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Low-frequency transonic flows past a thin airfoil

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LOW-FREQUENCY transonic flows past oscillating thin airfoils are considered. Three methods of analysis: 1) method of perturbations, 2) method of local mean invariation, 3) method of reduction to a constant-coefficient equation, are developed and illustrated.

Rozważono nisko-częstotliwościowy opływ przydźwiękowy cienkich płatów. Rozwinięto i zastosowano trzy metody analizy problemu: 1) metodę perturbacji, 2) metodę lokalnej inwariacji i 3) metodę redukcji do równań różniczkowych ze stałymi współczynnikami.

Рассмотрено низкочастотное околозвуковое обтекание тонких крыльев. Развиты и применены три метода анализа проблемы: 1) метод пертурбаций, 2) метод локальной инварианции и 3) метод сведения к дифференциальным уравнениям с постоянными коэффициентами.

1. Introduction

ONE AREA of applicability of unsteady transonic flow is the possibility of transonic supercritical airfoils with which a shockless mixed subsonic and supersonic flow may be realised. A few versions of the original "local linearisation" concept have been given by STAHARA and SPREITER [1], DOWELL [2], among others, to treat unsteady flows. In the present paper three methods of analysis:

1) method of perturbations,
2) method of local mean invariation,
3) method of reduction to a constant-coefficient equation, are developed and illustrated to treat a harmonically oscillating thin airfoil in transonic flow. The restriction is made to the low-frequency limit which is of relevance, for instance, in one-degree-of-freedom torsional flutter instability. Since the experimental works of NIEWLAND [3] and HOLDER [4] revealed the existence of shock-free transonic flows, in the following a shock-free case is treated.

2. Statement of the problem

For simplicity wing sections that are symmetric about the x-axis are considered. One has for two-dimensional potential flows past a thin airfoil

$$\varepsilon(\Phi_{\tilde{z}\tilde{z}}-\beta^2\Phi_{xx}-2M_\infty^2\Phi_{x\tilde{t}}-\varepsilon M_\infty^2\Phi_{\tilde{t}\tilde{t}}) = M_\infty^2[(\gamma-1)(\Phi_x+\varepsilon\Phi_t)(\Phi_{xx}+\varepsilon\Phi_{\tilde{z}\tilde{z}}) + 2\Phi_x\Phi_{xx}+2\varepsilon\Phi_{\tilde{z}}\Phi_{x\tilde{z}}+2(\varepsilon\Phi_x\Phi_{x\tilde{t}}+\varepsilon^2\Phi_{\tilde{z}}\Phi_{\tilde{z}\tilde{t}})], \tag{2.1}$$

$$z = f(x, \tilde{t}): \frac{\sqrt{\varepsilon}\,\Phi_{\tilde{z}}}{1+\Phi_x} = \left(\frac{\partial}{\partial x} + \varepsilon \frac{\partial}{\partial \tilde{t}}\right) f(x, \tilde{t}); \tag{2.2}$$

upstream:

$$\Phi_x, \Phi_{\tilde{z}} \to 0, \tag{2.3}$$

where

$$\tilde{z} = \sqrt{\varepsilon}\, z, \quad \tilde{t} = \varepsilon t, \quad \beta^2 = \frac{M_\infty^2 - 1}{\varepsilon} = 0(1), \quad \varepsilon \ll 1,$$

Φ is the velocity potential, M_∞ the free-stream Mach number, γ the ratio of specific heats, and $z = f(x, \tilde{t})$ the airfoil surface. All quantities have been nondimensionalised using the chord length c, and free-stream velocity V_∞. The spatial scalings are motivated by the fact that in the limit $M_\infty \to 1$, the linearised supersonic-flow theory (LIEPMANN and PUCKETT [5]) indicates that the disturbances are propagated almost undiminished to infinity in the z-direction but are confined to a small width in the x-direction. The physical significance of ε will become apparent when it is related to the thickness ratio τ.

