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Numerical analysis of heat flow in flash welding

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IN THE PAPER a finite element method is used for solving the nonlinear transient heat transfer problem of the axisymmetric flash welding. The variational derivation of the finite element matrices and the algorithm for solving the resulting system of nonlinear equations are discussed. Numerical illustrations prove the effectiveness of the approach.

Artykuł opisuje zastosowanie metody elementów skończonych do rozwiązania problemu nieliniowego przepływu ciepła przy zgrzewaniu elementów osiowosymetrycznych. W pracy analizowano wariacyjną postać równania i podano algorytm dla jego rozwiązania. Na zakończenie podano przykłady ilustrujące rozpatrywany proces.

Статья описывает применение метода конечных элементов для решения задачи нелинейного течения тепла при сварке осесимметричных элементов. В работе анализируется вариационный вид уравнения и приведен алгоритм для его решения. В заключении даются примеры иллюстрирующие рассматриваемый процесс.

1. Introduction

FLASH welding of metal rods is a resistance welding process wherein coalescence is produced, simultaneously over the entire area of abutting surfaces, by the heat resulting from resistance to the flow of electric current between the two surfaces and by a pressure applied to the rods after heating is essentially completed. In order to make a good flash weld, it is necessary that appropriate plastic zones be generated in the rods to be welded. These are very much dependent on the heat treatment experienced by the metal around the weld.

The present paper describes a numerical method for the analysis of transient temperature distribution in the vicinity of the weld for arbitrary axisymmetric rods. A common assumption in attempting to find an analytical solution to such a problem is the postulated temperature independence of all material properties. No such simplifications have to be done in the present approach: the finite element method used in its incremental form makes it possible to account for arbitrary variations of all material characteristics during the process.

2. Formulation of the heat transfer problem

The conventional (and adopted below) analytical treatment of fully axisymmetric problems reduces the analysis to the appropriate radial plane of axial symmetry yielding the description which is essentially two-dimensional. However, in the present paper the three-dimensional equations referred to rectangular Cartesian coordinates are discussed only as the development is aimed at more general applications. Furthers the way the equations of heat flow are specified to the axisymmetric case is considered standard and can be found in any of the available textbooks, [3] for instance.

The spatial problem of the transient heat flow is assumed to be described by the partial differential equation

$$(2.1)\quad \frac{\partial}{\partial x}\left(k_x \frac{\partial \theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial \theta}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial \theta}{\partial z}\right)+q_B = \varrho c \frac{\partial \theta}{\partial t}, \qquad (x, y, z) \in \Omega,$$

where $\theta(x, y, z, t)$ is the temperature of the body, k_x, k_y, k_z are the (possibly temperature dependent) thermal conductivities corresponding to the x, y, z Cartesian axes, t stands for the time variable, ϱ is the material density, c is the temperature-dependent material heat capacity and q_B is the rate of heat generated per unit volume (with the material heat capacity effect excluded, positive if the heat is put into the body).

The initial conditions are defined as

$$\theta(x, y, z, t_0) = \hat{\theta}_0(x, y, z) \quad \text{for} \quad t = t_0,$$

$\hat{\theta}_0$ is a given function whereas the boundary conditions can be written as

$$\theta(x, y, z, t)|_{\partial\Omega_\theta} = \hat{\theta}(x, y, z, t),$$

$$\lambda \frac{\partial \theta}{\partial n}(x, y, z, t)|_{\partial\Omega_q} = q_n(x, y, z, t),$$

where $\hat{\theta}$ is the environmental temperature of the surface area $\partial\Omega_\theta$, q_n is the boundary heat flow input on the area $\partial\Omega_q$, n is the normal to the boundary and λ is the body thermal conductivity. Specifically, the convection boundary conditions can be represented as

$$\lambda \frac{\partial \theta}{\partial n}(x, y, z, t)|_{\partial\Omega_c} = h(\hat{\theta}-\theta_s),$$

where h is the convection coefficient which may be temperature dependent, θ_s the boundary surface temperature and $\hat{\theta}$ the corresponding environmental temperature. Other boundary conditions (such as radiation boundary conditions) can easily be specified but are not needed in the development to follow.

Equation (2.1) is a nonlinear partial differential equation which expresses the heat flow "transient equilibrium" principle: the rate of heat transfer by conduction must be equal to the rate of heat generation. The solution of Eq. (2.1) comes down to tracing the temperature distributions at subsequent time instants taken from the time interval $[t_0, t^*]$.

3. Incremental finite element equations

As in the incremental finite element stress analysis, [4, 5], it is now assumed that the considered problem of nonlinear, transient heat flow has been already solved for all the time steps from the initial time t_0 to the "current" time t, inclusive, and that the solution (i.e. temperature distribution) for the time $t+\Delta t$ is required next. It is noted that the solution process for the next "transient equilibrium" position is typical and would be applied repetitively until the complete solution path has been solved for.

