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Peristaltic flow of non-Newtonian fluids containing small spherical particles(*)

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We consider the two-dimensional plane peristaltic flow of an incompressible non-Newtonian fluid containing solid spherical particles. Furthermore it is assumed that the wavelength of the peristaltic wave is large compared with the channel width, and the appropriate Reynolds number is small compared with unity. The non-Newtonian fluid is characterized by a power law model which describes dilatant and pseudoplastic fluid behaviour. The inertia-free partial differential equation system has been solved analytically. Velocity distributions of the fluid phase and particulate phase as well as pressure-volume flow relationships are given. The results are compared with the case of a single-phase fluid and it is seen that the various parameter have an influence on the velocity distributions and on the pressure-flow relationships.

Rozważamy dwuwymiarowy, płaski przepływ perystaltyczny nieściśliwego płynu nienewtonowskiego zawierającego stałe cząsteczki kuliste. Założono, że długość fali perystaltycznej jest duża w porównaniu z szerokością kanału, a odpowiednia liczba Reynoldsa jest dużo mniejsza od jedności. Płyn nienewtonowski opisany jest modelem potęgowym uwzględniającym pseudoplastyczne zachowanie się płynu. Układ równań różniczkowych cząstkowych nie uwzględniający sił bezwładności rozwiązano analitycznie. Podano rozkłady prędkości fazy ciekłej i fazy cząsteczkowej. Wyniki porównano z przypadkiem płynu jednofazowego i pokazano, że różne parametry mają wpływ na rozkłady prędkości i na zależność przepływu od ciśnienia.

Рассматриваем двумерное, плоское перистальтическое течение несжимаемой неньютоновской жидкости, содержавшей твердые сферические частицы. Предположено, что длина перистальтической волны является большой по сравнению с шириной канала, а соответствующее число Рейнольдса много меньше чем единица. Неньютоновская жидкость описана степенной моделью, учитывающей псевдопластическое поведение жидкости. Система дифференциальных уравнений в частных производных, не учитывающая сил инерции, решена аналитически. Приведены распределения скоростей жидкой фазы и фазы частиц. Результаты сравнены со случаем однофазной жидкости и показано, что разные параметры имеют влияние на распределения скорости и на зависимость течения от давления.

1. Introduction

In general, peristaltic pumping is characterized by the dynamic interaction of fluid flow with the movement of a flexible boundary. The dynamics of fluid transport by peristaltic motion of the confining walls has received careful study in the recent literature in both, the mechanical and physiological sciences. Peristaltic motion is one of the major mechanisms for fluid transport in many biological systems. Peristaltic pumping is the common mechanism for urine transport from kidney to bladder, food mixing and motility in the intestine, ejection of semen in male reproductive organs, and egg transport in female

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fallopian tubes. Technical roller and finger pumps using viscous fluids also operate according to this principle.

The fluid mechanics of peristaltic pumping have been studied for several years and reviews of some of the literature are presented by JAFFRIN and SHAPIRO [1] and RATH [2]. Most investigations about peristalsis refer to the human ureter [3–6]. In [7] the authors investigated a peristaltic flow through nonuniform channels. Most of the studies on peristaltic motion assumed Newtonian fluids of constant viscosity. HUNG and BROWN [8, 9] carried out an extensive experimental investigation to understand the mechanics of solid particle transport by peristaltic motion and they compared their computational two-dimensional flow model with the experimental data. KAIMAL [10] studied the peristaltic pumping of suspension of rigid particles in an Newtonian-liquid in a tube of arbitrary wave shape. Two-phase flow analysis without peristaltic wall contractions has been studied extensively by many authors [11], [12], [14]. The general study of solid particles-fluid flow systems is of importance in many fields, for example in biological fluid flow, in sediment transport and in chemical processing.

Most of all investigations on peristaltic flow are restricted to Newtonian fluids. However, RAJU and DEVANATHAN [13] studied a single-phase-peristaltic flow of a non-Newtonian fluid, considering blood as a power law fluid and they obtained the solution for the stream function as a power series in terms of the amplitude of deformation and evaluated the stream function and velocity components numerically. BECKER [15] studied a peristaltic flow of simple non-Netwonian fluids. No experimental or theoretical papers on the peristaltic flow of non-Netwonian two-phase mixtures have been published. On the other hand most of the liquids used by peristaltic pumping have non-Netwonian fluid behaviour. And in physiology as well as in chemical engineering there are a lot of flow problems which are connected with a peristaltic flow of non-Newtonian fluids containing small spherical particles.

The purpose of the present paper is to investigate a peristaltic flow of a non-Newtonian fluid containing small rigid spherical particles. The non-Newtonian fluid should be characterized by a power law model which describes dilatant, pseudoplastic and also Newtonian fluid behaviour. For our theoretical investigations it is assumed that the wavelength of the peristaltic wave is large compared with the mean half width of the channel. Furthermore we assume that the appropriate Reynolds number is small compared with unity. It should be noted that for our model arbitrary wave shapes are possible. This study should be helpful in regard to both, peristaltic pumping in physiology and to the engineering application of pumping solid-non-Newotnian fluid mixtures by peristalsis.

2. Mathematical formulation of the problem

We consider the peristaltic flow of an incompressible non-Newtonian fluid with distributed small rigid particles which are neutrally buoyant. The particles themselves, however will be assumed to consist either of a solid material or of drops or bubbles of a fluid immiscible with the suspending medium. The spherical particle size will also be assumed throughout to be very much smaller than the width of the channel and furthermore we

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neglect the Brownian motion and the lift due to the rotation of the particles. It is considered that the length of the peristaltic wave is large compared with the channel width and the appropriate Reynolds number is sufficiently small for the flow to be considered inertia-free. We consider a symmetric channel with flexible walls, where the shape of the walls is determined by the arbitrary function of the travelling peristaltic wave. We choose the non-Newtonian fluid of a power law model which represents dilatant, Newtonian and pseudoplastic fluids and we consider a two-dimensional laminar flow of the two-phase mixture.

