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Dynamics of a model of two-component medium

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TWO INFINITELY long, parallel elastic rods interact with each other, the forces of interaction being proportional to the difference of their axial velocities. The dispersion curve is determined, and two initial problems are solved. In the first problem the initial displacement distribution corresponds to Heaviside's function, and in the second one — to the case of contact of a uniformly deformed and an undeformed rod. The solutions are sought in the form of Fourier series. At the beginning of the process, for small values of time, the profile velocities in both the rods are equal to the corresponding propagation speeds, and for large values of time the wave profile propagates at the velocity different from the individual propagation speeds.

Dwa różne nieskończenie długie pręty sprężyste oddziaływują na siebie siłą proporcjonalną do różnicy ich prędkości. Wyznacza się krzywą dyspersyjną, a następnie rozwiązuje dwa zagadnienia początkowe. W pierwszym rozkład przemieszczeń jest taki jak w funkcji Heaviside'a, a w drugim odpowiadający kontaktowi pręta poddanego jednorodnemu odkształceniu i pręta nieodkształconego. Rozwiązań poszukuje się w postaci szeregów Fouriera. W chwilach bliskich zera prędkość profilu w pręcie pierwszym i drugim jest równa prędkości propagacji w tych prętach. Dla dużych czasów profil fali porusza się z prędkością inną niż prędkości propagacji w prętach.

Два разных бесконечно длинных упругих стержня воздействуют на себя силой пропорциональной разности их скорости. Определяется дисперсионная кривая, а затем решаются две начальные задачи. В первой распределение перемещений такое как в функции Хевисайда, а во второй распределение отвечающее контакту стержня подвергнутого однородной деформации и недеформируемого стержня. Решений ищется в виде рядов Фурье. В моментах близких нуля скорость профиля в первом и во втором стержнях равна скорости распространения в этих стержнях. Для больших времен профиль волны движется с другой скоростью, чем скорость распространения в стержнях.

THERE exists a large literature concerning the statics of multi-component media, but very few papers deal with the problem of dynamics of such media. Formal consideration of discontinuity waves is worthless since, as it was shown in [1], the speed of the wave profile is completely different from the speed of the wave propagation; that is why none of such papers will be quoted here.

1. Equation of the problem

Two parallel elastic rods of cross-sections S_1, S_2 are made of different materials with the respective elastic moduli E_1, E_2 and densities ϱ_1, ϱ_2. Let us consider the motion of the rods in the direction of their axes and disregard the motion in the transversal direction. The rods are assumed to exert forces on each other, and the forces are proportional to the difference of their velocities, Fig. 1. An experimental model of such a system may be represented by a rod placed in a thick-walled pipe filled with a viscous liquid, Fig. 2.

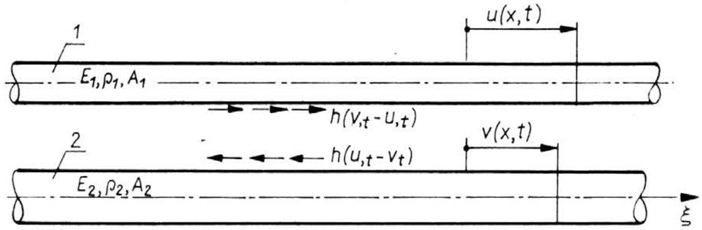

FIG. 1.

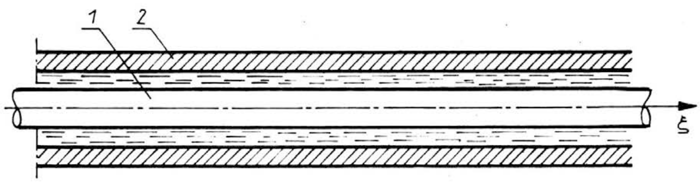

FIG. 2.

Displacements of the first rod are denoted by u, and in the second — by v. Elementary considerations yield the following equations of motion:

$$
\begin{aligned}
E_1 u_{,\xi\xi}+h(v_{,t}-u_{,t}) &= \varrho_1 u_{,tt},\\
E_2 v_{,\xi\xi}+h(u_{,t}-v_{,t}) &= \varrho_2 v_{,tt},
\end{aligned}
\tag{1.1}
$$

where h denotes the interaction coefficient. In order to simplify the considerations assume $\varrho_1 = \varrho_2 = \varrho$. By introducing a new coordinate

$$
x = \sqrt{\frac{\varrho}{E_1}}\,\xi,
\tag{1.2}
$$

and the parameters

$$
H = \frac{h}{\varrho}, \quad q^2 = \frac{E_2}{E_1},
\tag{1.3}
$$

the following system of equations is obtained

$$
\begin{aligned}
u_{,xx}+H(v_{,t}-u_{,t}) &= u_{,tt},\\
q^2 v_{,xx}+H(u_{,t}-v_{,t}) &= v_{,tt};
\end{aligned}
\tag{1.4}
$$

this is a set of linear second order differential equations. If $H = 0$, then the rods will be uncoupled. In such a case the disturbances in the first rod will be propagated with the speed $c_1 = 1$, and in the second one — with the speed $c_2 = q$.

