Archives of Mechanics • Archiwum Mechaniki Stosowanej • 28, 4, pp. 601-606, Warszawa 1976

Recovery creep model for a stationary state of plastic yielding

M. ŻÓRAWSKI (WARSZAWA)

THIS paper is aimed at deriving the theoretical foundations of the process of plastic yielding. This phenomenon is fairly well known in the case of a one-dimensional model and is based on the experimentally confirmed formulae and hypotheses. The paper is based on the general field equations of mobile dislocations; the relations derived are known from experiments concerning plastic yielding. Three-dimensional cases may also be treated in a similar manner.

Praca niniejsza ma za zadanie podanie podstaw teoretycznych dla zjawiska płynięcia plastycznego. Zjawisko to jest dość dobrze opracowane dla modelu jednowymiarowego i opiera się na wzorach i hipotezach potwierdzonych doświadczalnie. W pracy, opierając się na równaniach ogólnych pola ruchomych dyslokacji, otrzymano związki przyjmowane doświadczalnie a odnoszące się do stanu płynięcia plastycznego. Można więc podobnie otrzymać związki dla ciała trójwymiarowego.

Настоящая работа имеет целью приведение теоретических основ для явления пластического течения. Это явление довольно хорошо разработано для одномерной модели и опирается на формулы и гипотезы подтвержденные экспериментально. В работе, базируя на общих уравнениях поля подвижных дислокаций, получены соотношения накладываемые экспериментально и относящиеся к состоянию пластического течения. Итак можно аналогично получить соотношения для трехмерного тела.

THE idea of the model of recovery creep is the assumption that the strength of metals increases along with increasing deformation and decreases with time. If both processes occur simultaneously, the deformation produced is time-dependent and may serve as on adequate description for the processes in the first and second stages of yielding.

The model was originally proposed by BAILE and OROWAN [1, 2] and then developed by various authors [3, 4, 8]; it provides a real, experimentally verified process of yielding. The model is one-dimensional and takes into account the variations of dislocation density occurring during the process.

In the model proposed by MC LEAN [3], the activation energy of the process is expressed by the formulae

$$E = \nu(h\varepsilon - rt), \quad h = \frac{\partial\sigma}{\partial\varepsilon}, \quad r = -\frac{\partial\sigma}{\partial t},$$

$\partial\sigma/\partial\varepsilon$ denoting the hardening parameter and $\partial\sigma/\partial t$ — the creep velocity.

The strain rate may be written as

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp\left(-\frac{E}{kT}\right) \tag{1}$$

$\dot{\varepsilon}_0$ being the strain rate at $t = 0$.

In the state of stationary yielding $\dot{\varepsilon}$ = const and the activation energy is constant, thus both processes of creep and hardening compensate each other. In terms of dislocations we may say that during stationary yielding the dislocation density increases (the mean distance between the dislocation becomes smaller due to the increasing deformations and internal stresses); on the other hand the dislocation density decreases (the mean distance between the dislocation increases). This is connected with stretching of the material itself. The latter process is called the diffusion of dislocations. It should be stressed that the model is characterized by two unknown coefficients connected with the initial strain rate and with the mobility of diffusing dislocations.

This paper aims at demonstrating the theoretical foundations of that model on the basis of the dynamical theory of continuous distribution of dislocations in a continuous medium.

Let us start from the general field equations written in the form

$$\in^{KLM}\in^{PQR}\partial_L\partial_Q \underset{d}{\varepsilon}_{MR}+\frac{1}{2}\left(\in^{KLM}\in^{PRS}\partial_L\Omega_{RSM}+\in^{PLM}\in^{KRS}\partial_L\Omega_{RSM}\right)=0, \tag{2}$$

$$\nabla_L\nabla_M(\underset{d}{\sigma}^{KM}+\underset{p}{\sigma}^{KM}) = \varrho\ddot{U}^K_L, \tag{3}$$

$$\nabla_j(\underset{s}{\sigma}^{ij}+\underset{d}{\sigma}^{ij}) = \varrho\ddot{\underset{s}{u}}^i, \tag{4}$$

$$\nabla_{j'}(\underset{s}{\sigma}^{i'j'}+\underset{p}{\sigma}^{i'j'} = \varrho\ddot{\underset{p}{u}}^{i'} \tag{5}$$

with the following notations:

$\underset{s}{\sigma}^{ij}$ selfstresses (connected with the dislocation density),

$\underset{d}{\sigma}^{KL}$ apparent stresses introduced to the body so as to ensure the compatibility of elements in the relaxed state (in the equilibrium equations they play the role of body forces),

$\underset{p}{\sigma}^{KL}$ stresses due to external loading,

$\underset{d}{\varepsilon}_{KL}$ local (incompatible) strains produced by the dislocation density,

Ω_{KLM} dislocation density,

$\in^{KLM}$ Ricci tensor,

$\underset{s}{u}^i$ displacements of the medium produced by selfstresses (dislocations),

$\underset{p}{u}^i$ displacements produced by external loading,

U^K_L variation of the plastic distortion (of the non-holonomic system) due to external loading.