In general arbitrary wave shapes are possible, but at the moment we assume that the travelling waves are represented by an infinite sinusodial wave train function $h(x, t)$

(2.1) $$h(x, t) = a+b\sin[2\pi/\lambda(x-ct)],$$

where t is the time, a is the mean half width of the channel, b is the amplitude and λ the wavelength of the peristaltic wave. The constant wave speed is denoted by c. The x, y-coordinates are fixed in space and so they represent the laboratory frame of reference. The corresponding velocity components of the fluid velocity in the x and y-direction, measured in the laboratory frame are u and v, and the corresponding velocities of the particulate phase are u_p and v_p.

Now we introduce the relative wave frame of reference which moves in the x-direction with the constant wave speed c relative to the laboratory frame. The variables X and Y measured in the wave frame are defined by

(2.2) $$X = x-ct, \qquad Y = y.$$

The corresponding velocity components of the fluid are

(2.3) $$U = u-c, \qquad V = v,$$

and the components of the particulate velocity in the wave frame are defined by

(2.4) $$U_p = u_p-c, \qquad V_p = v_p.$$

The reason for introducing the wave frame is that the governing equations become stationary.

By means of the assumptions at the beginning of this paragraph we obtain the following inertia-free Navier–Stokes equations in the wave frame of reference:

(2.5) $$0 = -(1-\alpha)\frac{\partial P}{\partial X} - \alpha F_{D_u}(U-U_p)+\frac{\partial}{\partial Y}\tau_{xy},$$

(2.6) $$0 = -\alpha\frac{\partial P}{\partial X} + \alpha F_{D_u}(U-U_p),$$

(2.7) $$0 = -\alpha\frac{\partial P}{\partial Y} + \alpha F_{D_v}(V-V_p),$$

(2.8) $$0 = -(1-\alpha)\frac{\partial P}{\partial Y} - \alpha F_{D_v}(V-V_p)$$

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and the continuity equations

$$\frac{\partial}{\partial X}[(1-\alpha)U]+\frac{\partial}{\partial Y}[(1-\alpha)V] = 0, \tag{2.9}$$

$$\frac{\partial}{\partial X}[\alpha U_p]+\frac{\partial}{\partial Y}[\alpha V_p] = 0, \tag{2.10}$$

in which $F_{D_{u,v}}$ is the drag force of the particles in the fluid, α is the volume fraction of the particles, P is the pressure and τ_{xy} is the shear stress of the fluid and of course it is possible to introduce various rheological models which describe different non-Newtonian fluid behaviour. We use a simple rheological model which is called the power law fluid and it is given by OSTWALDE-DE WAELE [14] (see Eq. (2.11)).

$$\tau_{xy} = K\left|\frac{\partial U}{\partial Y}\right|^{n-1}\frac{\partial U}{\partial Y}. \tag{2.11}$$

The derivative of U with respect to Y is the shear rate, n is the flow behaviour index and K is the consistency index which depend upon n. It is well known that for $n \gtreqless 1$, this model represents the dilatant, Newtonian and pseudoplastic fluids, respectively.

The drag force of a moving rigid sphere in a power law fluid is given by KAWASE *et al.* [16, 17]

$$\begin{aligned} F_{D_u} &= \frac{9}{2}\left(\frac{27}{4}\right)^{\frac{n-1}{2}}\frac{K}{R^{n+1}}(U-U_p)^{n-1}\frac{-22n^2+29n+2}{n(n+2)(2n+1)}, \\ F_{D_v} &= \frac{9}{2}\left(\frac{27}{4}\right)^{\frac{n-1}{2}}\frac{K}{R^{n+1}}(V-V_p)^{n-1}\frac{-22n^2+29n+2}{n(n+1)(2n+1)}. \end{aligned} \tag{2.12}$$

This equation is obtained by an approximate solution of the equation of motion in the creeping flow regime, where R is the radius of the particles. Equation (2.12) shows that the drag force increases with a decrease of the flow index n.

Now we introduce dimensionless parameters and coordinates. Three important describing parameters of peristaltic flow are the ratio of amplitude ε, the wave number δ and a modified Reynolds number Re.

$$\varepsilon = b/a, \tag{2.13}$$

$$\delta = a/\lambda, \tag{2.14}$$

$$\mathrm{Re} = \frac{\varrho a^n c^{2-n}}{K}\delta. \tag{2.15}$$

For $n = 1$ the consistency index K is equal the shear viscosity μ_0 and so from Eq. (2.15) we get the appropriate Reynolds number for a Newtonian fluid. It is efficient to introduce the following dimensionless parameters and coordinates:

$$\hat{U} = U/c, \qquad \hat{U}_p = U_p/c, \tag{2.16}$$

$$\hat{V} = V\lambda/(ac), \quad \hat{V}_p = V_p\lambda/(ac), \tag{2.17}$$

$$\hat{X} = X/\lambda, \qquad \hat{Y} = Y/a, \tag{2.18}$$

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$$\tau = tc/\lambda, \qquad \hat{K} = \frac{K}{\mu_0}\left(\frac{c}{a}\right)^{n-1}, \tag{2.19}$$

$$\hat{P} = Pa^2/(\mu_0 c\lambda), \tag{2.20}$$

$$\hat{H} = h/a = 1+\varepsilon\sin(2\pi\hat{X}). \tag{2.21}$$

For the integration of the continuity and momentum equations we must satisfy the following boundary conditions. For the fluid phase we use

$$\hat{U} = -1 \quad \text{on} \quad \hat{Y} = \hat{H}, \tag{2.22}$$

$$\hat{V} = \frac{\partial\hat{H}}{\partial\tau} \quad \text{on} \quad \hat{Y} = \hat{H}, \tag{2.23}$$

$$\frac{\partial\hat{U}}{\partial Y} = 0 \quad \text{on} \quad \hat{Y} = 0, \tag{2.24}$$

$$\hat{V} = 0 \quad \text{on} \quad \hat{Y} = 0 \tag{2.25}$$

and for the particulate phase

$$\hat{U}_p = -1 \quad \text{on} \quad \hat{Y} = \hat{H}, \tag{2.26}$$

$$\hat{V}_p = \frac{\partial\hat{H}}{\partial\tau} \quad \text{on} \quad \hat{Y} = \hat{H}, \tag{2.27}$$

$$\frac{\partial\hat{U}_p}{\partial Y} = 0 \quad \text{on} \quad \hat{Y} = 0, \tag{2.28}$$

$$\hat{V}_p = 0 \quad \text{on} \quad \hat{Y} = 0. \tag{2.29}$$

3. Velocity profiles and pressure-flow rate relationships

Two important parameters of peristaltic flow are the dimensionless pressure rise per wavelength and the normalized time-mean volume flow of the two-phase mixture. For the rate of volume flow we need the velocities $\hat{U}$ and $\hat{U}_p$ in the $\hat{X}$-direction. An analytical integration of the momentum equations is possible by the fact that the pressure $\hat{P}$ is only a function of $\hat{X}$.