2. Weak discontinuity wave

Let us assume the existence of a time-dependent surface $\mathscr{S}$ described by the equation

$$
x = Ut,
\tag{2.1}
$$

at which the displacements $u(x, t)$ and $v(x, t)$, and their derivatives $u_{,x}, u_{,t}, v_{,x}, v_{,t}$ are continuous, and the higher order derivatives are discontinuous. Denoting by double brackets the jump at $\mathscr{S}$,

$$(2.2) \qquad [\![\,\cdot\,]\!] = \lim_{x\to Ut-0}(\,.\,) - \lim_{x\to Ut+0}(\,.\,),$$

we obtain the following compatibility conditions (cf. e.g., [2]):

$$(2.3) \qquad \begin{aligned} [\![u_{,xx}]\!] &= B_1, \quad [\![u_{,tt}]\!] = U^2B_1,\\ [\![v_{,xx}]\!] &= B_2, \quad [\![v_{,tt}]\!] = U^2B_2, \end{aligned}$$

B_1, B_2 denoting certain parameters. Substitution of expressions (2.3) into Eqs. (1.4) yields two equations

$$(2.4) \qquad \begin{aligned} (U^2-1)B_1 &= 0,\\ (U^2-q^2)B_2 &= 0, \end{aligned}$$

which must be satisfied simultaneously. It follows that the equalites must be satisfied:

$$(2.5) \qquad U^2 = 1, \quad B_2 = 0,$$

or

$$(2.6) \qquad U^2 = q^2, \quad B_1 = 0.$$

The discontinuity surface must propagate therefore either at velocity $U = 1$ or at velocity $U = q$. This result is of a great scientific value but, as it will be shown later, it yields no information on the real behaviour of the mechanical system discussed here.

The system considered is linear and, hence, the propagation speeds of strong discontinuity waves are the same as the weak discontinuity wave speeds determined above.

3. Sinusoidal wave

The solution of Eqs. (1.4) is sought in the form

$$(3.1) \qquad \begin{aligned} u &= Ae^{i(kx-\omega t)},\\ v &= pAe^{i(kx-\omega t)}, \end{aligned}$$

frequency ω and amplitudes A, pA being constant, independent of x and t. The wave number k is assumed to be known.

By substituting the expression (3.1) into Eqs. (1.4) we obtain the set of two equations in constants ω and p

$$(3.2) \qquad \begin{aligned} (\omega^2-k^2+i\omega H)-i\omega Hp &= 0,\\ -i\omega H+(\omega^2-k^2q^2+i\omega H)p &= 0. \end{aligned}$$

Non-zero solutions of the set exist provided its principal determinant vanishes. Denoting $\omega = iw$ we obtain the fourth degree algebraic equation

$$(3.3) \qquad w^4+2Hw^3+k^2(1+q^2)w^2+k^2(1+q^2)Hw+k^4q^2 = 0,$$

with real-valued coefficients and the single unknown $w(k)$. This equation has four solutions w_1, w_2, w_3, w_4. Hence, there exist four branches of the dispersion curve $\omega(k)$, and namely $\omega_1(k), \omega_2(k), \omega_3(k), \omega_4(k)$. For each q, $H \neq 0$ the values of $\omega(k)$ are imaginary or complex numbers. In Fig. 3 are shown the numerically determined real (solid line) and imaginary

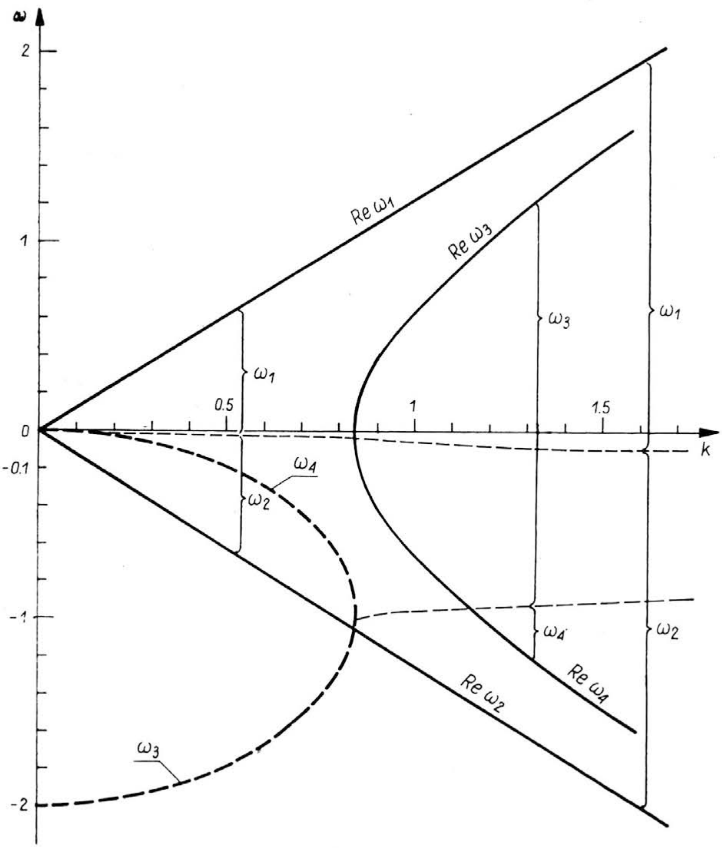

FIG. 3.