The pressure coefficient is given by

$$C_p = -2\Phi_x - 2\varepsilon\Phi_{\tilde{t}} + \varepsilon\beta^2\Phi_x^2 - \varepsilon\Phi_{\tilde{z}}^2 + 2\varepsilon M_\infty^2 \Phi_x \Phi_{\tilde{t}} + M_\infty^2 \varepsilon^2 \Phi_{\tilde{t}}^2. \tag{2.4}$$

3. Method of perturbations

Let

$$f(x, z, \tilde{t}) = \varepsilon^2 g(x) + \varepsilon^{5/2} h(x) e^{ik\tilde{t}}. \tag{3.1}$$

Seek a solution of the form

$$\Phi(x, \tilde{z}, \tilde{t}; \varepsilon) = \varepsilon^{3/2}\hat{\phi}(x, \tilde{z}) + [\varepsilon^2 \phi(x, \tilde{z}) + \varepsilon^{5/2}\phi'(x, \tilde{z})] e^{ik\tilde{t}} + o(\varepsilon^{5/2}). \tag{3.2}$$

Then, from Eqs. (2.1), (2.2), and (2.4), one obtains

$$\begin{gathered} \hat{\phi}_{\tilde{z}\tilde{z}} - \beta^2 \hat{\phi}_{xx} = 0, \\ \tilde{z} = 0: \hat{\phi}_{\tilde{z}} = g_x; \end{gathered} \tag{3.3}$$

$$\begin{gathered} \phi_{\tilde{z}\tilde{z}} - \beta^2 \phi_{xx} - 2M_\infty^2 ik\phi_x = 0, \\ \tilde{z} = 0: \phi_{\tilde{z}} = h_x; \end{gathered} \tag{3.4}$$

$$\begin{gathered} \phi'_{\tilde{z}\tilde{z}} - \beta^2 \phi'_{xx} - 2M_\infty^2 ik\phi'_x = (\gamma+1) M_\infty^2 (\hat{\phi}_x \phi_x)_x, \\ \tilde{z} = 0: \phi'_{\tilde{z}} = -\hat{\phi}_x h_x - \phi_x g_x; \end{gathered} \tag{3.5}$$

$$C_p = \varepsilon^{3/2}(-2\hat{\phi}_x) + \varepsilon^2(-2\phi_x) e^{ik\tilde{t}} + \varepsilon^{5/2}(-2\phi'_x) e^{ik\tilde{t}} + o(\varepsilon^{5/2}). \tag{3.6}$$

From Eq. (3.3)

$$\hat{\phi}(x, \tilde{z}) = -\frac{1}{\beta} g(x - \beta\tilde{z}). \tag{3.7}$$

From Eq. (3.4)

$$(3.8)\qquad \phi(x,\tilde{z}) = -\frac{1}{\beta}\int\limits_0^{x-\beta\tilde{z}} h_{x'}(x')e^{-i\frac{M_\infty^2 k}{\beta^2}(x-x')} J_0\left[\frac{M_\infty^2 k}{\beta^2}\sqrt{(x-x')^2-\beta^2\tilde{z}^2}\right]dx',$$

J_n being the Bessel function of order n.

Putting

$$(3.9)\qquad \phi'(x,\tilde{z}) = e^{-i\frac{M_\infty^2 k}{\beta^2}x}\tilde{\phi}'(x,\tilde{z})$$

and taking the Laplace transform

$$(310)\qquad \mathscr{L}[\tilde{\phi}'] \equiv \bar{\tilde{\phi}}' \equiv \int\limits_0^\infty e^{-sx}\tilde{\phi}'(x,\tilde{z})dx,$$

Eq. (3.5) becomes

$$(3.11)\qquad \bar{\tilde{\phi}}'_{\tilde{z}\tilde{z}}-\beta^2 s^2\bar{\tilde{\phi}}' = -\frac{(\gamma+1)}{\beta}M_\infty^2 s e^{-\beta s\tilde{z}}\mathscr{L}[g_x\phi_x],$$

from which,

$$(3.12)\qquad \tilde{\phi}'(s,\tilde{z}) = A(s)e^{-\beta s\tilde{z}}+\left[\frac{\gamma+1}{2\beta^2}M_\infty^2(g_x\phi_x)\right]\tilde{z}e^{-\beta s\tilde{z}}.$$