In order to consistently set up the finite element matrices, the heat flow equilibrium at the time $t+\Delta t$ is formulated as a variational problem of the form

$$\int_{\Omega} \delta \boldsymbol{\theta}'^{T\, t+\Delta t}\mathbf{k}^{t+\Delta t}\boldsymbol{\theta}'\, d\Omega = \delta^{t+\Delta t}Q + \int_{\partial\Omega_c} \delta\theta_s{}^{t+\Delta t}h({}^{t+\Delta t}\hat{\theta} - {}^{t+\Delta t}\theta_s)\, d(\partial\Omega),$$

where

$$\boldsymbol{\theta}'^{T} = \left[\frac{\partial\theta}{\partial x}, \frac{\partial\theta}{\partial y}, \frac{\partial\theta}{\partial z}\right],$$

$$\mathbf{k} = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix},$$

$\delta\, {}^{t+\Delta t}Q$ is the virtual work of the external heat flow input to the system at the time $t+\Delta t$ which in our case reads

$$\delta^{t+\Delta t}Q = \int_{\Omega} \delta\theta({}^{t+\Delta t}q_B - {}^{t+\Delta t}C\, {}^{t+\Delta t}\dot{\theta})\, d\Omega, \qquad C = c\cdot\varrho$$

and for an arbitrary function of space and time coordinates $f(x, y, z, t)$ the shortened notation is used $f(x, y, z, t) = {}^{t}f$, with the spatial coordinates x, y, z being omitted for simplicity. We introduce further the obvious relationships

$$\begin{aligned} {}^{t+\Delta t}\theta &= {}^{t}\theta + \Delta\theta, \\ {}^{t+\Delta t}\boldsymbol{\theta}' &= {}^{t}\boldsymbol{\theta}' + \Delta\boldsymbol{\theta}', \\ {}^{t+\Delta t}h &= {}^{t}h + \Delta h, \\ {}^{t+\Delta t}C &= {}^{t}C + \Delta C, \end{aligned}$$

and assume the following equations to hold approximately:

$$\begin{aligned} {}^{t+\Delta t}\mathbf{k}\, {}^{t+\Delta t}\boldsymbol{\theta}' &\cong {}^{t}\mathbf{k}({}^{t}\boldsymbol{\theta}' + \Delta\boldsymbol{\theta}'), \\ {}^{t+\Delta t}h({}^{t+\Delta t}\hat{\theta} - {}^{t+\Delta t}\theta_s) &\cong {}^{t}h({}^{t+\Delta t}\hat{\theta} - {}^{t}\theta_s - \Delta\theta_s), \\ {}^{t+\Delta t}C\, {}^{t+\Delta t}\dot{\theta} &\cong {}^{t}C({}^{t}\dot{\theta} + \Delta\dot{\theta}). \end{aligned}$$

The above approximations result in the incrementally linearized equation of the form

$$\begin{aligned} \int_{\Omega} \delta\boldsymbol{\theta}'^{T}\, {}^{t}\mathbf{k}\Delta\boldsymbol{\theta}' d\Omega + \int_{\partial\Omega_c} \delta\theta_s\, {}^{t}h\Delta\theta_s d(\partial\Omega) &= \int_{\Omega} \delta\theta({}^{t+\Delta t}q_B - {}^{t+\Delta t}C\, {}^{t+\Delta t}\dot{\theta})\, d\Omega \\ &- \int_{\Omega} \delta\boldsymbol{\theta}'^{T}\, {}^{t}\mathbf{k}\, {}^{t}\boldsymbol{\theta}' d\Omega + \int_{\partial\Omega_c} \delta\theta_s\, {}^{t}h({}^{t+\Delta t}\hat{\theta} - {}^{t}\theta_s)\, d(\partial\Omega) \end{aligned}$$

which yields further the equation

$$\begin{aligned} \text{(3.1)}\quad \int_{\Omega} \delta\boldsymbol{\theta}'^{T}\, {}^{t}\mathbf{k}\Delta\boldsymbol{\theta}' d\Omega + \int_{\partial\Omega_c} \delta\theta_s\, {}^{t}h\Delta\theta_s d(\partial\Omega) &= \int_{\Omega} \delta\theta(\Delta q_B - {}^{t}C\Delta\dot{\theta})\, d\Omega + \int_{\partial\Omega_c} \delta\theta_s\, {}^{t}h\Delta\hat{\theta}\, d(\partial\Omega) \\ &+ \int_{\Omega} \delta\theta({}^{t}q_B - {}^{t}C\, {}^{t}\dot{\theta})\, d\Omega - \int_{\Omega} \delta\boldsymbol{\theta}'^{T}\, {}^{t}\mathbf{k}\, {}^{t}\boldsymbol{\theta}' d\Omega + \int_{\partial\Omega_c} \delta\theta_s\, {}^{t}h({}^{t}\hat{\theta} - {}^{t}\theta_s)\, d(\partial\Omega) \end{aligned}$$

and, finally, denoting by ${}^tJ$ the last three integrals on the right-hand side of Eq. (3.1), the equation

$$(3.2)\quad \int_{\Omega} \delta\boldsymbol{\theta}'^T\, {}^t\mathbf{k}\Delta\boldsymbol{\theta}' d\Omega + \int_{\partial\Omega_c} \delta\theta_s\, {}^th\Delta\theta_s d(\partial\Omega) = \int_{\Omega} \delta\theta(\Delta q_B - {}^tC\Delta\dot{\theta})d\Omega + \int_{\partial\Omega_c} \delta\theta_s\, {}^th\Delta\hat{\theta}\, d(\partial\Omega) + {}^tJ.$$