(dashed line) parts of the function $\omega_r(k)$, $r = 1, 2, 3, 4$ for $H = 1$. For large k the real part of $\omega(k)$ is proportional to k, and

$$\begin{aligned} \mathrm{Re}\omega_1 &= -\mathrm{Re}\omega_2 = q, \\ \mathrm{Re}\omega_3 &= -\mathrm{Re}\omega_4 = 1. \end{aligned} \tag{3.4}$$

For $H = 0$ the Eq. (3.3) is a biquadratic equation

$$w^2+k^2(1+q^2)w^2+k^4q^2 = 0, \tag{3.5}$$

and we have

$$\omega_1 = -\omega_2 = q, \quad \omega_3 = -\omega_4 = 1. \tag{3.6}$$

Such system is dispersionless and its dynamics is very simple. In the considerations to follow it will be assumed that $H \neq 0$.

From the theory of algebraic equations it is known that the roots of the fourth degree Eq. (3.3) are identical with the roots of two quadratic equations

$$\begin{aligned} &w^2+(2H+M)\frac{w}{2}+\left(y+\frac{2Hy-k^2(1+q^2)}{M}\right) = 0, \\ &w^2+(2H-M)\frac{w}{2}\left(y+\frac{2Hy-k^2(1+q^2)}{-M}\right) = 0. \end{aligned} \tag{3.7}$$

Here y is an arbitrary solution of the algebraic equation of the third degree

$$(3.8)\quad 8y^3-4k^2(1+q^2)y^2+4k^2[H^2(1+q^2)-2k^2q^2]y+4k^4q^2[k^2(1-q^2)-H^2]-k^4H^2(1+q^2)^2 = 0,$$

and parameter M is defined by the formula

$$(3.9)\quad M = 2\sqrt{\eta}, \quad \eta = 2y+H^2-k^2(1+q^2).$$

Equation (3.8) is of the third degree, and so it must possess at least one real-valued root y. This conclusion enables further analysis of the solutions of Eqs. (3.7). Without going into detailed calculations, let us present the final conclusion concerning the frequency ω. There exists such $\bar{k}$ depending on H that for $k > \bar{k}$ Eqs. (3.7) have complex conjugate roots, and the corresponding frequencies ω have the forms

$$(3.10)\quad \begin{aligned} \omega_1 &= \alpha_1+i\beta_1, & \omega_3 &= \alpha_2+i\beta_2,\\ \omega_2 &= -\alpha_1+i\beta_1, & \omega_4 &= -\alpha_2+i\beta_2. \end{aligned}$$

However, for $k \leqslant \bar{k}$ one of the Eqs. (3.7) has real-valued roots, and the second one — complex conjugate roots. In such a case

$$(3.11)\quad \begin{aligned} \omega_1 &= \alpha_1+i\beta_1, & \omega_3 &= i\gamma_3,\\ \omega_2 &= -\alpha_1+i\beta_1, & \omega_4 &= i\gamma_4. \end{aligned}$$

If $H = 1$, then $\bar{k} \approx 0.84$, Fig. 3.

In compliance with Eq. (3.2), the coefficient p is found from any of the formulae

$$(3.12)\quad p = 1+\frac{\omega^2-k^2}{i\omega H}, \quad p = -\frac{\omega^2-k^2}{\omega^2-k^2q^2}.$$

In the case of very small values of H, Eq. $(3.12)_2$ is more expedient since it doesn't contain the 0/0-type ratio. Using the formulae (3.10), (3.11) we obtain for $k > \bar{k}$

$$(3.13)\quad \begin{aligned} p_1 &= \varphi_1+i\psi_1, & p_3 &= \varphi_2+i\psi_2,\\ p_2 &= \varphi_1-i\psi_1, & p_4 &= \varphi_2-i\psi_2, \end{aligned}$$

$$(3.14)\quad \begin{aligned} \varphi_1 &= 1+\frac{\beta_1}{H}\left(1+\frac{k^2}{\alpha_1^2+\beta_1^2}\right), & \psi_1 &= \frac{\alpha_1}{H}\left(-1+\frac{k^2}{\alpha_1^2+\beta_1^2}\right)\\ \varphi_2 &= 1+\frac{\beta_2}{H}\left(1+\frac{k^2}{\alpha_2^2+\beta_2^2}\right), & \psi_2 &= \frac{\alpha_2}{H}\left(-1+\frac{k^2}{\alpha_2^2+\beta_2^2}\right). \end{aligned}$$

