, 0)$. The displacement u_3 and the rotations ω_1, ω_2 arise in the semi-infinite space due to the loading which acts on the boundary $x_2 = 0$: the moment u_{22}.

Introducing the elastic potentials ϕ, ψ connected with the rotations, we obtain

$$\omega_1 = \phi_{,1} - \psi_{,2}, \quad \omega_2 = \phi_{,2} + \psi_{,1}. \tag{4.1}$$

By substituting Eqs. (4.1) into Eqs. (2.7), we find the functions ϕ, ψ and u_3 satisfying the differential equations

$$\begin{gathered}\left(\nabla^2 - v_2^2 - \frac{1}{c_3^2}\frac{\partial^2}{\partial t^2}\right)\phi = 0,\\ \left(\nabla^2 - v_0^2 - \frac{1}{c_4^2}\frac{\partial^2}{\partial t^2}\right)\psi - su_3 = 0,\\ \left(\nabla^2 - \frac{1}{c_2^2}\frac{\partial^2}{\partial t^2}\right)u_3 + p\nabla^2\psi = 0.\end{gathered} \tag{4.2}$$

We have introduced the notations

$$v_2^2 = \frac{4\alpha}{2\gamma+\beta}, \quad c_3^2 = \frac{2\gamma+\beta}{2}, \quad s = \frac{2\alpha}{\gamma+\varepsilon}, \quad p = \frac{2\alpha}{\mu+\alpha}, \quad v_0 = \frac{4\alpha}{\gamma+\varepsilon}.$$

Eliminating ψ(or u_3) from the second and third equations of (4.2), we arrive at the partial differential equation

$$\left[\left(\nabla^2 - \frac{1}{c_2^2}\frac{\partial^2}{\partial t^2}\right)\left(\nabla^2 - v_0^2 - \frac{1}{c_4^2}\frac{\partial^2}{\partial t^2}\right) + v_1^2\nabla^2\right](\psi, u_3) = 0 \tag{4.3}$$

which is identical with Eq. (3.3).

Since an observer is always with the load, he would see the load as stationary. Introducing Galilean transformation, (2.10), Mach numbers $\mathsf{M}_i = \frac{U}{c_i}$, $i = 2, 3, 4$ and the parameters

$$\begin{aligned}\beta_i^2 &= 1 - \frac{U^2}{c_i^2} = 1 - \mathsf{M}_i^2 \quad (i = 2, 3, 4) \quad \text{if} \quad \mathsf{M}_i < 1,\\ \beta_i'^2 &= \left(\frac{U^2}{c_i^2} - 1\right) = \mathsf{M}_i^2 - 1, \quad \text{if} \quad \mathsf{M}_i > 1\end{aligned} \tag{4.4}$$

and assuming $c_2 > c_3 > c_4$ and c_3, c_4 to be very small, we obtain from the first equation of (4.2) and (Eq. 4.3) the following partial differential equations:

$$\begin{gathered}\left(\beta_3'^2\frac{\partial^2}{\partial x_1'^2} - \frac{\partial^2}{\partial x_2'^2} + v_2^2\right)\phi = 0,\\ \left[\left\{\beta_2'^2\frac{\partial^2}{\partial x_1'^2} - \frac{\partial^2}{\partial x_2'^2}\right\}\left\{\beta_4'^2\frac{\partial^2}{\partial x_1'^2} - \frac{\partial^2}{\partial x_2'^2} + v_1^2\right\} + v_0^2\left(\frac{\partial^2}{\partial x_1'^2} + \frac{\partial^2}{\partial x_2'^2}\right)\right](\psi, u_3) = 0\end{gathered} \tag{4.5}$$

if $M_1 > 1$, $M_i > 1$ (supersonic case) and $M_1 < 1$, $M_i > 1$ (transonic case), and

$$(4.6)\quad \begin{gathered} \left(\beta_3'^2 \frac{\partial^2}{\partial x_1'^2} - \frac{\partial^2}{\partial x_2'^2} + v_2^2\right)\phi = 0, \\ \left[\left\{\beta_2'^2 \frac{\partial^2}{\partial x_1'^2} + \frac{\partial^2}{\partial x_2'^2}\right\}\left\{\beta_4'^2 \frac{\partial^2}{\partial x_1'^2} - \frac{\partial^2}{\partial x_2'^2} + v_0^2\right\} - v_1^2\left(\frac{\partial^2}{\partial x_1'^2} + \frac{\partial^2}{\partial x_2'^2}\right)\right](\psi, u_3) = 0 \end{gathered}$$

if $M_2 < 1$, $M_4 > M_3 > 1$ (subsonic case).