With Eqs. (2.11) and (2.12), Eqs. (2.5) and (2.6) may be integrated twice with respect to $\hat{Y}$ using the boundary conditions (2.22), (2.24), (2.26) and (2.28). As a result we get velocity profiles for the fluid velocity in the $\hat{X}$-direction (measured in the wave frame).

$$U = -1-\frac{n}{n+1}\left[\frac{1}{\hat{K}}\frac{d\hat{P}}{d\hat{X}}\right]^{1/n}\left(\hat{H}^{\frac{n+1}{n}}-\hat{Y}^{\frac{n+1}{n}}\right) \tag{3.1}$$

and for the particulate velocity in the $\hat{X}$-direction

$$\hat{U}_p = -1-\frac{n}{n+1}\left[\frac{1}{\hat{K}}\frac{d\hat{P}}{d\hat{X}}\right]^{1/n}\left(\hat{H}^{\frac{n+1}{n}}-\hat{Y}^{\frac{n+1}{n}}\right) - \left(\frac{2}{9}\right)^{1/n}\left(\frac{27}{4}\right)^{\frac{1-n}{2n}}\hat{R}\left(\frac{\hat{R}}{\hat{K}}\right)^{1\,n}\left[\frac{n(n+2)(2n+1)}{-22n^2+29n+2}\right]^{1/n}\left(\frac{d\hat{P}}{d\hat{X}}\right)^{1/n}. \tag{3.2}$$

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For the case of a Newtonian fluid ($n = 1$) containing spherical particles we get from Eqs. (3.1) and (3.2) parabolic velocity profiles.

Now we calculate the pressure rise per wavelength and the time-mean volume flow. The rate of volume flow $\hat{Q}$ measured in the wave frame of reference is a constant that varies neither with time nor with the position along the axis of the channel.

The volume flow $\hat{Q}$ is calculated as the sum of the fluid volume flow and the particulate volume flow.

$$\hat{Q} = \hat{Q}_{\text{Fluid}} + \hat{Q}_{\text{Particle}}. \tag{3.3}$$

It is important to mention that for the calculation of volume flow arbitrary initial concentration profiles $\alpha(\hat{Y})$ are possible.

$$\hat{Q} = \int_0^{\hat{H}} (1-\alpha(\hat{Y}))\hat{U}d\hat{Y} + \int_0^{\hat{H}} \alpha(\hat{Y})\hat{U}_p d\hat{Y}. \tag{3.4}$$

For the parabolic concentration profile

$$\alpha(\hat{Y}) = \alpha_0[1-(\hat{Y}/\hat{H})^2] \tag{3.5}$$

the volume flow is calculated as

$$\hat{Q} = -\hat{H} - \frac{n}{2n+1}\hat{H}^{\frac{2n+1}{n}}\left[\frac{1}{\hat{R}}\frac{d\hat{P}}{d\hat{X}}\right]^{1/n} - \frac{2}{3}\left(\frac{2}{9}\right)^{1/n}\left(\frac{27}{4}\right)^{\frac{1-n}{2n}}\left(\frac{n(n+2)(2n+1)}{-22n^2+29n+2}\right)\alpha_0 \hat{R}^{\frac{n+1}{n}}\hat{H}\left[\frac{1}{\hat{K}}\frac{d\hat{P}}{d\hat{X}}\right]^{1/n}. \tag{3.6}$$

It may be noted that for a concentration profile $\alpha(\hat{Y}) = \text{const} = \alpha_0$, the factor 2/3 in Eq. (3.6) is replaced by the factor 1.

Now we change the system of reference. The instantaneous volume flow $\hat{q}$ is given by the following relationship:

$$\hat{q} = \int_0^{\hat{H}} (1-\alpha(\hat{Y}))(\hat{U}+1)d\hat{Y} + \int_0^{\hat{H}} \alpha(\hat{Y})(\hat{U}_p+1)d\hat{Y}, \tag{3.7}$$

because $\hat{U} = \hat{u}-1$ and $\hat{U}_p = \hat{u}_p - 1$. The interpretation of Eq. (3.7) leads to

$$\hat{q} = \hat{Q} + \hat{H}. \tag{3.8}$$

For the peristaltic flow, the time-mean volume flow $\bar{q}$ in the x-direction is a quantity of practical interest. In our theory arbitrary wave functions $\hat{H}$ are possible. On the other hand if we want to calculate the time average flow we must use a special function for $\hat{H}$, for example we use Eq. (2.21). For the time-mean flow we integrate the instantaneous volume flow over one period. If $\hat{T}$ is the dimensionless period of the peristaltic wave, we get for the time average flow the following relation:

$$\bar{q} = \frac{1}{\hat{T}}\int_0^{\hat{T}} \hat{q}d\tau = \hat{Q}+1. \tag{3.9}$$

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After normalizing the time-mean volume flow we get

$$\bar{\bar{q}} = \frac{\bar{q}}{\varepsilon} = \frac{1}{\varepsilon}(\hat{Q}+1). \tag{3.10}$$

The other important parameter of a peristaltic flow is the dimensionless pressure rise per wavelength, and for this characteristic parameter we should know the pressure gradient in the $\hat{X}$-direction. With Eqs. (3.10) and (3.6) we get

$$\bar{q}\varepsilon - 1 = -\hat{H} - \frac{n}{2n+1}\hat{H}^{\frac{2n+1}{n}}\left[\frac{1}{\hat{K}}\frac{d\hat{P}}{d\hat{X}}\right]^{1/n} - \frac{2}{3}\left(\frac{2}{9}\right)^{1/n}\left(\frac{27}{4}\right)^{\frac{1-n}{2n}}\left(\frac{n(n+2)(2n+1)}{-22n^2+29n+2}\right)^{1/n}\alpha_0\hat{R}^{\frac{n+1}{n}}\hat{H}\left[\frac{1}{\hat{K}}\frac{d\hat{P}}{d\hat{X}}\right]^{1/n} \tag{3.11}$$

using the parabolic concentration profile from Eq. (3.5). It may be noted that Eq. (3.11) changes for $n = 1$ to

$$\left[\frac{d\hat{P}}{d\hat{X}}\right]_{n=1} = -\frac{\bar{q}\varepsilon - 1 + \hat{H}}{\frac{1}{3}\hat{H}^3/\hat{K} + \frac{2}{3}\alpha_0\frac{2}{9}\hat{R}^2\hat{H}/\hat{K}} \tag{3.12}$$

and for $\alpha_0 = 0$ (single-phase flow) Eq. (3.12) is equal to SHAPIRO's result [3].