For $k < \bar{k}$ the formulae for p_1, p_2 remain unchanged, while p_3, p_4 are determined by the formulae

$$(3.15)\quad \begin{aligned} p_3 &= 1+\frac{\gamma_3}{H}+\frac{k^2}{H\gamma_3},\\ p_4 &= 1+\frac{\gamma_4}{H}+\frac{k^2}{H\gamma_4}. \end{aligned}$$

The solutions of Eqs. (1.4) are the following displacements

$$(3.16)\quad \begin{aligned} u &= A_r e^{i(kx-\omega_r(k)x)},\\ v &= p_r A_r e^{i(kx-\omega_r(k)x)}, \end{aligned}$$

where ω_r, $r = 1, 2, 3, 4$ are determined by Eqs. (3.10) or (3.11), and p_r — by Eqs. (3.13) or (3.15). Superposition of the solutions of Eqs. (3.16) makes it possible to solve the initial problem (Sect. 5). Observe that the phase and group velocities, U_p and U_g, corresponding to the solutions (3.16)

$$U_p = \frac{\omega(k)}{k}, \qquad U_g = \frac{d\omega(k)}{dk} \tag{3.17}$$

are complex and different at different branches of the dispersion curve $\omega(k)$.

4. Case $q^2 = 2$

Having in view the solution of the initial problem, let us now analyze in detail the case $q^2 = 2$; it corresponds to the case when the ratio of propagation speeds in both the rods equals $\sqrt{2}$. Substitution of $q^2 = 2$ into Eq. (3.8) and simple transformations lead to the following equation for η (y was found from Eq. $(3.9)_2$ and substituted into Eq. (3.8))

$$L(\eta) \equiv \eta^3 + (6h^2 - 3H^2)\eta^2 + (k^4 - 6k^2H^2 + 3H^4)\eta - H^6 = 0. \tag{4.1}$$

One root of this equation lies within the interval

$$0 < \eta < H^2,$$

since $L(0) = -H^6 < 0$, and $L(H^2) = H^2 k^2 > 0$. This root must be determined numerically. With $H = 1$ the curve $\eta(k^2)$ is shown in Fig. 4.

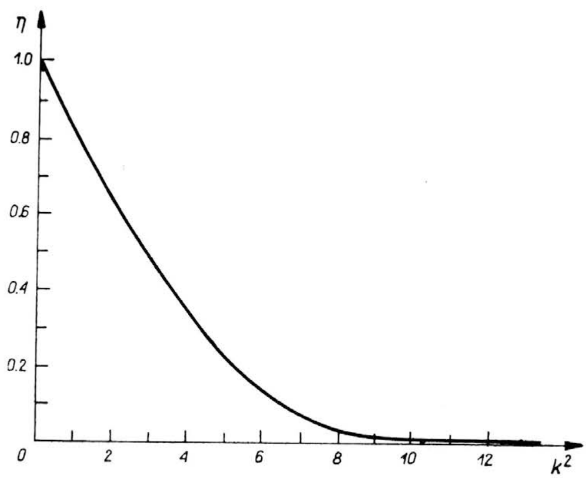

FIG. 4.

Equations (3.7) are reduced to two equations

$$w^2 + (H \pm \sqrt{\eta})w + \frac{1}{2}\left[3k^2 - H^2 + \eta \pm H\frac{-H^2+\eta}{\sqrt{\eta}}\right] = 0. \tag{4.2}$$

which enable us to determine w and $\omega = iw$. Denote

$$\begin{aligned} Q_1 &= 6k^2 + \eta - 3H^2 + 2H^3/\sqrt{\eta}, \\ Q_2 &= 6k^2 + \eta - 3H^2 - 2H^3/\sqrt{\eta}. \end{aligned} \tag{4.3}$$

For each k we have $Q_1 > 0$. If $Q_2 > 0$, then

$$\omega_{1,2} = \pm\alpha_1 + i\beta_1, \quad \omega_{3,4} = \pm\alpha_2 + i\beta_2,$$

(4.4)
$$\alpha_1 = \frac{1}{2}\sqrt{Q_1}, \quad \beta_1 = -\frac{1}{2}(H - \sqrt{\eta}),$$

$$\alpha_2 = \frac{1}{2}\sqrt{Q_2}, \quad \beta_2 = -\frac{1}{2}(H + \sqrt{\eta}).$$

If $Q_2 < 0$, the formulae for α_1 and β_1 remain unchanged, while

(4.5)
$$\omega_3 = i\gamma_3, \quad \gamma_3 = -\frac{1}{2}(H + \sqrt{\eta}) + \frac{1}{2}\sqrt{-Q_2},$$

$$\omega_4 = i\gamma_4, \quad \gamma_4 = -\frac{1}{2}(H + \sqrt{\eta}) - \frac{1}{2}\sqrt{-Q_2}.$$

The parameter $\bar{k}$ introduced in the preceding section corresponds to $Q_2 = 0$. Coefficients $\varphi_1, \psi_1, \varphi_2, \psi_2, p_3$ which correspond to the above functions are determined by the formulae (3.14), (3.15).