We note that the term ${}^tJ$ expresses nothing more but the heat flow equilibrium at the time t and should theoretically be put equal to zero. However, an important feature of all incremental formulations is that at the beginning of an incremental step the solution obtained so far is, due ot the approximate nature of the solution algorithm, not exact. It goes without saying that such residuals arising after each incremental step can accumulate unless the algorithm is endowed with some appropriate numerical improvements. Thus the inclusion of the equilibrium imbalance at the beginning of a step may greatly improve the solution for the next increment.

We further note that the right-hand side of Eq. (3.2) is a function of $\Delta\dot{\theta}$ which in order to obtain the unknown function $\Delta\theta(x, y, z)$ must be approximated using a time integration scheme.

Equation (3.2) has been written for a three-dimensional problem; for a two-dimensional situation only the two appropriate coordinates x, y are employed.

4. Finite element equations

On the basis of Eq. (3.2) the governing equations for heat transfer analysis of a solid idealized by a system of finite elements can easily be derived. In the analysis to follow isoparametric finite element discretization is employed, in which we describe the geometry of an element e shown in Fig. 1 by the expansions, [4],

$$(4.1)\quad \begin{aligned} x^{(e)}(r, s) &= \sum_{i=1}^{N} h_i(r, s)x_i^{(e)}, \\ y^{(e)}(r, s) &= \sum_{i=1}^{N} h_i(r, s)y_i^{(e)}, \end{aligned}$$

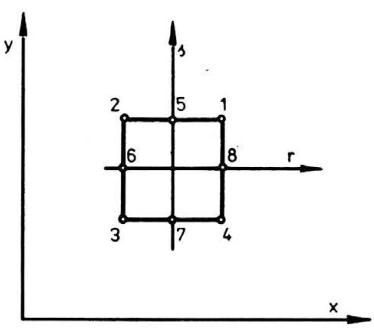

FIG. 1. Numbering of element nodes.

where the h_i are the element interpolation functions given in [4], N is the number of element nodal points (which can alternatively be taken as 4, 5, 6, 7 or 8), $x_i^{(e)}$, $y_i^{(e)}$ are the coordinates of the nodal point i with respect to the global coordinates x, y while r, s stand for local coordinates of the element considered. Eight nodes per element are consequently assumed in the present investigation.

The temperature interpolation involves the same functions $h_1, h_2, \ldots, h_8$ and is given in terms of nodal temperatures $\theta_1^{(e)}, \theta_2^{(e)} \ldots, \theta_8^{(e)}$ as

$$\theta^{(e)}(r, s, t) = \sum_{i=1}^{8} h_i(r, s)\theta_i^{(e)}(t). \tag{4.2}$$

The last relation written in matrix notation reads

$$\theta^{(e)}(r, s, t) = \mathbf{b}_{1\times 8}^{(e)}(r, s)\boldsymbol{\theta}_{8\times 1}^{(e)}(t), \tag{4.3}$$

where the row interpolation vector is given by

$$\mathbf{b}_{1\times 8}^{(e)} = [h_1, h_2, \ldots, h_8] \tag{4.4}$$

while the column vector of the nodal temperatures is defined as

$$\boldsymbol{\theta}_{8\times 1}^{(e)} = \{\theta_1^{(e)}, \theta_2^{(e)}, \ldots, \theta_8^{(e)}\}. \tag{4.5}$$

It is seen that

$$\frac{\partial}{\partial r}\theta^{(e)}(r, s, t) = \sum_{i=1}^{8} \frac{\partial}{\partial r} h_i(r, s)\theta_i^{(e)}(t), \tag{4.6}$$

$$\frac{\partial}{\partial s}\theta^{(e)}(r, s, t) = \sum_{i=1}^{8} \frac{\partial}{\partial s} h_i(r, s)\theta_i^{(e)}(t), \tag{4.7}$$

hence

$$\begin{bmatrix} \dfrac{\partial\theta^{(e)}}{\partial r} \\[2mm] \dfrac{\partial\theta^{(e)}}{\partial s} \end{bmatrix}_{2\times 1} = \mathbf{B}_{2\times 8}(r, s)\boldsymbol{\theta}_{8\times 1}^{(e)}(t), \tag{4.8}$$

where $\mathbf{B}(r, s)$ is the 2×8 matrix of derivatives of the interpolation functions. The chain rule relating x, y to r, s derivatives is written as

$$\begin{bmatrix} \dfrac{\partial\theta}{\partial r} \\[2mm] \dfrac{\partial\theta}{\partial s} \end{bmatrix}^{(e)} = J \begin{bmatrix} \dfrac{\partial\theta}{\partial x} \\[2mm] \dfrac{\partial\theta}{\partial y} \end{bmatrix}^{(e)} \tag{4.9}$$

in which

$$J = \begin{bmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial y}{\partial r} \\[2mm] \dfrac{\partial x}{\partial s} & \dfrac{\partial y}{\partial s} \end{bmatrix}. \tag{4.10}$$