The pressure change over one wavelength Δp_λ, also called pressure rise, is the same whether measured in moving or stationary coordinates. Δp_λ is given by

$$\Delta p_\lambda = \int_0^1 \frac{d\hat{P}}{d\hat{X}}d\hat{X}. \tag{3.13}$$

Using Eq. (3.11) for $d\hat{P}/d\hat{X}$ we solve Eq. (3.13) numerically. Up to this point we have not made any restriction to the values of the volume fraction of the particles, we assumed that $0 \leqslant \alpha \leqslant 1$. However, in many flows of practical interest, the volumetric concentration of the particles is small. Therefore, we assume now that α is small compared with unity. By means of this assumption it is possible to calculate the velocity component in the Y-direction because from Eq. (2.9) we obtain for $\alpha \ll 1$

$$\hat{V} = -\int_0^{\hat{Y}} \frac{\partial \hat{U}}{\partial \hat{X}}\partial \hat{Y} \tag{3.14}$$

and from Eqs. (2.7) and (2.8) it could be seen that $\hat{V} = \hat{V}_p$. With the help of Eq. (3.1) and using the boundary conditions (2.23) and (2.25), we get for the velocities in the $\hat{Y}$-direction

$$\hat{V} = \hat{V}_p = -\frac{2n+1}{n+1}\frac{d\hat{H}}{d\hat{X}}\left[\frac{\hat{Y}}{\hat{H}} - \frac{n}{2n+1}\left(\frac{\hat{Y}}{\hat{H}}\right)^{\frac{2n+1}{n}}\right] - \frac{n}{n+1}\left[\frac{1}{K}\frac{d\hat{P}}{d\hat{X}}\right]^{1/n}\frac{d\hat{H}}{d\hat{X}}\left(\hat{H}^{1/n}\hat{Y} - \hat{H}^{-1}\hat{Y}^{\frac{2n+1}{n}}\right). \tag{3.15}$$

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4. Results and discussion

The purpose of this paper is to calculate velocity distributions of the fluid and particular phase as well as pressure-flow rate relationships as functions of the volume flow, pressure rise, amplitude ratio and rheological parameter of the liquid. For the calculation of the various parameters of peristaltic flow we have taken the following quantities: $c = 3$ cm/s, $\lambda = 8$ cm, $\varrho = 1$ g/cm^3, $a = 8 \cdot 10^{-2}$ cm, $R = 8 \cdot 10^{-4}$ cm, $\alpha_0 = 0.1$. First we calculate the pressure gradient from Eq. (3.11). For the numerical integration of Eq. (3.13) we used a modified Simpson method given by ROMBERG [18]. Two important parameters of peristaltic flow are the pressure rise per wavelength Δp_λ and the time-mean volume flow $\bar{\bar{q}}$. As we see from Eqs. (3.11) and (3.13), the pressure rise is a nonlinear function of the time-average volume flow. Figure 1 shows the relationships between the normalized pressure rise per wavelength and the time-mean volume flow as well as the influence of the rheological parameter and the influence of the ratio of amplitude on the pressure-flow relationships. It should be noted that the pressure rise is normalized by the pressure rise for zero time-mean flow. From Fig. 1 we see that only for $n = 1$, which corresponds to a

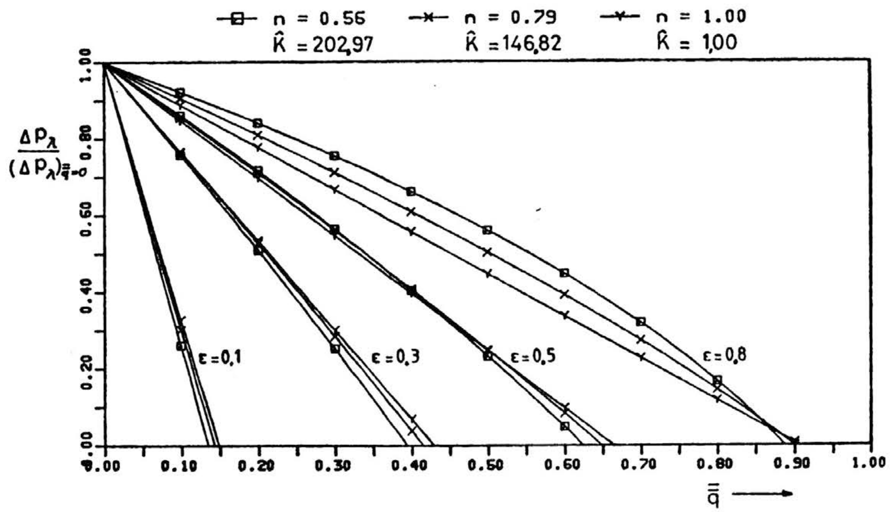

FIG. 1. Normalized pressure rise per wavelength versus dimensionless time mean flow.

Newtonian fluid containing spherical particles, we have a linear function between Δp_λ and $\bar{\bar{q}}$. In general, the volume flow rate increases with the increasing amplitude ratio ε. For example, at $\varepsilon = 0.8$ we see that with decreasing values of the flow behaviour it increases while retaining the same pressure rise.