Upon inverting

$$(3.13)\qquad \tilde{\phi}'(x,\tilde{z}) = A(x-\beta\tilde{z})+\frac{\gamma+1}{2\beta^2}M_\infty^2\tilde{z}[g(x-\beta\tilde{z})]_x[\phi(x-\beta\tilde{z},0)]_x.$$

Using Eq. (3.5), one obtains

$$(3.14)\qquad \tilde{\phi}'(x,\tilde{z}) = -\frac{2}{\beta^2}e^{-i\frac{M_\infty^2 k}{\beta^2}x}\left[-\int\limits_0^{x-\beta\tilde{z}}\beta\left(e^{i\frac{M_\infty^2 k}{\beta^2}\xi}+\frac{\gamma+1}{4\beta^2}M_\infty^2\right)g_\xi\phi_\xi d\xi\right.$$
$$\left.-\frac{\gamma+1}{4}M_\infty^2\tilde{z}\{g(x-\beta\tilde{z})\}_x\{\phi(x-\beta\tilde{z},0)\}_x\right].$$

The pressure coefficient on the airfoil is given by

$$(3.15)\qquad C_p = \varepsilon^{3/2}\left(2\frac{g_x}{\beta}\right)-\varepsilon^2(2\phi_x)e^{ik\tilde{t}}+\varepsilon^{5/2}\left[\frac{4iM_\infty^2 k}{\beta^2}e^{-i\frac{M_\infty^2 k}{\beta^2}x}\int\limits_0^x\left(e^{i\frac{M_\infty^2 k}{\beta^2}\xi}\right.\right.$$
$$\left.\left.+\frac{\gamma+1}{4\beta^2}M_\infty^2\right)g_\xi\phi_\xi d\zeta-\frac{4}{\beta}\left(1+\frac{\gamma+1}{4\beta^2}M_\infty^2 e^{i\frac{M_\infty^2\beta}{\beta^2}x}\right)g_x\phi_x\right]+o(\varepsilon^{5/2}).$$

Consider a parabolic arc airfoil in pitching oscillation so that

$$(3.16)\qquad g(x) = x(1-x),\quad h(x) = x$$

one obtains for the lift coefficient

$$C_l = 4\frac{\varepsilon^2}{\beta} e^{ik\tilde{t}} \int_0^1 e^{-i\frac{M_\infty^2 k}{\beta^2}(1-x')} J_0\left[\frac{M_\infty^2 k}{\beta^2}(1-x')\right] dx' \tag{3.17}$$

$$+\varepsilon^{5/2}\frac{8}{\beta^2} e^{ik\tilde{t}} \int_0^1 \left[1+e^{i\frac{M_\infty^2 k}{\beta^2}(\xi-1)} + \frac{\gamma+1}{4\beta^2} M_\infty^2 \left(e^{-i\frac{M_\infty^2 k}{\beta^2}\xi} + e^{-\frac{M_\infty^2 k}{\beta^2}}\right)\right]$$

$$\times\left[1+\int_0^\xi \left(-iJ_0\left\{\frac{M_\infty^2 k}{\beta^2}(\xi-x')\right\} + J_1\left\{\frac{M_\infty^2 k}{\beta^2}(\xi-x')\right\}\frac{M_\infty^2 k}{\beta^2} e^{-i\frac{M_\infty^2 k}{\beta^2}(\xi-x')} dx\right]\right.$$

$$\times(1-2\xi)d\xi + o(\varepsilon^{5/2}).$$

VAN DYKE [6] gave a calculation for the supersonic counterpart of this problem where M_∞ is fixed and $\tau \to 0$, i.e. β is large. The present result is shown along with that of Van Dyke in Fig. 1.

The present result, as well as that of Van Dyke, breaks down for $M_\infty = 1$. Next, therefore, methods that are capable of yielding better results at $M_\infty = 1$ are developed.