Inverting the Jacobian operator J, we obtain

$$\begin{bmatrix} \dfrac{\partial\theta}{\partial x} \\ \dfrac{\partial\theta}{\partial y} \end{bmatrix}^{(e)} = \frac{1}{\det J}\begin{bmatrix} \dfrac{\partial y}{\partial s} & -\dfrac{\partial y}{\partial r} \\ -\dfrac{\partial x}{\partial s} & \dfrac{\partial x}{\partial r} \end{bmatrix}\begin{bmatrix} \dfrac{\partial\theta}{\partial r} \\ \dfrac{\partial\theta}{\partial s} \end{bmatrix}^{(e)}, \tag{4.11}$$

where the Jacobian determinant is

$$\det J = \frac{\partial x}{\partial r}\frac{\partial y}{\partial s} - \frac{\partial x}{\partial s}\frac{\partial y}{\partial r}. \tag{4.12}$$

Note that Eqs. (4.2) ÷ (4.12) hold also for temperature increments.

The finite element incremental heat flow equations are derived by substituting the interpolations (4.1), (4.2) into Eq. (3.2). We obtain

$$({}^t\mathbf{K}^k + {}^t\mathbf{K}^c)\Delta\boldsymbol{\theta}^{(N)} = \Delta\mathbf{Q}(\Delta\dot{\boldsymbol{\theta}}^{(N)}) + \Delta\mathbf{Q}^c + {}^t\mathbf{J}^*, \tag{4.13}$$

where

$$ {}^t\mathbf{K}^k = \sum_{e=1}^{E}\Big(\int_{\Omega_e}\mathbf{B}^T\,{}^t\mathbf{k}\mathbf{B}\,d\Omega\Big), \tag{4.14}$$

$$ {}^t\mathbf{K}^c = \sum_{e-1}^{E}\Big(\int_{\partial\Omega_c}{}^th\mathbf{H}^{sT}\mathbf{H}^s d(\partial\Omega)\Big), \tag{4.15}$$

$$\begin{aligned} \Delta\mathbf{Q}(\Delta\dot{\boldsymbol{\theta}}^{(N)}) &= \sum_{e=1}^{E}\Big(\int_{\Omega_e}\mathbf{H}^T\Delta q_B\,d\Omega - \Big(\int_{\Omega_e}\mathbf{H}^T\mathbf{H}\,{}^tC\,d\Omega\Big)\,\Delta\dot{\boldsymbol{\theta}}^{(N)}\Big) \\ &= \sum_{e=1}^{E}\Big(\int_{\Omega_e}\mathbf{H}^T\Delta q_B\,d\Omega - {}_{(e)}^{\ t}\mathbf{C}\Delta\dot{\boldsymbol{\theta}}^{(N)}\Big), \end{aligned} \tag{4.16}$$

$$\Delta\mathbf{Q}^c = \sum_{e=1}^{E}\Big(\int_{\Omega_e}\mathbf{H}^{sT}\mathbf{H}^s\,{}^th\Delta\hat{\boldsymbol{\theta}}^{(N)}d\Omega\Big), \tag{4.17}$$

$$ {}^t\mathbf{J}^* = \sum_{e=1}^{E}\int_{\Omega_e}\Big(\mathbf{H}^T\,{}^tq_B\,d\Omega - {}_{(e)}^{\ t}\mathbf{C}\,{}^t\dot{\theta}^{(N)} - {}^t\mathbf{K}^k\,{}^t\boldsymbol{\theta}^{(N)} + \Big(\int_{\partial\Omega_e}\mathbf{H}^{sT}\mathbf{H}^s\,{}^th\,d(\partial\Omega)\,({}^t\hat{\boldsymbol{\theta}}^{(N)} - {}^t\boldsymbol{\theta}^{(N)})\Big)\Big). \tag{4.18}$$

Vectors $\mathbf{H}$ and $\mathbf{H}^s$ and the matrix $\mathbf{B}$ define the temperatures and temperature gradients within the element or on its boundary as a function of the nodal point temperatures,

$$\begin{aligned} \theta(x,y,t) &= \mathbf{H}(x,y)\,{}^t\boldsymbol{\theta}^{(N)}, \\ \theta_s(x,y,t) &= \mathbf{H}^s(x,y)\,{}^t\boldsymbol{\theta}^{(N)}, \\ \boldsymbol{\theta}'(x,y,t) &= \mathbf{B}(x,y)\,{}^t\boldsymbol{\theta}^{(N)}, \end{aligned}$$

which can easily be determined from Eq. (4.2). The symbol $\sum_{e=1}^{E}$ implies the summation over all finite elements and the column vector

$$ {}^t\boldsymbol{\theta}^{(N)} = \{{}^t\theta_1^{(N)}, {}^t\theta_2^{(N)}, \ldots, {}^t\theta_{NE}^{(N)}\}$$