In Fig. 2 we see the dimensionless pressure rise per wavelength for zero time-mean flow as a function of the amplitude ratio and the rheological parameter of the liquid. It may be noted that for $\varepsilon = 0$ we have no peristalsis and the pressure rise tends to zero and for $\varepsilon = 1$ we have a complete occlusion and the pressure rise tends to infinity. For the given rheological parameter the pressure rise of a Newtonian two-phase mixture is much less than the pressure rise of a non-Newtonian mixture while retaining the same

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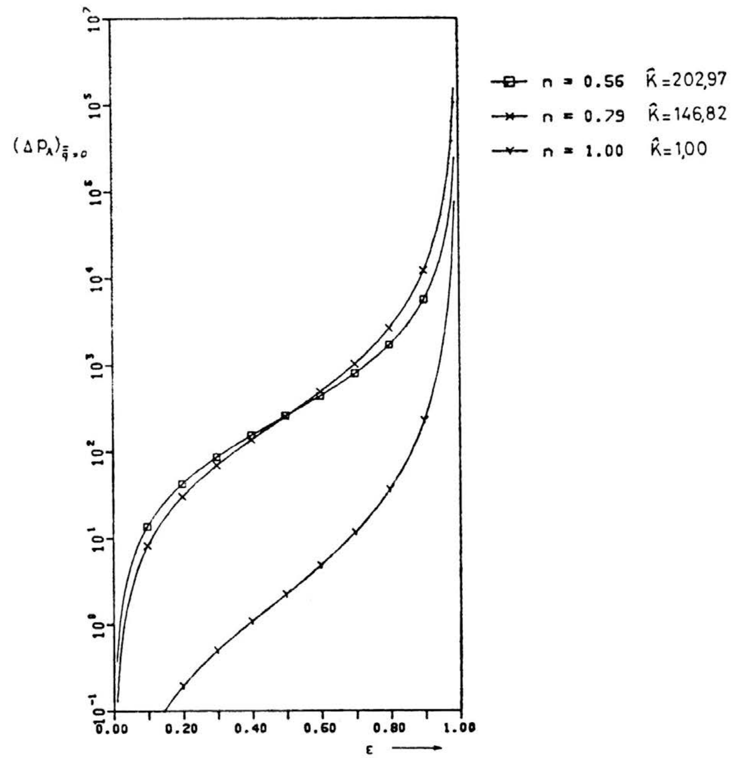

FIG. 2. Dimensionless pressure rise per wavelength for zero time mean flow as a function of the ratio of amplitude.

amplitude ratio. Now we take a look at details of fluid motion. Figure 3 shows the axial velocity profiles of the fluid velocity and the particulate velocity in the laboratory frame of reference for an amplitude ratio of $\varepsilon = 0.8$ and a time-mean flow of $\bar{\bar{q}} = 0.8$.

We see that there is no significant difference between the fluid velocity and the particulate velocity for the scale we have used. A variation of the exponent n and the quantity $\hat{K}$ causes the velocity distribution to deviate from a parabolic shape. In Fig. 4 we see the lateral and axial fluid velocity distribution in the fixed frame for an amplitude ratio of $\varepsilon = 0.8$ while in Fig. 5 we see the same quantities for a ratio of amplitude of $\varepsilon = 0.3$. We see that the maximum and minimum values of the velocities in Fig. 4 are greater than the corresponding values in Fig. 5. From Figs. 4 and 5 we see also that at the moving wall we have satisfied the boundary condition. It may be noted that at the axis of symmetry of the channel the lateral velocity is zero for all values of X/λ because of symmetry.

Now we take a look at the velocity fields. Figure 6 shows the field of the fluid velocity in the relative wave frame of reference. For the given amplitude ratio of $\varepsilon = 0.8$ we see the interesting trapping phenomenon. This trapping phenomenon forms a pair of vortices. We see that there are two stagnation points at the axis of symmetry. In general, trapping is usually identified by the existence of stagnation points. It should be noted that the vortices will not be observed in the fixed frame. Figure 7 shows the trapping phenomenon for a Newtonian fluid and we see only small differences between the fluid velocity fields

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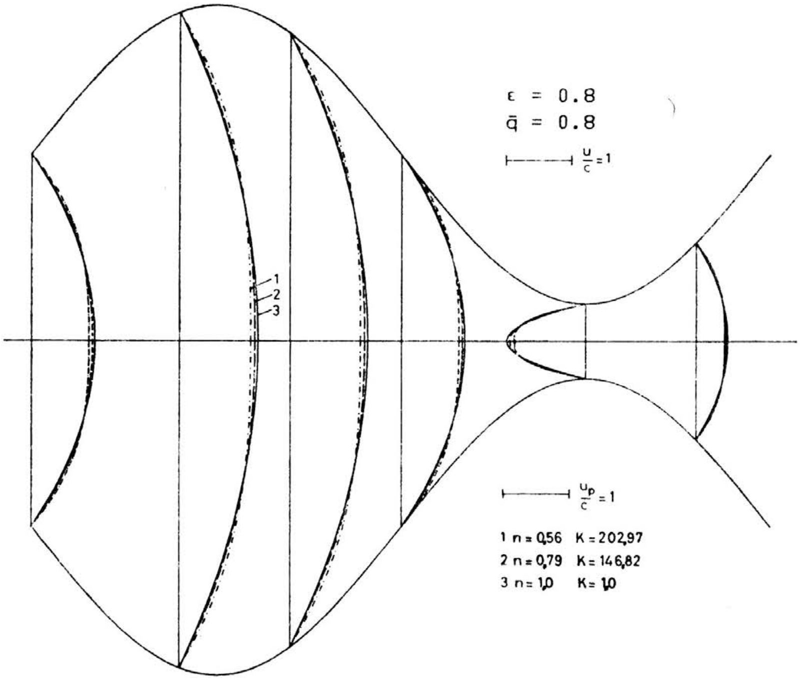

FIG. 3. Distribution of the axial fluid velocity and particulate velocity in the laboratory frame of reference.