Case I. $M_1 > 1$, $M_i > 1$ (supersonic case) $M_1 < 1$, $M_i > 1$ (transonic case).

In this case we assume the general solution of Eqs. (4.5) in the form

$$\phi = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\lambda)e^{-i\sigma x_2'}e^{i\lambda x_1'}d\lambda,$$

$$(4.7)\quad \psi = \frac{1}{2\pi}\int_{-\infty}^{\infty}\{B(\lambda)e^{-i\lambda_1 x_2'} + C(\lambda)e^{-i\lambda_2 x_2'}\}e^{-i\lambda x_1'}d\lambda,$$

$$u_3 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\{B_1(\lambda)e^{-i\lambda_1 x_2'} + C_1(\lambda)e^{-i\lambda_2 x_2'}\}e^{i\lambda x_1'}d\lambda,$$

where

$$\sigma^2 = \beta_3'^2\lambda^2 - v_2^2,$$

$$\lambda_{1,2}^2 = \frac{1}{2}\Big[\{(\beta_4'^2 + \beta_2'^2)\lambda^2 + v_1^2 - v_0^2\} \mp \sqrt{\{(\beta_4'^2 + \beta_2'^2)\lambda^2 + v_1^2 - v_0^2\} - 4\{\beta_2'^2\lambda^2(\beta_4'^2\lambda^2 - v_0^2) - v_1^2\lambda^2\}}\Big].$$

Introducing ψ and u_3 into the third equation of (4.2), we obtain the relations

$$(4.8)\quad B_1 = \hat{\varkappa}_1 B, \quad C_1 = \hat{\varkappa}_2 C,$$

where

$$\hat{\varkappa}_1 = -p\frac{\lambda^2 + \lambda_1^2}{\lambda_1^2 - \lambda^2\beta_2'^2}, \quad \hat{\varkappa}_2 = -p\frac{\lambda^2 + \lambda_2^2}{\lambda_2^2 - \lambda^2\beta_2'^2}.$$

Applying the boundary conditions (2.12), one obtains

$$A = i\lambda\beta_{11}Q(\lambda)/\Delta, \quad B = i\lambda\beta_{22}Q(\lambda)/\Delta, \quad C = i\lambda\beta_{33}Q(\lambda)/\Delta,$$

where

$$\beta_{11} = (\alpha_{22}\alpha_{33} - \alpha_{23}\alpha_{32}), \quad \beta_{22} = \alpha_{23}\alpha_{31} - \alpha_{21}\alpha_{33},$$

$$\beta_{33} = \alpha_{21}\alpha_{32} - \alpha_{22}\alpha_{31},$$

$$\Delta = \alpha_{11}(\alpha_{22}\alpha_{33} - \alpha_{23}\alpha_{32}) + \alpha_{12}(\alpha_{23}\alpha_{31} - \alpha_{21}\alpha_{33}) + \alpha_{13}(\alpha_{21}\alpha_{32} - \alpha_{22}\alpha_{31}),$$

$$(4.9)\quad \begin{gathered} \alpha_{11} = (2\gamma + \beta)\sigma^2 + \beta\lambda^2, \quad \alpha_{12} = -2\gamma\lambda\lambda_1, \quad \alpha_{13} = -2\gamma\lambda\lambda_2, \\ \alpha_{21} = 2\gamma\sigma\lambda, \quad \alpha_{22} = (\gamma + \varepsilon)\lambda_1^2 - (\gamma - \varepsilon)\lambda^2, \quad \alpha_{23} = (\gamma + \varepsilon)\lambda_2^2 - (\gamma - \varepsilon)\lambda^2, \\ \alpha_{31} = 2\alpha i\lambda, \quad \alpha_{32} = \{(\mu + \alpha)\hat{\varkappa}_1 + 2\alpha\}i\lambda_1, \quad \alpha_{33} = \{(\mu + \alpha)\hat{\varkappa}_2 + 2\alpha\}i\lambda_2, \end{gathered}$$

$i\lambda Q(\lambda)$ is the Fourier Transform of the concentrated moment $l\delta(x_1')$.