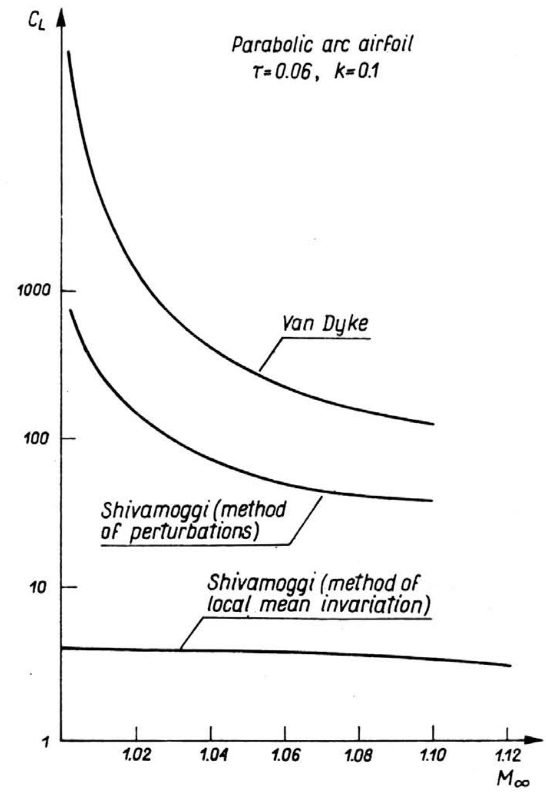

FIG. 1.

4. Method of local mean invariation

Let

$$f(x, \tilde{t}) = \varepsilon^{3/2} g(x) + \varepsilon^{5/2} h(x) e^{ik\tilde{t}}; \tag{4.1}$$

seek a solution of the form

$$\Phi(x, \tilde{z}, \tilde{t}, \varepsilon) = \varepsilon \hat{\phi}(x, \tilde{z}) + \varepsilon^2 \phi(x, \tilde{z}) e^{ik\tilde{t}} + o(\varepsilon^2); \tag{4.2}$$

then, one obtains from Eqs. (2.1), (2.2), and (2.3)

$$\begin{gathered} \hat{\phi}_{\tilde{z}\tilde{z}} - \beta^2 \hat{\phi}_{xx} - M_\infty^2 (\gamma+1) \hat{\phi}_x \hat{\phi}_{xx} = 0, \\ \tilde{z} = 0\colon\ \hat{\phi}_{\tilde{z}} = g_x; \end{gathered} \tag{4.3}$$

$$\begin{gathered} \phi_{\tilde{z}\tilde{z}} - \beta^2 \phi_{xx} - 2M_\infty^2 ik\phi_x - M_\infty^2 (\gamma+1)(\hat{\phi}_x \phi_x)_x = 0, \\ \tilde{z} = 0\colon\ \phi_{\tilde{z}} = h_x; \end{gathered} \tag{4.4}$$

$$C_p = \varepsilon(-2\hat{\phi}_x) + \varepsilon^2(-2\phi_x) e^{ik\tilde{t}} + o(\varepsilon^2). \tag{4.5}$$

Equation (4.3) is solved by using the method of local linearisation due to SPREITER and ALKSNE [7]. In this method $\hat{\phi}_{xx}$ is held constant in the nonlinear term only until a formal solution is obtained, but $\hat{\phi}_{xx}$ is then allowed to vary with x.

Let,

$$\lambda^2 = \beta^2 + M_\infty^2 (\gamma+1) \hat{\phi}_x > 0 \tag{4.6}$$

so that, from Eq. (4.3)

$$-\lambda^2 \hat{\phi}_{xx} + \hat{\phi}_{\tilde{z}\tilde{z}} = 0. \tag{4.7}$$

Now one assumes that λ varies slowly enough so that it can be considered as a constant in the initial stages of the calculations. Thus one obtains from Eq. (4.7)

$$\hat{\phi}_x(x, 0) = -\frac{g_x}{\lambda} \tag{4.8}$$

from which

$$\hat{\phi}_{xx}(x, 0) = -\frac{g_{xx}}{\lambda}, \tag{4.9}$$

so that upon replacing λ by Eq. (4.6) and solving for $\hat{\phi}_x$,

$$\hat{\phi}_x = -\frac{1}{M_\infty^2(\gamma+1)} \left\{ \beta^3 - \left[-\frac{3M_\infty^2(\gamma+1) g_x}{2} + C \right]^{2/3} \right\}, \tag{4.10}$$

where C is a constant of integration. In order to determine C, let

$$x = 0\colon\ \hat{\phi}_x = 0. \tag{4.11}$$

This amounts to modelling the steady flow as shown in Fig. 2. The subsonic region ahead of the sonic point poses difficulties in analytical approaches, and similar steps have been taken by STAHARA and SPREITER [1], DOWEL [2] to get around this difficulty.