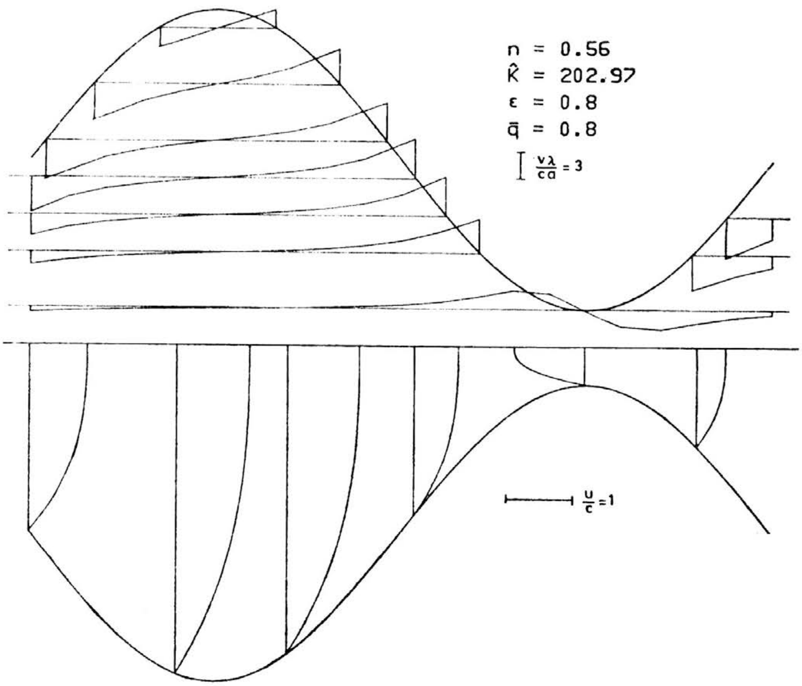

FIG. 4. Lateral and axial fluid velocity distribution in the laboratory frame.

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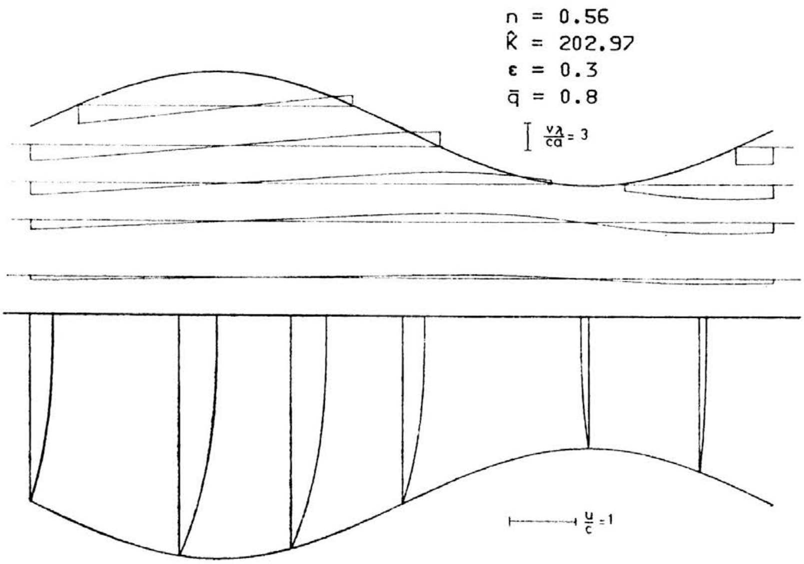

FIG. 5. Lateral and axial fluid velocity distribution in the laboratory frame.

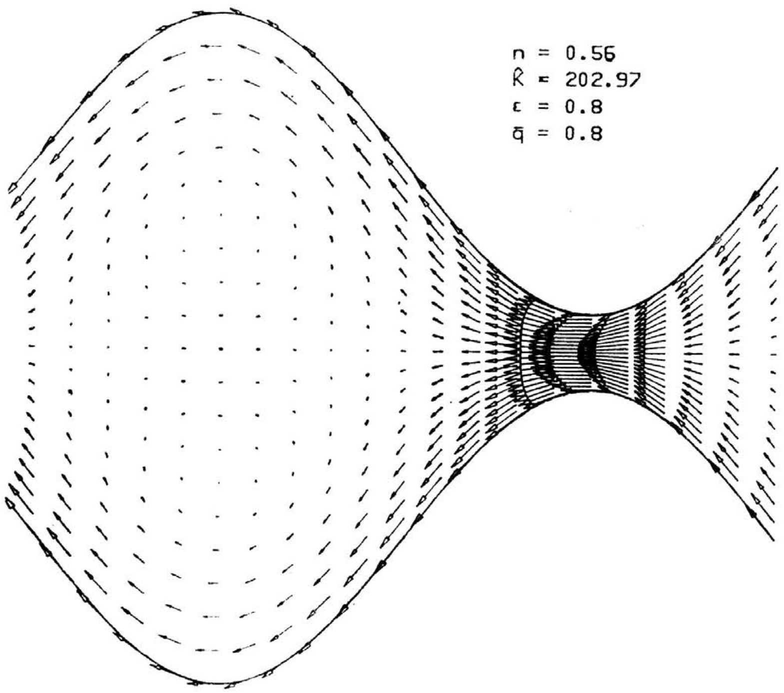

FIG. 6. Velocity field of the fluid velocity showing the trapping phenomenon in the wave frame.

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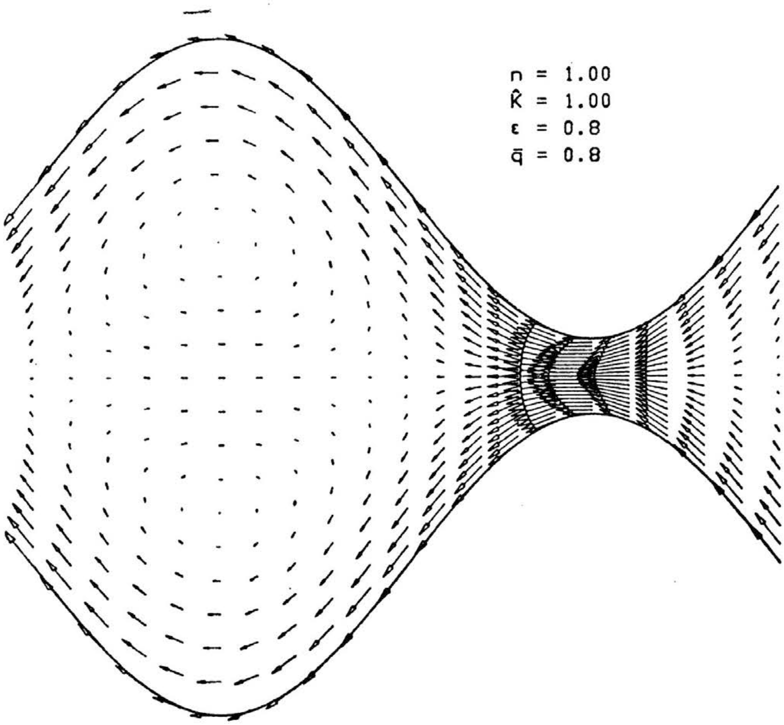

FIG. 7. Velocity field of the fluid velocity showing the trapping phenomenon in the wave frame.