stands for the nodal point temperatures at all nodal points in the discretized region. The volume and surface integrations in Eqs. (4.15) ÷ (4.18) are effectively carried out for the axisymmetric finite elements by using the Gauss integration formula. We have for a one-radian part of the specimen, for instance,

$$\int_{0}^{1}\int_{-1}^{+1}\int_{-1}^{+1} \mathbf{B}^T(r,s)\,{}^t\mathbf{k}(r,s)\mathbf{B}(r,s)\det J(r,s)\,R(r,s)\,d\theta\,dr\,ds$$

$$= \int_{-1}^{1}\int_{-1}^{1} \mathbf{B}^T(r,s)\,{}^t\mathbf{k}(r,s)\mathbf{B}(r,s)\det J(r,s)\,R(r,s)\,dr\,ds$$

$$= \sum_{i,j} \alpha_{ij}\mathbf{B}^T(r_i,s_j)\,{}^t\mathbf{k}(r_i,s_j)\mathbf{B}(r_i,s_j)\det J(r_i,s_j)\,R(r_i,s_j),$$

where the summations extend over all i and j specified in the given Gauss formula and the matrices are evaluated at the sampling points.

Equation (4.13) is the fundamental matrix equation describing the axisymmetric heat flow problem in the incremental form. We note that for a given temperature distribution at time t the incremental nodal temperatures can still not be found unless we use some approximation for the temperature rate in the vector $\Delta\mathbf{Q}$, Eq. (4.16). We transform Eq. (4.13) to the form

$$({}^t\mathbf{K}^k+{}^t\mathbf{K}^c)\Delta\boldsymbol{\theta}^{(N)} = -{}^t\mathbf{C}\,{}^{t+\Delta t}\dot{\boldsymbol{\theta}}^{(N)}+{}^t\mathbf{F}-{}^t\mathbf{K}^k\,{}^t\boldsymbol{\theta}^{(N)},$$

where

$${}^t\mathbf{F} = \sum_{d=1}^{E}\Big(\int_{\Omega_e}\mathbf{H}^T\,{}^{t+\Delta t}q_B\,d\Omega+\int_{\partial\Omega_c}\mathbf{H}^{sT}\mathbf{H}^s\,{}^th({}^{t+\Delta t}\hat{\theta}^{(N)}-{}^t\theta^{(N)})\,d(\partial\Omega)\Big)$$

or, further, to the form

$$(4.19)\qquad {}^t\mathbf{K}\,{}^{t+\Delta t}\boldsymbol{\theta}^{(N)}+{}^t\mathbf{C}\,{}^{t+\Delta t}\dot{\boldsymbol{\theta}}^{(N)} = {}^{t+\Delta t}\mathbf{Q},$$

where

$${}^t\mathbf{K} = {}^t\mathbf{K}^k+{}^t\mathbf{K}^c,$$

$${}^{t+\Delta t}\mathbf{Q} = \sum_{e=1}^{E}\Big(\int_{\Omega_e}\mathbf{H}^T\,{}^{t+\Delta t}q_B\,d\Omega+\int_{\partial\Omega_e}\mathbf{H}^{sT}\mathbf{H}^s\,{}^th\,{}^{t+\Delta t}\hat{\boldsymbol{\theta}}^{(N)}\Big)$$

is the vector of thermal nodal loads while $\mathbf{C}$ is the global heat capacity matrix,

$$\mathbf{C} = \sum_{e=1}^{E} {}_{(e)}\mathbf{C}.$$

Equation (4.19) is the system of nonlinear ordinary differential equations which must be solved for the subsequent values of the incremental nodal temperatures.

Using the Euler backward method we obtain, from Eqs. (4.19),

$${}^t\mathbf{C}\frac{1}{\Delta t}({}^{t+\Delta t}\boldsymbol{\theta}^{(N)}-{}^t\boldsymbol{\theta}^{(N)})+{}^t\mathbf{K}\,{}^{t+\Delta t}\boldsymbol{\theta}^{(N)} = {}^{t+\Delta t}\mathbf{Q},$$

which yields

$$ {}^{t}\mathbf{K}^{*}\Delta\boldsymbol{\theta}^{(N)} = {}^{t+\Delta t}\mathbf{Q}^{*} \tag{4.20}$$

with the effective system matrix

$$ {}^{t}\mathbf{K}^{*} = \frac{1}{\Delta t}\,{}^{t}\mathbf{C}+{}^{t}\mathbf{K}$$

and the effective load vector

$$ {}^{t+\Delta t}\mathbf{Q}^{*} = {}^{t+\Delta t}\mathbf{Q}-{}^{t}\mathbf{K}\,{}^{t}\boldsymbol{\theta}^{(N)}.$$

Equation (4.20) can be used for the step-by-step evaluation of $\Delta\boldsymbol{\theta}^{(N)}$. The accumulation formula for temperatures at the step $(t, t+\Delta t)$ follows as

$$ {}^{t+\Delta t}\boldsymbol{\theta}^{(N)} = {}^{t}\boldsymbol{\theta}^{(N)}+\Delta\boldsymbol{\theta}^{(N)}$$

and then, after appropriate updating of the material parameters, the next step calculations are carried out similarly. In this way the solution for temperatures can be advanced in time resulting in the whole history of the process.