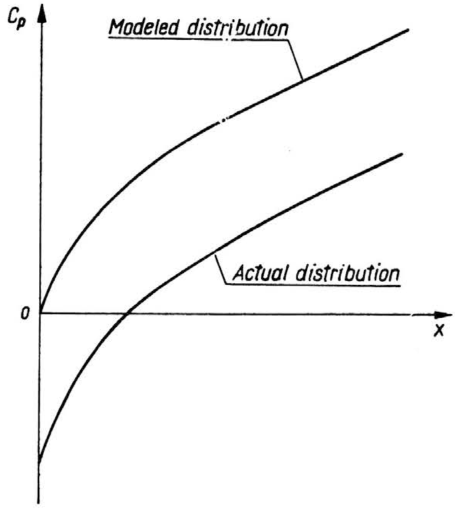

FIG. 2.

In analogy with the local linearisation hypothesis, a local mean invariation hypothesis is now introduced to solve Eq. (4.4). Thus Eq. (4.4) is replaced in a small region by an equation with constant coefficients, and then introducing for the latter different values for different points in the field. Mathematically, $\hat{\phi}_x$ and $\hat{\phi}_{xx}$ are considered constant until a formal solution for ϕ is obtained, but are then allowed to vary with x. Hence one obtains

$$\phi = -\frac{1}{\sqrt{\beta^2+M_\infty^2(\gamma+1)\hat{\phi}_x}} \int\limits_0^{x-\sqrt{\beta^2+M_\infty^2(\gamma+1)\hat{\phi}_x}\tilde{z}} dx' h_{x'}(x') e^{-i\zeta(x-x')} J_0[\xi(x-x')], \tag{4.12}$$

where

$$\zeta(x) = \frac{M_\infty^2\left[k-\frac{i(\gamma+1)}{2}\right]\hat{\phi}_{xx}}{\beta^2+M_\infty^2(\gamma+1)\hat{\phi}_x}\, x,$$

$$\xi(x) = \frac{M^2\left[k-\frac{i(\gamma+1)}{2}\right]\hat{\phi}_{xx}}{\beta^2+M_\infty^2(\gamma+1)\hat{\phi}_x}\sqrt{x^2-\{\beta+M_\infty^2(\gamma+1)\hat{\phi}_x\}\tilde{z}^2}\,.$$

Using Eqs. (4.5) and (4.12), one obtains for a parabolic-arc airfoil in pitching oscillations, Eq. (3.16),

$$C_L = \frac{4\varepsilon^2}{[\beta^2+3M_\infty^2(\gamma+1)]^{1/3}} \int\limits_0^1 e^{-i\eta(1-x')} J_0[\eta(1-x')]dx', \tag{4.13}$$

where

$$\eta(y) = \frac{M_\infty^2[k-i(\gamma+1)\{3M_\infty^2(\gamma+1)y+\beta^3\}^{-1/3}}{[3M_\infty^2(\gamma+1)y+\beta^3]^{2/3}}\, y.$$

The present result is shown Fig. 1 as C_L vs. M_∞ for $k = 0.1$, $\tau = 0.06$. In Fig. 3 the present result C_L vs. k for $M_\infty = 1$, $\tau = 0.06$ is shown along with those of the linearised supersonic-flow theory, and DOWELL [2].