Hence, in view of Eqs. (4.9) and (4.7), making use of Eqs. (4.1), we finally obtain

$$\omega_1 = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\lambda^2\beta_{11}}{\Delta}e^{-i\sigma x_2'}+\left\{\frac{\beta_{22}\lambda_1}{\Delta}e^{-i\lambda_1 x_2'}+\frac{\beta_{33}\lambda_2}{\Delta}e^{-i\lambda_2 x_2'}\right\}\lambda\right]Q(\lambda)e^{i\lambda x_1'}d\lambda,$$

(4.10)
$$\omega_2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{\lambda\sigma\beta_{11}}{\Delta}e^{-i\sigma x_2'}-\left(\frac{\beta_{22}}{\Delta}e^{-i\lambda_1 x_2'}+\frac{\beta_{33}}{\Delta}e^{-i\lambda_2 x_2'}\right)\lambda^2\right]Q(\lambda)e^{i\lambda x_1'}d\lambda,$$

$$u_3 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\hat{\varkappa}_1\beta_{22}e^{-i\lambda_1 x_2'}+\hat{\varkappa}_2\beta_{33}e^{-i\lambda_2 x_2'}\right)\frac{i\lambda Q(\lambda)}{\Delta}e^{i\lambda x_1'}d\lambda.$$

Knowing the functions ω_1, ω_2 and u_3, we can determine the stresses and couple-stresses from the formulae (2.9).

In the particular case, $\alpha \to 0$, we obtain the following independent partial differential equations for ϕ, ψ and u_3:

(4.11)
$$\left(\beta_3'^2\frac{\partial^2}{\partial x_1'^2}-\frac{\partial^2}{\partial x_2'^2}\right)\phi = 0,$$
$$\left(\beta_4'^2\frac{\partial^2}{\partial x_1'^2}-\frac{\partial^2}{\partial x_2'^2}\right)\psi = 0,$$
$$\left(\beta_2'^2\frac{\partial^2}{\partial x_1'^2}-\frac{\partial^2}{\partial x_2'^2}\right)u_3 = 0.$$

The solutions of Eqs. (4.11) are

(4.12)
$$\phi = \frac{1}{2\pi}\int_{-\infty}^{\infty}A^0(\lambda)e^{-i\sigma_0 x_2'}e^{i\lambda x_1'}d\lambda,$$
$$\psi = \frac{1}{2\pi}\int_{-\infty}^{\infty}B^0(\lambda)e^{-\lambda_2^0 x_2'}e^{i\lambda x_1'}d\lambda,$$
$$u_3 = \frac{1}{2\pi}\int_{-\infty}^{\infty}C^0(\lambda)e^{-\lambda_2^0 x_2'}e^{i\lambda x_1'}d\lambda,$$

where $A^0(\lambda)$, $B^0(\lambda)$ and $C^0(\lambda)$ are found from the boundary condition (2.12) as

$$A^0(\lambda) = \frac{i\alpha_{22}^0 Q(\lambda)}{\Delta^0\lambda},\quad B^0(\lambda) = -i\alpha_{21}Q(\lambda)/\Delta^0\lambda,\quad C^0(\lambda) = 0,\quad \sigma = \beta_3'\lambda,$$
$$\lambda_1^0 = \beta_4'\lambda,\quad \lambda_2^0 = \beta_2'\lambda,\quad \Delta^0 = \alpha_{11}^0\alpha_{22}-\alpha_{12}^0\alpha_{21},\quad \alpha_{11}^0 = (2\gamma+\beta)\beta_3'+\beta,$$
$$\alpha_{12}^0 = -2\gamma\beta_4',\quad \alpha_{21}^0 = 2\gamma\beta_3',\quad \alpha_{22}^0 = (\gamma+\varepsilon)\beta_4'^2-(\gamma-\varepsilon).$$