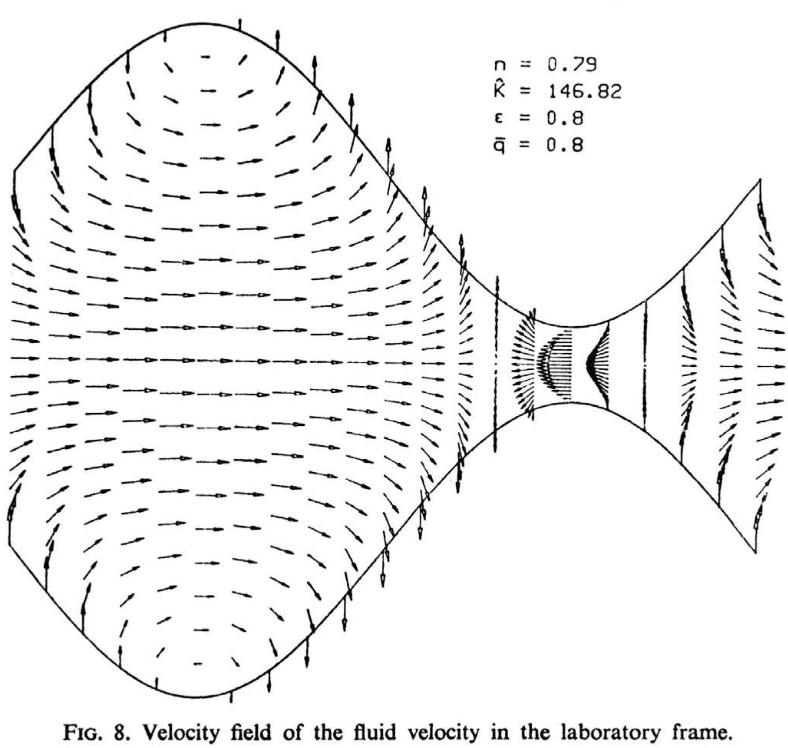

FIG. 8. Velocity field of the fluid velocity in the laboratory frame.

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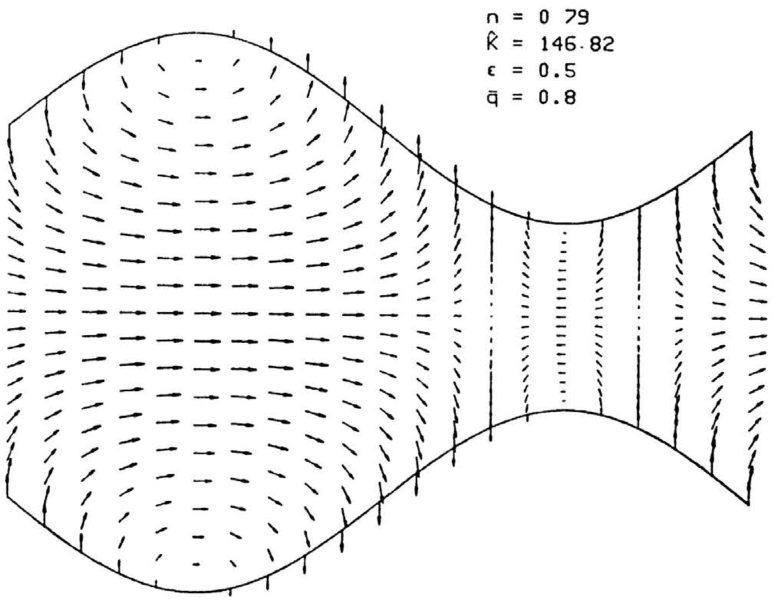

FIG. 9. Velocity field of the fluid velocity in the laboratory frame.

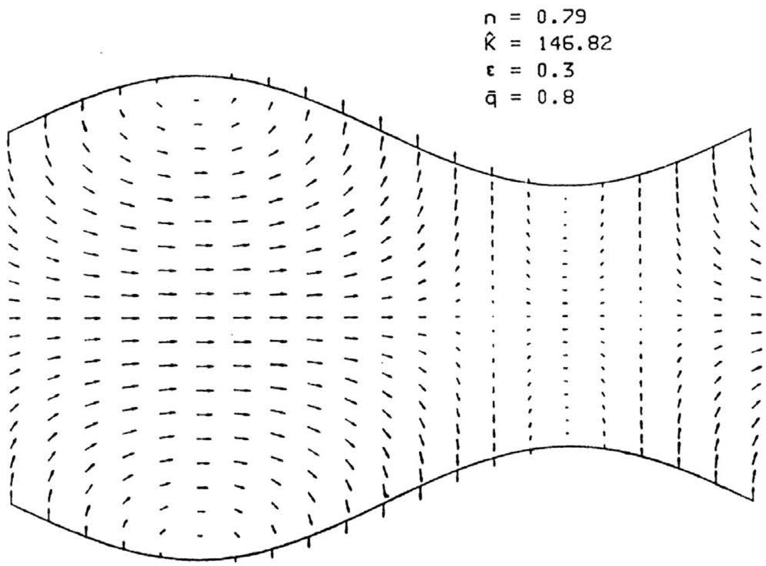

FIG. 10. Velocity field of the fluid velocity in the laboratory frame.

given by Figs. 6 and 7. Figures 8, 9 and 10 show us the velocity fields in the fixed frame for three different values of the amplitude ratio. We see that there is no trapping phenomenon in the laboratory frame. Figures 11 and 12 show the fluid velocity fields in the wave frame of reference. We see that at the widest part of the channel we have a maximum of the velocity and at the narrowest part we have a minimum of the velocity.