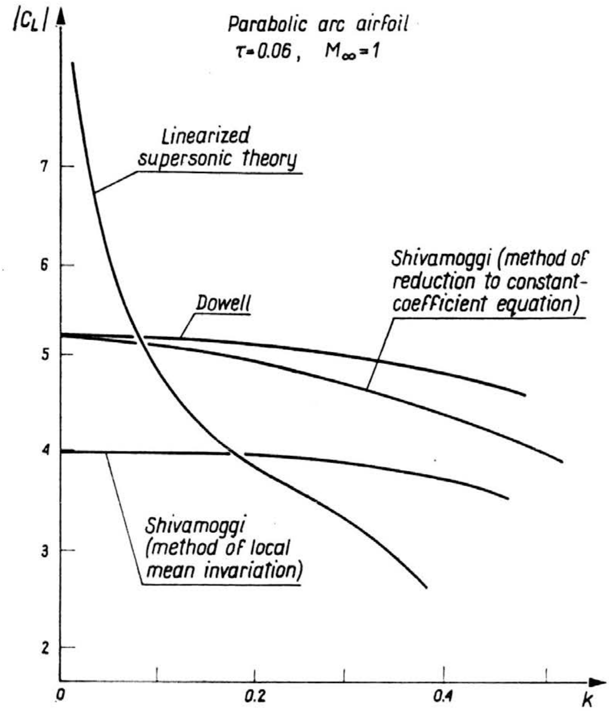

FIG. 3.

5. Method of reduction to a constant-coefficient equation

Putting

$$\Phi = \varepsilon\tilde{\phi} \tag{5.1}$$

in Eq. (2.1), one obtains to $O(\varepsilon)$

$$\{\beta^2 + M_\infty^2(\gamma+1)\tilde{\phi}_x\}\tilde{\phi}_{xx} - \tilde{\phi}_{\tilde{z}\tilde{z}} + 2M_\infty^2\tilde{\phi}_{x\tilde{t}} = 0. \tag{5.2}$$

Reverting to the unstretched variables

$$\{(M_\infty^2 - 1) + M_\infty^2(\gamma+1)\Phi_x\}\Phi_{xx} - \Phi_{zz} + 2M_\infty^2\Phi_{xt} = 0. \tag{5.3}$$

Let

$$f(x, t) = g(x) + h(x)e^{ikt}, \quad k \ll 1, \tag{5.4}$$

$$\Phi(x, z, t) = \hat{\phi}(x, z) + \phi(x, z)e^{ikt} \tag{5.5}$$

so that upon neglecting the higher harmonics, one obtains

$$(5.6)\qquad (1-M_\infty^2)\hat{\phi}_{xx}+\hat{\phi}_{zz}-M_\infty^2(\gamma+1)\hat{\phi}_x\hat{\phi}_{xx} = 0, \quad z = 0:\ \hat{\phi}_z = g_x;$$

$$(5.7)\qquad (1-M_\infty^2)\phi_{xx}+\phi_{zz}-2M_\infty^2 ik\phi_x = M_\infty^2(\gamma+1)(\hat{\phi}_x\phi_x)_x, \quad z = 0:\ \phi_z = h_x.$$

It turns out in the following that the solution to Eq. (5.6), given by Eq. (4.10), is not proper to use for $M_\infty = 1$. Note from Eq. (4.9) that, for $M_\infty = 1$, $\hat{\phi}_{xx}$ is infinite at a point where $\hat{\phi}_x = 0$ (sonic point), if g_{xx} does not vanish rapidly enough at the same point (and for the parabolic-arc airforl it does not). This did not pose any problem in the evaluation of Eq. (4.12) but a different method of constructing a solution to Eq. (5.6) is necessary if a physically realistic value for $\hat{\phi}_{xx}$ at the sonic point is needed per se.

For $M_\infty = 1$, following SPREITER and ALKSENE [7], let

$$(5.8)\qquad \lambda^2 = (\gamma+1)\hat{\phi}_{xx} > 0$$

so that from Eq. (5.6)

$$(5.9)\qquad \hat{\phi}_{zz}-\lambda^2\hat{\phi}_x = 0.$$

Again one assumes that λ varies slowly enough so that it can be considered as a constant in the initial stages of the calculations. Thus one obtains from Eq. (5.9)

$$(5.10)\qquad \hat{\phi}_x(x,0) = -\frac{1}{\sqrt{\pi}\,\lambda}\left[\frac{d}{dx}\int_0^x \frac{g_\xi}{\sqrt{x-\xi}}\,d\xi\right].$$