Let us now pass to the construction of the solutions. In accordance with the definitions introduced above, we obtain the following solutions:

(4.6)
$$\begin{aligned} u &= e^{\beta_1 t} e^{i(kx - \alpha_1 t)}, & v &= (\varphi_1 + i\psi_1) e^{i(kx - \alpha_1 t)}, \\ u &= e^{\beta_1 t} e^{i(-kx + \alpha_1 t)}, & v &= (\varphi_1 - i\psi_1) e^{i(-kx + \alpha_1 t)}, \\ u &= e^{\beta_1 t} e^{i(kx + \alpha_1 t)}, & v &= (\varphi_1 - i\psi_1) e^{i(kx + \alpha_1 t)}, \\ u &= e^{\beta_1 t} e^{i(-kx - \alpha_1 t)}, & v &= (\varphi_1 + i\psi_1) e^{i(-kx - \alpha_1 t)}. \end{aligned}$$

Following the scheme $[(1)+(2)-(3)-(4)] : 2$, $[(1)-(2)+(3)-(4)] : 2i$, we obtain two real-valued solutions

(4.7)
$$\begin{aligned} u_1 &= e^{\beta_1 t} c_1, & v_1 &= e^{\beta_1 t}(\varphi_1 c_1 - \psi_1 s_1), \\ u_2 &= e^{\beta_1 t} s_1, & v_2 &= e^{\beta_1 t}(\psi_1 c_1 + \varphi_1 c_1), \end{aligned}$$

where

(4.8)
$$\begin{aligned} c_1 &= \cos(kx - \alpha_1 t) - \cos(kx + \alpha_1 t), \\ s_1 &= \sin(kx - \alpha_1 t) + \sin(kx + \alpha_1 t). \end{aligned}$$

If $Q_2 > 0$, a similar reasoning yields two further solutions

(4.9)
$$\begin{aligned} u_3 &= e^{\beta_2 t} c_2, & v_3 &= e^{\beta_2 t}(\varphi_2 c_2 - \psi_2 s_2), \\ u_4 &= e^{\beta_2 t} s_2, & v_4 &= e^{\beta_2 t}(\psi_2 c_2 + \varphi_2 c_2), \end{aligned}$$

where

(4.10)
$$\begin{aligned} c_2 &= \cos(kx - \alpha_2 t) - \cos(kx + \alpha_2 t), \\ s_2 &= \sin(kx - \alpha_2 t) + \sin(kx + \alpha_2 t). \end{aligned}$$

If $Q_2 < 0$, we obtain

(4.11)
$$\begin{aligned} u_3 &= (e^{\gamma_3 t} - e^{\gamma_4 t}) \sin kx, & v_3 &= (p_3 e^{\gamma_3 t} - p_4 e^{\gamma_4 t}) \sin kx, \\ u_4 &= (e^{\gamma_3 t} + e^{\gamma_4 t}) \sin kx, & v_4 &= (p_3 e^{\gamma_3 t} + p_4 e^{\gamma_4 t}) \sin kx. \end{aligned}$$

Having in view the initial problem, let us now determine the displacements and speeds at the instant $t = 0$. In accordance with Eqs. (4.7) and (4.9) we obtain with $Q_2 > 0$

$$\begin{bmatrix} u_r(x,0) \\ v_r(x,0) \\ \dot{u}_r(x,0) \\ \dot{v}_r(x,0) \end{bmatrix} = \begin{bmatrix} A_{1r} \\ A_{2r} \\ A_{3r} \\ A_{4r} \end{bmatrix} \sin kx \tag{4.12}$$

matrix A_{ir} being determined by the formula

$$A_{ir} = 2\begin{bmatrix} 0, & 1, & 0, & 1 \\ -\psi_1, & \varphi_1, & -\psi_2, & \varphi_2 \\ \alpha_1, & \beta_1, & \alpha_2, & \beta_2 \\ -\beta_1\psi_1+\alpha_1\varphi_1, & \beta_1\varphi_1+\alpha_1\psi_1, & -\beta_2\psi_2+\alpha_2\varphi_2, & \beta_2\varphi_2+\alpha_2\psi_2 \end{bmatrix}. \tag{4.13}$$

If $Q_2 < 0$ then, in accordance with Eqs. (4.7) and (4.11), we obtain

$$A_{ir} = 2\begin{bmatrix} 0, & 1, & 0, & 1 \\ -\psi_1, & \varphi_1, & \dfrac{p_3-p_4}{2}, & \dfrac{p_3+p_4}{2} \\ \alpha_1, & \beta_1, & \dfrac{\gamma_3-\gamma_4}{2}, & \dfrac{\gamma_3+\gamma_4}{2} \\ -\beta_1\psi_1+\alpha_1\varphi_1, & \beta_1\varphi_1+\alpha_1\psi_1, & \dfrac{\gamma_3p_3-\gamma_4p_4}{2}, & \dfrac{\gamma_3p_3+\gamma_4p_4}{2} \end{bmatrix}. \tag{4.14}$$

It should be stressed that A_{ir} depends on the wave number k.