The equations were formulated in [5]. The essential problem is to figure out the description of a continuous medium defined by a reference frame and, simultaneously, by a non-holonomic base with each deformation of the reference frame deforming the non-holonomic bases and, conversely, each deformation of the bases deforming the frame of reference. To derive the above equations, the way of reasoning as proposed by Kröner [6] was used;

and so was the assumption that for non-holonomic bases the equations of motion (second law of Newtonian mechanics) is satisfied by the force and acceleration increments,

$$d\mathbf{F} = d(m\mathbf{a}).$$

Here, **dF** is the increment of the force acting on the element, and **a** — the acceleration increment of the element; both magnitudes are written in the non-holonomic system.

This assumption yields Eq. (2) which describes the variation of the non-holonomic system during the motion. The entire deformation process takes place in Euclidean space, and thus the curvature tensor vanishes both in the coordinate system and in the non-holonomic system. The aholonomy object Ω_{KLM} describes the dislocation density and hence the disordering of the non-holonomic bases is caused by the existence of dislocations. This description is synonymous with expressing the dislocation density in terms of the torsion tensor. In the case of $\Omega_{KLM} = 0$, with the non-holonomic bases forming a coordinate system, the equations reduce to the classical equations of continuous media.

The dependence of $\underset{d}{\sigma}^{KL}$ on $\underset{d}{\varepsilon}_{KL}$ holds true for the medium in a relaxed state (without any stresses or dislocations), and so it may be assumed — at least in the first phase of deformation — in the linear form

(6)
$$\underset{j}{\sigma}^{KL} = E^{KLMN} \underset{d}{\varepsilon}_{MN}.$$

The one-dimensional state may be obtained by assuming a single component of the strain tensor $\underset{d}{\varepsilon}_{MN}$ to be non-zero and under the condition that the non-holonomic bases are symmetrically deformed, i.e.,

$$\varepsilon = \underset{d}{\varepsilon}_{I\,II} \neq 0 \quad \text{and} \quad \partial_I = \partial_{II}.$$

The dislocation density will be equal to $\Omega = \Omega_{I\,II\,I} = -\Omega_{I\,II\,II} \neq 0$, the remaing components being zero. The latter assumption means that the number of dislocations with Burgers vectors parallel to the axes I and II is the same in each element.

From Eq. (2), we obtain the relation

$$\partial_I \varepsilon = -4\Omega$$

or an equivalent condition

$$\partial_{II} \varepsilon = -4\Omega.$$

If $\underset{d}{\sigma}_{I\,II} = \underset{d}{\sigma} = \varepsilon/\mu$, then by using in Eq. (4) the condition $\partial_1 \underset{d}{\sigma} = -\partial_1 \underset{s}{\sigma} \rightarrow \partial_I \underset{d}{\sigma} = -$ $-\partial_I \underset{s}{\sigma} = -\partial_I \sigma$ we obtain

$$\partial_I \sigma = 4\mu\Omega$$

or

(7)
$$d\sigma = 4\mu\Omega dX,$$

X being a non-holonomic system, dX can not be integrated in the medium. However, by ssuming the non-holonomic base vectors to have the lengths a, a being a multiple of the crystal lattice constant and of the order of magnitude of the dislocations spacing, Eq. (7) may be integrated locally in its base to yield

$$\sigma = 4\mu\Omega a. \tag{8}$$

Here it has been assumed that the dislocations existing in a square of sides a are uniformly distributed over the entire surface. Hence the dislocation density is constant on the surface $a\times a$ what does not mean, however, that it is constant in the whole body. The role of variability of the dislocation density is taken over by the parameter a.

On the other hand, in a square $a\times a$ the relation between Burgers vector and the dislocation density has the form

$$|b^I| = |b^{II}| = b = \Omega dF = \Omega a^2$$

whence

$$a = b^{1/2}\Omega^{-1/2}$$

and so Eq. (8) yields the result

$$\sigma = 4\mu(b\Omega)^{1/2}. \tag{9}$$

It is the experimentally confirmed relation between the self-stresses and the dislocation density. It has been stated before that in a lattice cell of side a, the dislocation density is constant, while the mean distance between the dislocations a may vary with the space variables and time.