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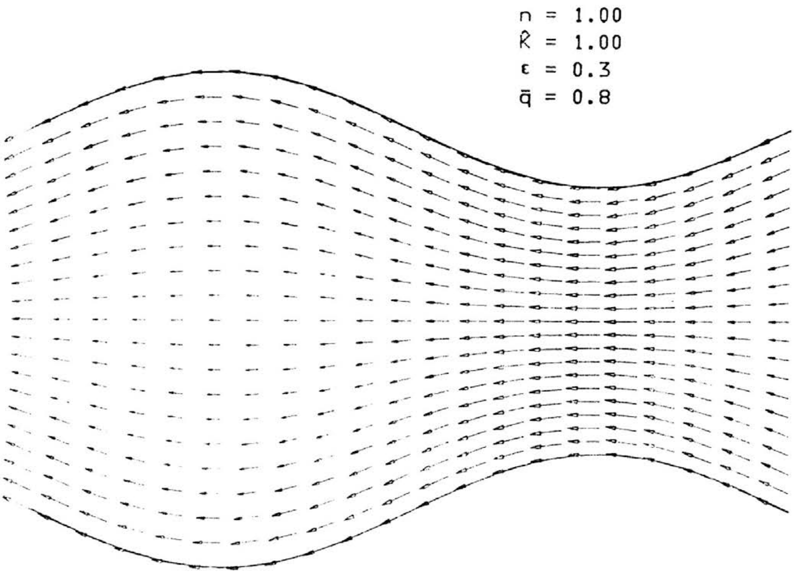

FIG. 11. Velocity field of the fluid velocity in the wave frame.

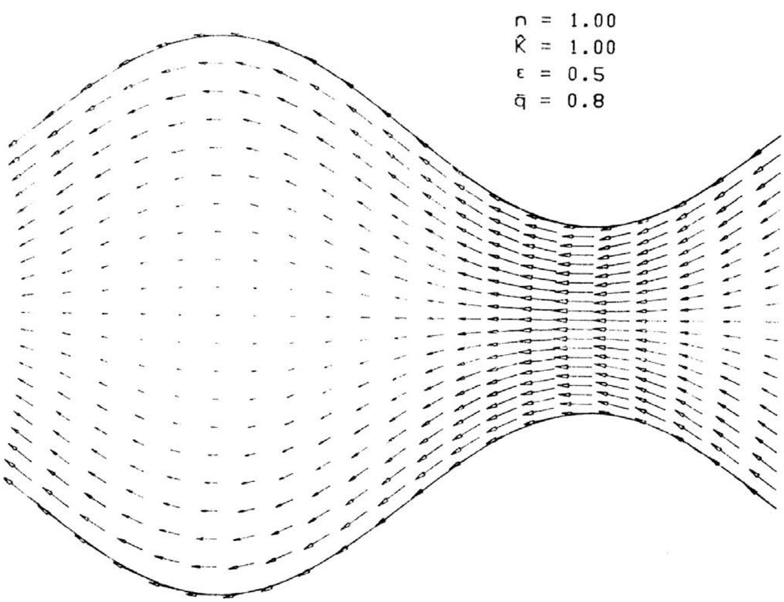

FIG. 12. Velocity field of the fluid velocity in the wave frame.

It may be noted that the rheological parameter $n = 0.56$, $\hat{K} = 202.97$ corresponds to a 0.5% PEO solution while $n = 0.79$, $\hat{K} = 146.82$ corresponds to a 0.5% Carbopol in 10% NaOH.

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In conclusion, we mention that this analysis is the first attempt made in literature to understand the peristaltic motion of non-Newtonian fluids containing small rigid particles and so further aspects and developments of these problems can be investigated.

References

1. M. Y. JAFFRIN and A. H. SHAPIRO, *Peristaltic pumping*, Annual Rev. Fluid Mechanics, **3**, 13–36, 1971.
2. H. J. RATH, *Peristaltische Strömungen*, Springer-Verlag, Berlin, Heidelberg, New York 1980.
3. A. H. SHAPIRO, M. Y. JAFFRIN and S. L. WEINBERG, *Peristaltic pumping with long wavelength at low Reynolds number*, J. Fluid Mech., **37**, 799–825, 1969.
4. H. J. RATH, *Peristaltic flow trough a lobe-shaped tube*, Intern. J. Mech. Sci., **24**, 6, 359–367, 1982.
5. O. H. MAHRENHOLTZ and R. U. ZIMMERMANN, *The influence of wave form on peristaltic transport*, Biorheology, **15**, 501–510, 1978.
6. Y. C. FUNG and C. S. YIH, *Peristaltic transport*, J. Appl. Mech. Trans. ASME, **35**, 669–675, 1968.
7. B. B. GUPTA and V. SHESHADRI, *Peristaltic pumping in non-uniform tubes*, J. Biomechanics, **9**, 105–109, 1976.
8. T. K. HUNG and T. D. BROWN, *Solid particle motion in two-dimensional persitaltic flows*, J. Fluid Mech., **73**, 77–96, 1967.
9. T. D. BROWN and T. K. HUNG, *Computational and experimental investigations of two-dimensional nonlinear peristaltic flows*, J. Fluid Mech., **83**, 249–273, 1974.
10. M. R. KAIMAL, *Peristaltic pumping of a Newtonian fluid with particles suspended in it at low Reynolds number under long wavelength*, J. Appl. Mech. Trans ASME, **45**, 32–36, 1978.
11. J. HAPPEL and H. BRENNER, *Low Reynolds number hydrodynamics*, Noordhoff Intern. Publishing, Leyden, Netherlands 1973.
12. S. L. SOO, *Fluid dynamics of multiphase systems*, Baisdell, Publishing Co., Waltham, Mass. 1967.
13. K. J. RAJU and R. DEVANATHAN, *Peristaltic motion of a non-Newtonian fluid*, Rheologica Acta, **11**, 170–178, 1972.
14. R. B. BIRD, W. E. STEWARD and E. N. LIGHTFOOT, *Transport phenomena*, John Wiley and Sons, New York 1960.
15. E. BECKER, *Simple non-Newtonian fluid flows*, Advances in Appl. Mech., **20**, 177–266, 1980.
16. Y. KAWASE and J. J. ULBRECHT, *Newtonian fluid sphere with rigid or mobile interface in a shear-thinning liquid: drag and mass transfer*, Chem. Eng. Commun., **8**, 213–231, 1981.
17. Y. KAWASE and J. J. ULBRECHT, *The effect of surfacfant on terminal velocity of and mass transfer from a fluid sphere in a non-Newtonian fluid*, The Canadian Journal of Chemical Engineering, **60**, February 1982.
18. R. ZURMÜHL, *Praktische Mathematik*, Springer Verlag 1965.

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