5. Initial problem

Let us assume that the displacements and their time rates at $t = 0$ are known; under this condition let us find the displacements at $t > 0$. Let us consider two cases only. In the first case

$$\begin{aligned} u(x,0) &= v(x,0) = -\frac{4}{\pi}\left(\sin k_0x + \frac{\sin 3k_0x}{3} + \frac{\sin 5k_0x}{5} + \dots\right), \\ \dot{u}(x,0) &= \dot{v}(x,0) = 0. \end{aligned} \tag{5.1}$$

In the neighbourhood of zero this function represents the Heaviside function. In the second case

$$\begin{aligned} u(x,0) = v(x,0) = -\frac{4}{\pi\alpha}\Big(&\sin\alpha\sin k_0x + \frac{1}{3^2}\sin 3\alpha\sin 3k_0x \\ &+ \frac{1}{s^2}\sin 5\alpha\sin 5k_0x + \dots\Big), \end{aligned} \tag{5.2}$$

$$\dot{u}(x,0) = \dot{v}(x,0) = 0.$$

The graphs of functions (5.1) and (5.2) are shown in Fig. 5. Both the functions are periodic, but let us concentrate upon the motion of the wave profile which at instant t

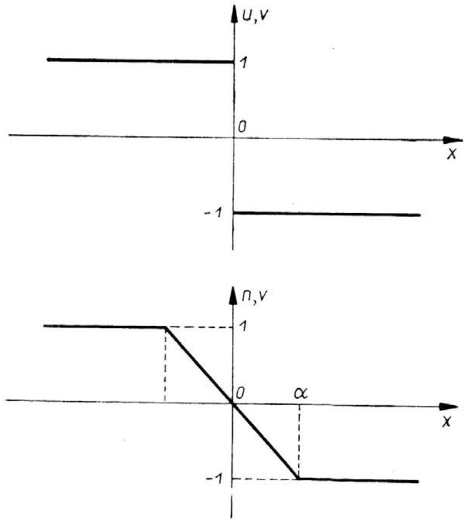

FIG. 5.

lies close to the point $x = 0$ (in the case of (5.1)), or close to $x = \alpha$ (in the case of (5.2)).

The calculations will be confined to a finite number of terms of the Fourier series. It should be stressed that, due to a very complex form of the dispersion formula, the application of the Fourier transform is impractical. The transforms would have to be determined numerically thus obscuring the physical sense of the results.

Let us fix the value of k_0 and assume

$$k = Nk_0. \tag{5.3}$$

Coefficients X_1, X_2, X_3, X_4 are selected so as to satisfy the following equations

$$A_{i1}X_1 + A_{12}X_2 + A_{13}X_3 + A_{i4}X_4 = (1, 1, 0, 0). \tag{5.4}$$

Obviously, $A_{ir} = A_{ir}(N)$, $X_r = X_r(N)$. The displacements

$$\overset{(N)}{u} = \sum_{r=1}^{4} u_r X_r, \qquad \overset{(N)}{v} = \sum_{r=1}^{4} v_r X_r, \tag{5.5}$$

are, at instant $t = 0$, equal to $\sin Nk_0 x$ and their time derivatives vanish since Eqs. (5.4) are satisfied. Suitable summation will then lead to a solution fulfilling the conditions (5.1) or (5.2).

In particular, by assuming

$$\begin{aligned} u(x, t) &= -\frac{4}{\pi}\left(\overset{(1)}{u} + \frac{1}{3}\overset{(3)}{u} + \frac{1}{5}\overset{(5)}{u} + \ldots + \frac{1}{K}\overset{(K)}{u}\right), \\ v(x, t) &= -\frac{4}{\pi}\left(\overset{(1)}{v} + \frac{1}{3}\overset{(3)}{v} + \frac{1}{5}\overset{(5)}{v} + \ldots + \frac{1}{K}\overset{(K)}{v}\right), \end{aligned} \tag{5.6}$$