Hence, in view of Eqs. (4.12) and (4.1), we have

$$\omega_1 = \frac{l}{\Delta^0}\{\alpha'_{21}\beta'_4 I(x'_1-\beta'_4x'_2)-\alpha^0_{22}I(x'_1-\beta'_3x'_2)\},$$

(4.13)
$$\omega_2 = \frac{l}{\Delta^0}\{\alpha^0_{21}I(x'_1-\beta'_4x'_2)+\alpha^0_{22}\beta'_3 I(x'_1-\beta'_3x'_2)\},$$

$$u_3 = 0.$$

The formulae (4.13) indicate the rotation field $\boldsymbol{\omega}(\omega_1, \omega_2, 0)$ in a hypothetical medium in which no displacement occurs. This field is produced by the action of concentrated moment $l\delta(x'_1)$ on the boundary $x'_1 = 0$.

It is also observed from Eqs. (4.13) that the disturbances are marked by two Mach waves

$$x'_1-\beta'_3x'_2 \quad \text{and} \quad x'_1-\beta'_4x'_2.$$

Case II. $M_2 < 1, \quad M_4 > M_3 > 1.$

In this case the solution to the partial differential Eqs. (4.6) may be taken in the form

$$\phi = \frac{1}{2\pi}\int A(\lambda)e^{-i\sigma x'_2}e^{i\lambda x'_1}d\lambda,$$

(4.14)
$$\psi = \frac{1}{2\pi}\int\{B(\lambda)e^{-i\lambda_1x'_2}+C(\lambda)e^{-i\lambda_2x'_2}\}e^{i\lambda x'_1}d\lambda,$$

$$u_3 = \frac{1}{2\pi}\int\{B_1(\lambda)e^{-i\lambda_1x'_2}+C_1(\lambda)e^{-i\lambda_2x'_2}\}e^{i\lambda x'_1}d\lambda,$$

where

$$\sigma^2 = (\beta'^2_3\lambda^2-v^2_2),$$

$$\lambda^2_{1,2} = \frac{1}{2}\Big[\{\lambda^2(\beta'^2_4-\beta^2_2)-v^2_0+v^2_1\} \mp \sqrt{\{\lambda^2(\beta'^2_4-\beta^2_2)-v^2_0+v^2_1\}^2+4\{\beta^2_2\lambda^2(\lambda^2\beta'^2_4-v^2_0)+\lambda^2v^2_1\}}\Big].$$

In view of Eqs. (4.14) and the third equation of (4.2), we obtain the relations

(4.15)
$$B_1 = \hat{\varkappa}_1 B, \quad C_1 = \hat{\varkappa}_2 C,$$

where

$$\hat{\varkappa}_1 = -p(\lambda^2+\lambda^2_1)/(\beta^2_2\lambda^2+\lambda^2_1), \quad \hat{\varkappa}_2 = -p(\lambda^2+\lambda^2_2)/(\beta^2_2\lambda^2+\lambda^2_2),$$

$A(\lambda)$, $B(\lambda)$ and $C(\lambda)$ are obtained from the boundary conditions (2.12) and are given by the expression (4.9), only replacing the values of λ_1, λ_2 and $\hat{\varkappa}_1$, $\hat{\varkappa}_2$ by those given in Eqs. (4.14) and (4.15), respectively. Hence, the rotation components ω_1, ω_2 and the displacement component u_3 are given by the same expression (4.10) where the values of λ_1, λ_2, $\hat{\varkappa}_1$, $\hat{\varkappa}_2$, are given by Eqs. (4.14) and (4.15).

In the particular case, $\alpha \to 0$, we obtain the same rotational field $\boldsymbol{\omega}(\omega_1, \omega_2, 0)$ given by Eqs. (4.13) in the hypothetical medium in which no displacement occurs.

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