We note that the Euler backward algorithms is unconditionally stable which assures that any errors at time t, which may be due to round-off in the computer, do not grow in the subsequent integration. This does not eliminate the accuracy problems which clearly depend on Δt so that in general some iterative procedure is needed. This has not been attempted in the present paper, though, and it is believed to be justified by "mild" nonlinearities in the analyses performed.

5. Sample solutions

5.1. Flash welding of two metal rods

The two semi-infinite rods shown in Fig. 2 are subjected to the flash welding. At the place of abutment the heat source is given by the following equation:

$$ Q = J^{2}\cdot R\cdot t,$$

where J is the current intensity and R resistance at the place of contact. It is noted that R depends on the diameter of the rods, compressive force applied, properties of the material and time.

Steel rods of the diameter ϕ 12 mm and density 7800 kg/m^3 are considered. The heat flux at the place of contact of the rods is assumed to be equal to $24\cdot 10^6$ W/m^2, which approximately amounts to 600 cal/cm^{2}s. The material properties are given in Table 1 for steel and Table 2 for the surrounding air (with air density taken as 1.29 kg/m^3). The surface film conductance is taken as 0.25 W/m^2.

Two kinds of the analysis are carried out with different inclusion of the surrounding air effects, Figs. 2 and 3. The discretization meshes in the corresponding steel and air areas are shown in Fig. 3. Due to the existing symmetries, one quarter of the area is considered only for each case. The results obtained for both the coarse and fine meshes and both the boundary conditions were almost coincident and are shown in Figs. 4 and 5.

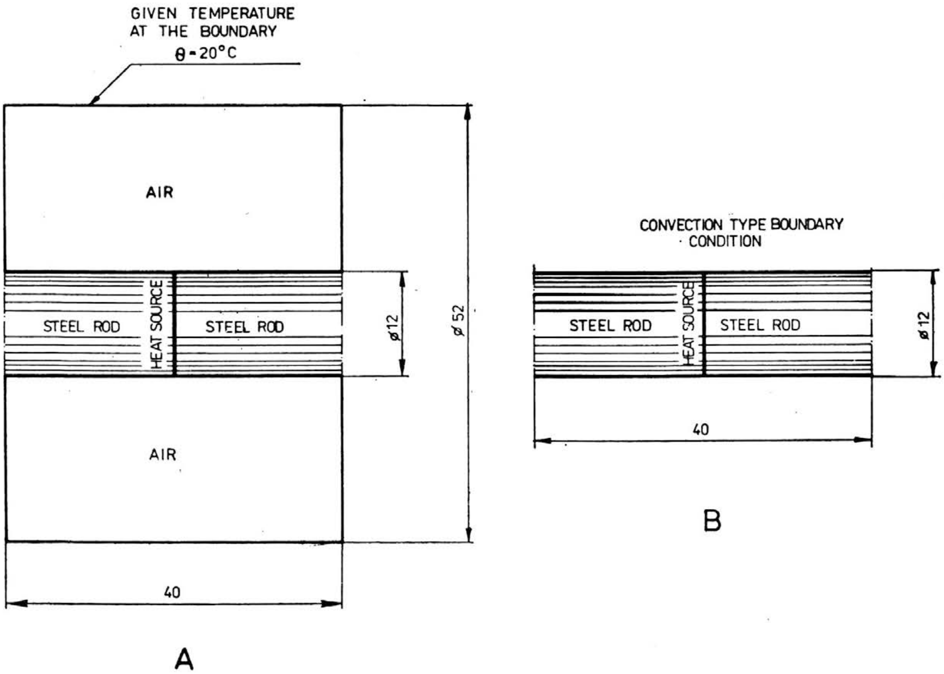

FIG. 2. Heat boundary conditions in flash welding of two rods; A — surrounding air included, B — surrounding air excluded.

Table 1.

Temperature [°C]	0	500	800	900	1600
Thermal conductivity $\left[\frac{J}{m \cdot K \cdot s}\right]$	50	50	50	50	50
Specific heat $\left[\frac{J}{kg \cdot K}\right]$	510	1030	1300	680	680

Table 2

Temperature [°C]	0	1600
Thermal conductivity $\left[\frac{J}{s \cdot m \cdot K}\right]$	0.23	0.23
Specific heat $\left[\frac{J}{kg \cdot K}\right]$	1005	1100