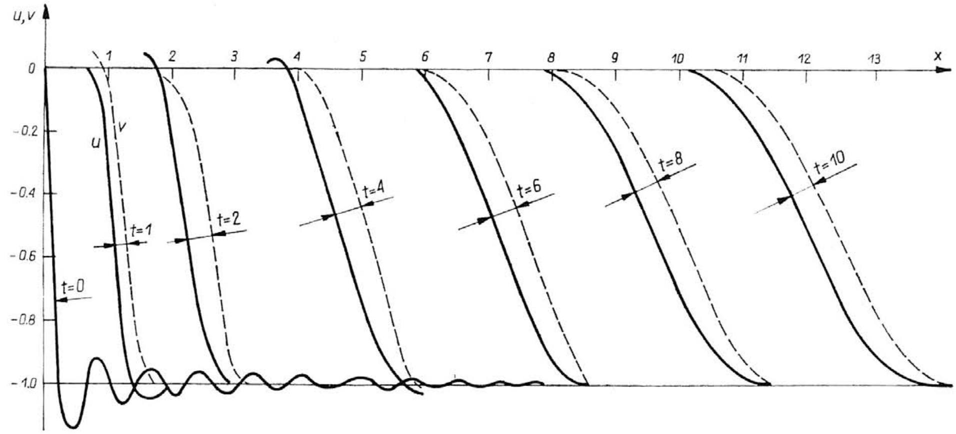

FIG. 6.

we obtain the solution satisfying the initial conditions (5.1). The wave profiles at times $t = 0, 1, 2, 4$ for $H = 1$, $k_0 = 0.1$ are shown in Fig. 6. The last term of (5.6) taken into account corresponds to $N = K = 75$. The solid line represents $u(x, t)$ and the dashed line — $v(x, t)$. Points of the second rod are reached by the disturbances earlier than those of the first rod. From the measurements made at half height of the profile it follows, however, that the profile speeds for large values of time ($t > 2$) are practically the same

$$U_u \approx U_v \approx 1.23.$$

At small times $U_u < U_v$. The corresponding measurement yield the following speeds

$$U_u \approx 1, \quad U_v \approx 1.40.$$

Let us observe that the discontinuity wave velocities determined for $q^2 = 2$ are $U = 1$ and $U = \sqrt{2} \approx 1.414$.

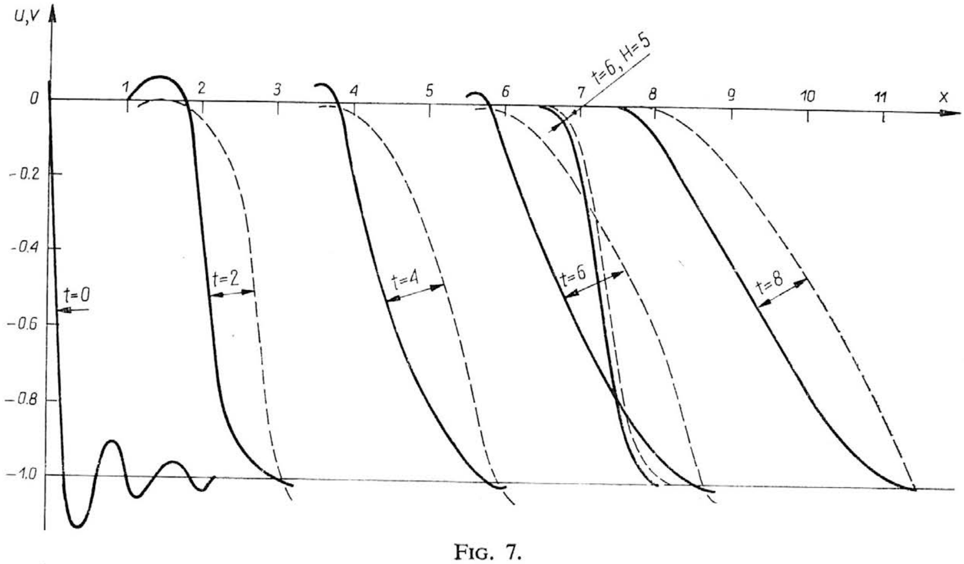

FIG. 7.

The wave profile deforms, its slope becoming more gentle with increasing time. To visualize the effect of viscosity upon the process of smoothing the profile, the subsequent profiles for $H = 0.5$ and $H = 5$ (at time $t = 8$ only) are shown in Fig. 7. The viscosity is seen to increase the profile distortion.

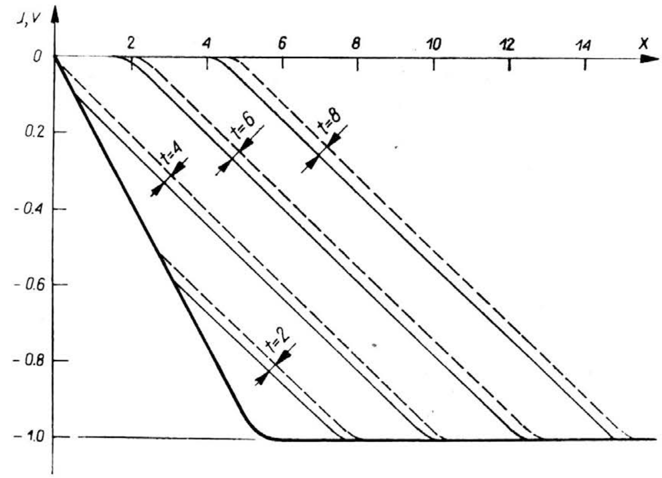

FIG. 8.