Upon replacing λ by Eq. (5.8) and solving for $\hat{\phi}_x$,

$$(5.11)\qquad \hat{\phi}_x = \left[\frac{3}{\pi(\gamma+1)}\int_{x^*}^{x}\left\{\frac{d}{dx}\int_0^{x'}\frac{g_\xi}{\sqrt{x-\xi}}\,d\xi\right\}^2 dx'\right]^{1/3},$$

where

$$x = x^*:\ \hat{\phi}_x = 0 \quad \text{or} \quad \frac{d}{dx}\int_0^x \frac{g_\xi}{\sqrt{x-\xi}}\,d\xi = 0.$$

For a parabolic-arc airfoil, Eq. (3.16), this gives

$$(5.12)\qquad x^* = \frac{1}{4}.$$

Using Eqs. (3.16) and (5.12), one obtains

$$(5.13)\qquad \phi_x = \left[\frac{3}{\pi(\gamma+1)}\left(\ln 4x-8x+8x^2+\frac{3}{2}\right)\right]^{1/3}.$$

In order to solve Eq. (5.7), let

$$(5.14)\qquad \phi(x,z) = \psi(x,z)+g(x,z),$$

where

$$(5.15) \qquad (1-M_\infty^2)\psi_{xx}+\psi_{zz}+-2M_\infty^2 ik\psi_x = {}^k\psi_x$$

so that

$$(5.16) \qquad g_{zz} = \frac{\partial}{\partial x}[\{(M_\infty^2-1)+M_\infty^2(\gamma+1)\hat{\phi}_x\}g_x + 2M_\infty^2 ikg]+[M_\infty^2(\gamma+1)(\hat{\phi}_x\psi_x)_x-{}^k\psi_x].$$

Near the surface of the airfoil, as a first approximation, one drops the term on the left hand side (a similar step has been taken by HOSOKAWA [8] for the steady problem) so that one obtains

$$(5.17) \qquad g_x = -\frac{\int\limits_{x^*}^{x} a(x')dx'}{b(x)} + \frac{2ik/(\gamma+1)}{b(x)}\int\limits_{x^*}^{x}\frac{\int\limits_{x^*}^{x} a(x'')dx''}{b(x')} e^{\int\limits_{x}^{x'}\frac{2ik/(\gamma+1)}{b(x'')}dx''}dx',$$

where

$$a(x) = (\hat{\phi}_x\psi_x)_x - \frac{K}{M_\infty^2(\gamma+1)}\psi_x,$$

$$b(x) = \hat{\phi}_x - \frac{1-M_\infty^2}{M^2(\gamma+1)}.$$

If,

$$(5.18) \qquad x = x_1:\ b(x) = 0,$$

g_x will not be well-behaved unless $x_1 = x^*$, and $a(x^*) = 0$.

Thus

$$(5.19) \qquad K = \frac{M_\infty^2(\gamma+1)}{\psi_x|_{x=x^*}}[(\hat{\phi}_x\psi_x)_x|_{x=x^*}],$$

$$(5.20) \qquad x = x^*:\ \phi_x = \psi_x,$$

i.e. the sonic point corresponding to the steady flow is the same for ψ as for ϕ. Therefore ψ would be a valid approximation in the neighbourhood of $x = x^*$, at least on the airfoil surface.

Note that for $M_\infty = 1$,

$$(5.21) \qquad K = (\gamma+1)\hat{\phi}_{xx}|_{x=x^*},$$

which is the same as that obtained by HOSOKAWA [8] for the steady case! Earlier MAEDER and THOMMEN [9] had advocated such a choice for the steady case on grounds of better correlation with experimental results.