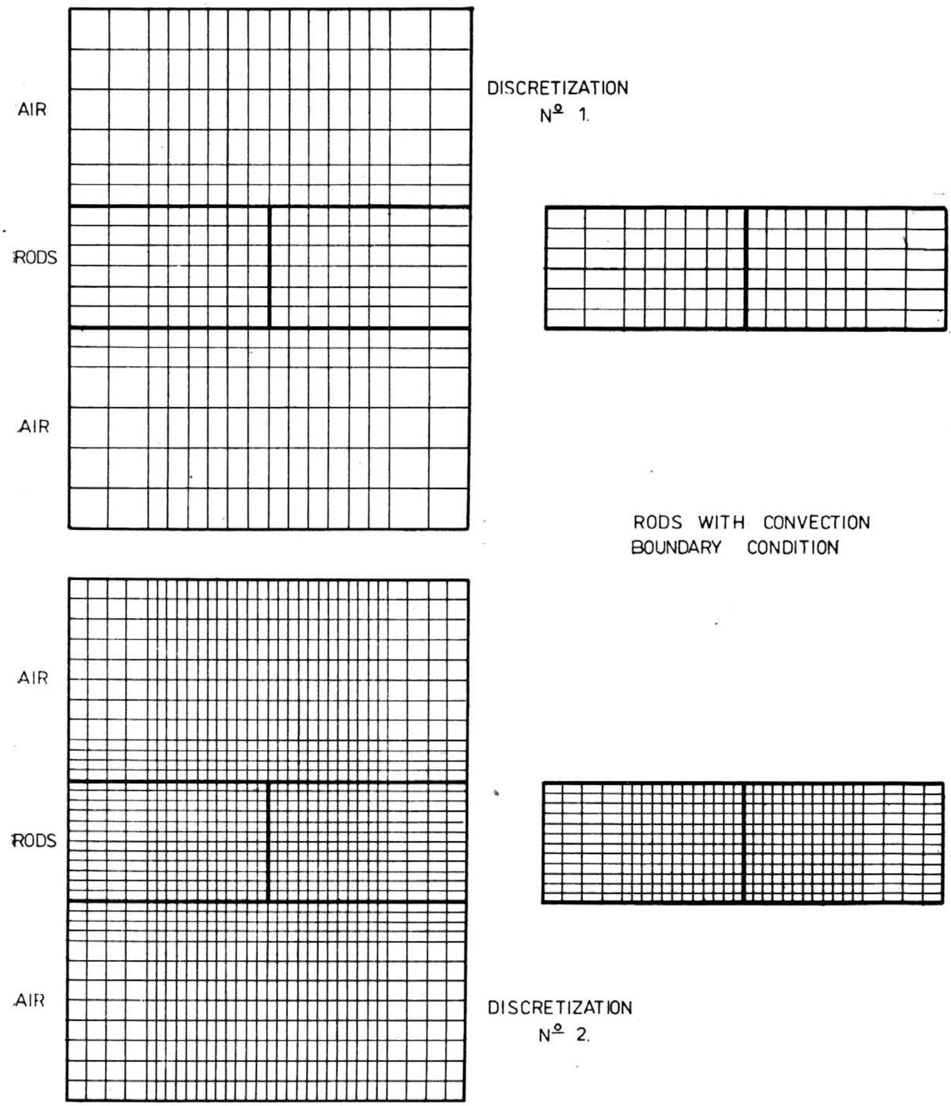

FIG. 3. Types of discretization.

The results given in Fig. 4 refer to the time instants of $t = 1$ s, $t = 2$ s, $t = 3$ s while those shown in Fig. 5 are for $t = 3$ s.

5.2. Flash welding of two metal pipes

The second example concerns the flash welding problem of two steel pipes with the same material properties as before, Fig. 6. On the basis of the previous results one discretization mesh and the convection-type boundary conditions are assumed to yield sufficiently accurate results in the present case. The temperature distribution along the pipe axis is shown for three time instants of 1, 2 and 3 seconds.

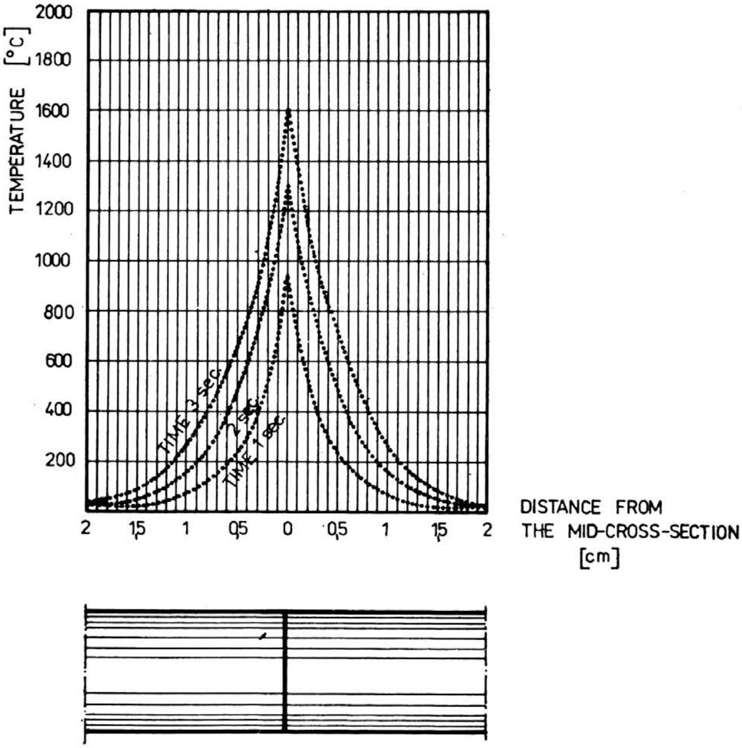

FIG. 4. Temperature distribution along the rod axis.