The assumption of stationary yielding requires that

$$\underset{s}{\ddot{u}^i} = 0 \quad \text{and} \quad \underset{p}{\ddot{u}^i} = 0.$$

Then, from Eqs. (3)–(5) it follows that

$$-2\partial_M \underset{s}{\sigma}^{KL}{}_{,L} = \varrho \ddot{U}^K_L.$$

By differentiating the above equation, considering its skew-symmetric part and taking into account the relations

$$\partial_{[M}\partial_{P]}\Phi = -\Omega_{PM}{}^s\partial_s\Phi$$

and

$$\partial_{[P}U^K_{M]} = \Omega_{PM}{}^K,$$

we obtain

$$-2\Omega_{PM}{}^s\partial_s(\underset{s}{\sigma}^{KL}{}_{,L}) = \varrho\ddot{\Omega}_{PM}{}^K.$$

Passing to the one-dimensional model, we have

$$-2\Omega\partial_I \underset{s}{\sigma} = \varrho\ddot{\Omega}.$$

By means of Eq. (7) this relation may be written in the form

$$-8\mu\Omega\partial_I\Omega = \varrho\ddot{\Omega}$$

or

$$-\partial_I(\Omega^2) = \frac{\varrho}{4\mu}\ddot{\Omega}. \tag{10}$$

The assumption of plastic yielding for the non-holonomic system yields the conclusion

$$dX^K = \alpha^{(K)} dt$$

what means that the rate of deformation of the non-holonomic lattice is constant in time, i.e. the shear deformation increments are also constant in time.

Equation (10) may then be written in the form

$$(11) \qquad \frac{\partial \Omega}{\partial t} = -4 \frac{\mu}{\varrho \alpha} \Omega^2, \qquad \alpha^{(K)} = \alpha.$$

This equation has been experimentally confirmed, the coefficient $4\mu/\varrho\alpha$ being defined as the coefficient of mobility of dislocations. This relation describes the process of recovery creep.

The increments of selfstresses with time t resulting from the recovery creep as described by Eq. (11) may be calculated from Eq. (9) by differentiation and by applying Eq. (11),

$$(12) \qquad \frac{\partial \sigma}{\partial t} = 4\mu b^{1/2} \frac{1}{2\sqrt{\Omega}} \frac{d\Omega}{dt} = -\frac{8\mu^2 b^{1/2} \Omega^{3/2}}{\varrho \alpha}.$$

Using the relation $a = b^{1/2}\Omega^{-1/2}$ as will as Eq. (11) we obtain a change of dislocation spacing (the cell $a \times a$) in time produced by the diffusion of dislocations. It may be written in the form

$$(13) \qquad \frac{da}{dt} = \frac{2b\mu}{\varrho\alpha} \frac{1}{a}.$$

This relation was also experimentally determined [7] as

$$\frac{da}{dt} = M \frac{\tau}{a},$$

where τ denoted the linear tension of a dislocation line and M — a certain coefficient depending on the dislocation mobility.

The increment of self-stresses produced by the increasing dislocation density is obtained directly from Eq. (9),

$$(14) \qquad \frac{\partial \sigma}{\partial \Omega} = 2\mu b^{1/2} \Omega^{-1/2}$$

and $\partial\sigma/\partial\varepsilon$ is found to have the form

$$\frac{\partial \sigma}{\partial \varepsilon} = \frac{\partial \sigma}{\partial \Omega} \frac{\partial \Omega}{\partial U} \frac{\partial U}{\partial \varepsilon}.$$

Here

$$(15) \qquad \begin{gathered} \varepsilon = \frac{1}{2}(U_{12} + U_{21}) = U, \qquad U_{12} = U_{21} = U, \\ \Omega_{I\,II}^{\;I} = \Omega = \partial_I U_{II}^I - \partial_{II} U_I^I = \partial_I U, \qquad U_I^I = U_{II}^{II} = 0. \end{gathered}$$

Equations (1), (12), (14), (15) constitute a complete set of equations describing the process of plastic yielding.

The corresponding three-dimensional model would be much more complicated, nevertheless the general procedure should be the same as in the case considered here (general field equations constituting the basis of considerations), and the general field equations seem to describe properly the phenomenon of yielding.

This paper was aimed, however, at demonstrating the theoretical foundations of the plastic yielding model, and thus, our considerations have been confined to such relations which could be experimentally verified, i.e., to one-dimensional processes.

References

1. R. W. Bailey, J. Inst. Metals, **35**, 27, 1926.
2. E. Orowan, J. West Scotland Iron Steel Inst., **54**, 45, 1946–1947.
3. C. McLean, Rep. Progress Physics, **29**, 1, 1966.
4. S. K. Mitra, D. McLean, Proc. Roy. Soc., **A295**, 288, 1966.
5. M. Zórawski, Théorie mathématique des dislocations, Dunod, Paris 1967.
6. E. Kröner, Arch. Rat. Mech. Anal., 1960.
7. J. Friedel, Dislocations, Pergamon Press, 1964.
8. R. Lagneborg, Metal Sci. J., **3**, 161, 1969.

POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.

Received October 3, 1975.