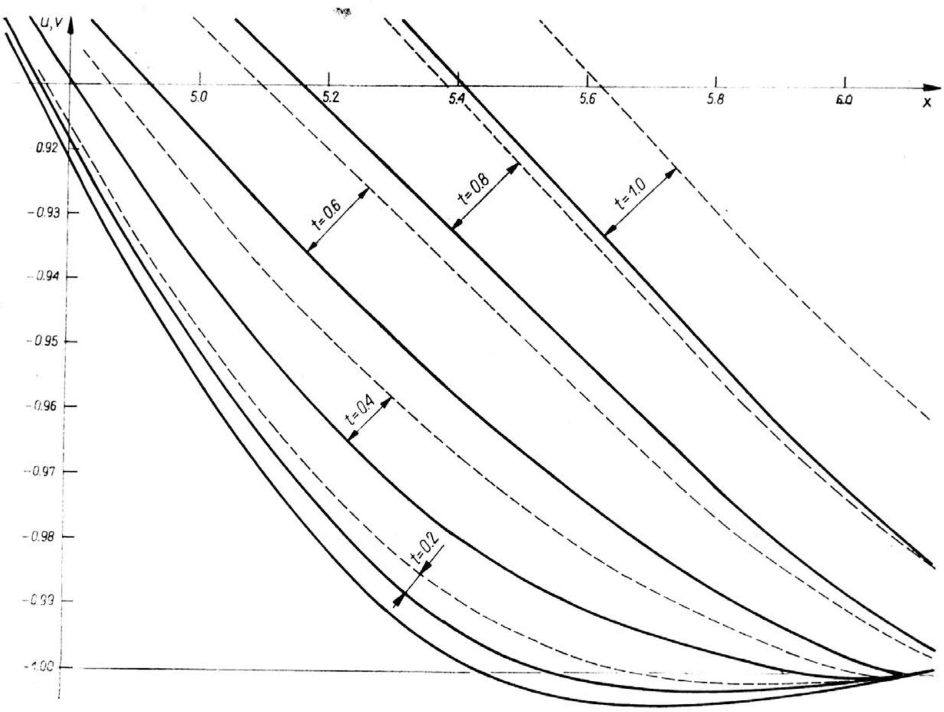

FIG. 9.

Assuming

$$u(x,t) = -\frac{4}{\pi\alpha}\left(\overset{(1)}{u}\sin\alpha + \frac{1}{3^2}\overset{(3)}{u}\sin 3\alpha + \ldots + \frac{1}{K^2}\overset{(K)}{u}\sin K\alpha\right), \tag{5.7}$$

$$v(x,t) = -\frac{4}{\pi\alpha}\left(\overset{(1)}{v}\sin\alpha + \frac{1}{3^2}\overset{(3)}{v}\sin 3\alpha + \ldots + \frac{1}{K^2}\overset{(K)}{v}\sin K\alpha\right),$$

we obtain the solution of the initial problem (5.2). With $\alpha = \pi/6$ the series is alternating and converges rapidly. The profiles shown in Fig. 8 are obtained under the assumptions $k_0 = 0.1$, $H = 1$, $K = 39$; dashed lines correspond to v, and solid lines — to u. In order to proceed with the analysis in the case of small times we may use the diagrams in Fig. 9 which present the enlarged neighbourhood of point $(5.5, -1)$. Wave profiles for $t = 5$

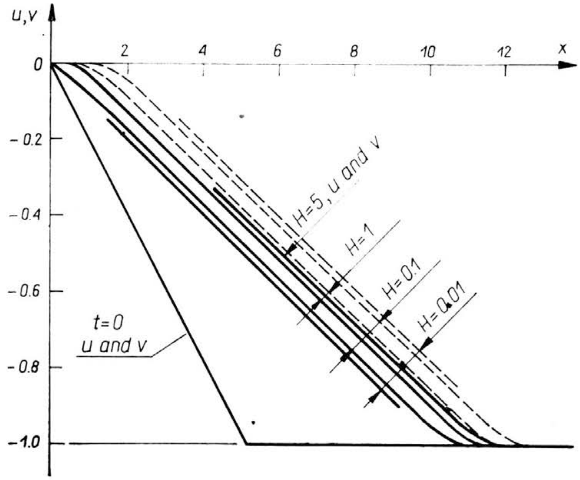

FIG. 10.

and various viscosities H are shown in Fig. 10. At low viscosities the displacements u and v differ from each other considerably, and at high viscosities $u \approx v$. For $H = 5$ the profiles u and v are practically identical.

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