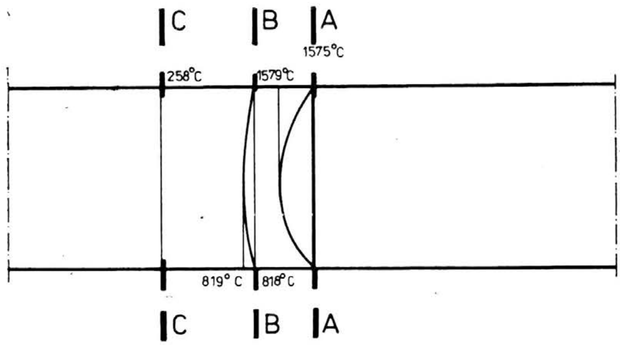

FIG. 5. Temperature distribution along the cross-section diameter (time 3 s);
A—A at the place of contact,
B—B at the distance of 4 mm from A—A,
C—C at the distance of 10 mm from A—A.

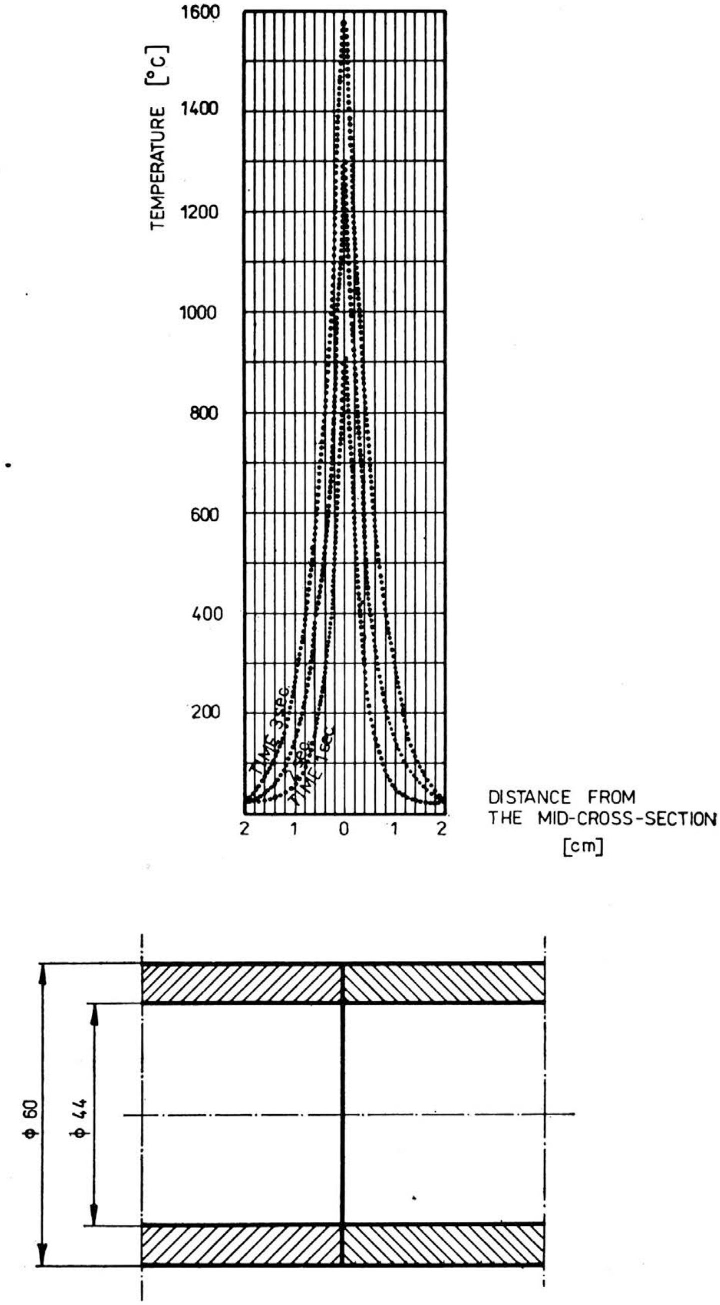

FIG. 6. Temperature distribution along the pipe axis.

Conclusions

1. The finite element method proves once more to be an effective tool in analysing complex problems of nonlinear thermal conductivity. The results obtained clearly indicate that the program can be used for further studies on more advanced problems of heat transfer.

2. The calculations performed confirm the known experimental results [1, 2], concerning the temperature distribution in rods subjected to flash welding. As expected, the cross-sectional temperature variations are small. At the distance of approximately two diameters from the place of contact, the steel temperature equals that of the surrounding air.

3. The method used makes it possible to analyse effectively more complex problems of the flash welding such as welding of different materials and elements of different shapes. Such analysis will be undertaken by the authors in subsequent publications.

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