For $M_\infty = 1$, Eq. (5.15) gives

$$(5.22) \qquad \psi_x = \frac{1}{\sqrt{\pi(K+2ik)}}\frac{d}{dx}\left[\int\limits_0^x \frac{h_\xi}{\sqrt{x-\xi}}d\xi\right].$$

For a pitching oscillation, $h(x) = x$, this gives

(5.23)
$$\psi_x = \frac{1}{\sqrt{\pi(K+2ik)}}\frac{1}{\sqrt{x}}$$

and

(5.24)
$$C_p = -2\psi_x,$$

(5.25)
$$\Delta C_p = C_p^+ - C_p^- = 2C_p = -4\psi_x,$$

(5.26)
$$C_L = \int_0^1 \Delta C_p dx = \frac{8}{\sqrt{\pi(K+2ik)}}.$$

The present results — C_L vs. k, for $M_\infty = 1$, $\tau = 0.06$, are shown, respectively, in Figs 3. and 4, along with those of STAHARA and SPREITER [1], DOWEL [2]. Again the present result, being adequate for low frequencies, shows departures at high frequencies from that due to DOWELL [2].

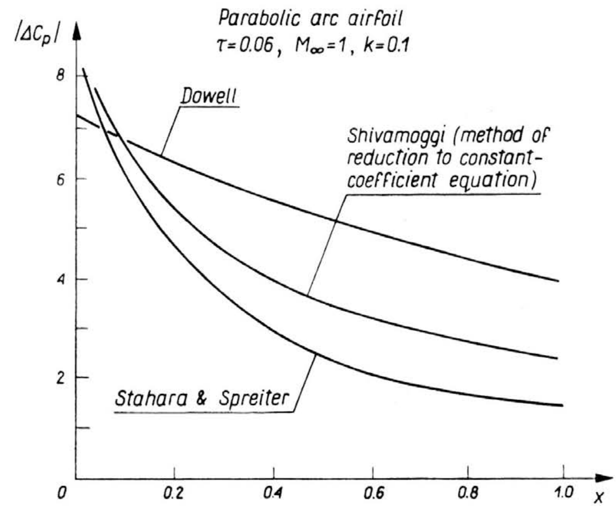

FIG. 4.

For a Guderley airfoil

(5.27)
$$\phi_x = 0.37(x-0.4)$$

so that one obtains, upon including the correction term g_x this time,

(5.28)
$$C_p = -2(\psi_x+g_x) = \frac{1/\sqrt{4}}{\sqrt{(K+2ik)}}\left[\frac{2}{1+\sqrt{\bar{x}}}\right.$$
$$\left. = \frac{|\bar{x}-1|^{-\beta}}{(\bar{x}-1)}\int_0^{\bar{x}-1} dz|z|^\beta\left\{\frac{(1+z)^{-1/2}-1}{(1+z)^{1/2}+1}\right\}\right],$$

where

$$\bar{x} = \frac{x}{0.4}, \quad \beta = \frac{2ik/0.37}{\gamma+1}.$$

The present result $|C_p|$ vs. x, for $M_\infty = 1$, $k = 0.5$, $\tau = 0.06$ is shown in Fig. 5 along with that of STAHARA and SPREITER [1].

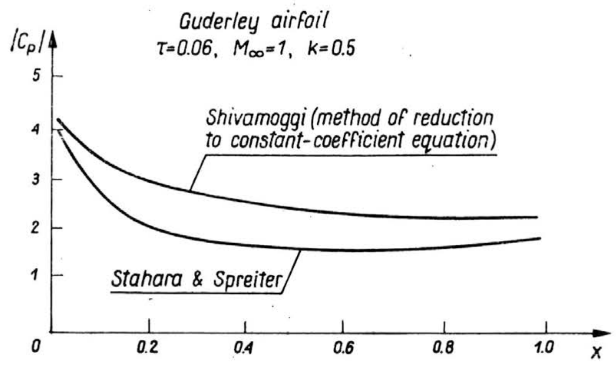

FIG. 5.

6. Conclusions

The method of perturbations gives better results than the supersonic theory due to VAN DYKE [6], but it fails at $M_\infty = 1$ like the latter. The method of local mean invariation and the method of reduction to a constant-coefficient equation, on the other hand, yield meaningful results at $M_\infty = 1$. The points in favour of the method of reduction to a constant-coefficient equation have

(i) a more sound rationale,
(ii) simplicity (only a slide-rule calculation required),
(iii) greater accuracy.

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