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A model for the prediction of time-average quantities in fluid-solid mixtures (*)

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THIS WORK presents the development of a model for fluid-solid mixtures in pipelines. The two-phase mixture is modeled as a space-variable density fluid. The steady state Navier–Stokes equations are then solved in cylindrical or Cartesian coordinates. Given that the flow regime is almost always turbulent, Reynolds stresses appear in the flow; appropriate closure equations based on the mixing length theory are used in order to express these stresses. The model predicts time-average quantities such as velocity distributions (for both symmetric and asymmetric suspensions), pressure losses due to friction and solids concentration distributions. Comparisons of the predicted results with experimental data for pneumatic conveying systems and slurries give good agreement.

Praca przedstawia budowę modelu przepływu mieszanin „ciecz–ciało stałe" w rurociągach. Mieszaninę dwufazową modeluje się jako ciecz o gęstości zmiennej ze współrzędnymi przestrzennymi. Równania ruchu ustalonego Naviera–Stokesa rozwiązano w cylindrycznym i kartezjańskim układzie współrzędnych. Przy założeniu że przepływ jest prawie zawsze burzliwy, pojawiają się w nim naprężenia Reynoldsa, które wyrazić można za pomocą teorii długości mieszania. Model umożliwia przewidywanie uśrednionych po czasie wartości takich parametrów jak rozkład prędkości (dla zawiesin symetrycznych lub niesymetrycznych), straty ciśnienia spowodowane tarciem oraz rozkład koncentracji składnika stałego. Uzyskano dobrą zbieżność przewidywań teoretycznych z wynikami doświadczalnymi w przypadku pneumatycznych urządzeń przenośnikowych.

Работа представляет строение модели течения смесей „жидкость — твердое тело" в трубопроводах. Двухфазную смесь моделируется как жидкость с переменной плотностью с пространственными координатами. Уравнения установившегося движения Навье–Стокса решены в цилиндрической и декартовой системах координат. При предположении, что течение почти всегда турбулентное, появляются в ним напряжения Рейнольдса, которые можно выразить при помощи теории длины смешивания. Модель дает возможность предсказывать усредненные во времени значения таких параметров, как распределение скорости (для симметричных и несимметричных взвесей), потери давления, вызванные трением и распределение концентрации твердого компонента. Получено хорошее совпадение теоретических предсказываний с экспериментальными результатами в случае пневматических транспортных устройств.

Nomenclature

Latin

- A pipe cross-sectional area,
- d particle equivalent diameter,
- D pipe diameter,
- f friction factor,
- g gravitational acceleration,
- G mass flux,

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k pipe roughness,
I_1, I_2 dimensionless integrals,
l mixing length,
m constant,
$\dot{m}$ mass flow rate,
P pressure,
r radial distance,
r_0 radius of pipe,
Re Reynolds number,
u longitudinal velocity,
v transverse velocity,
V average velocity,
V^* shear velocity,
y coordinate (transverse),
z coordinate (longitudinal).

Greek

α parameter for density,
η boundary layer coordinate,
$\varkappa$ mixing length constant,
μ dynamic viscocity,
ν kinematic viscocity,
τ shear stress,
ϱ density,
ϕ area fraction.

Superscripts

s superficial,
$-$ average (time or space),
$'$ fluctuation,
$*$ dimensionless.

Subscripts

f fluid,
G gas,
m mean,
s solids,
u velocity,
w wall,
ϱ density.

1. Introduction

THE TRANSPORT of solid matter via carrier fluids (gas or liquid) is an old and efficient technique of solids transportation. The pneumatic transport and slurry transport of various solids have been used industrially since the 1920's. However, it is recently that both of these modes of transportation have received wide engineering attention, because of their low cost relative to the conventional ways of solid transport. The design of the components used for the transportation of solids in a carrier fluid is primarily based on experimental data and correlations. The theoretical analyses are still very few and they pertain exclusively to either pneumatic or slurry systems.

Among the experimental studies relevant to both gas-solids and liquid-solids mixtures, the one by ROSE and DUCKWORTH [16] contains a great deal of experimental data and widely used correlations for acceleration lengths, pressure gradients and other average quantities of interest. As regards the experimental studies on slurry flows, the data derived by NEWIT *et al.* [11], GAESSLER [3] and WASP *et al.* [23] are still used for design purposes. A recent survey of the pressure loss data by LAZARUS and NEILSON [9] developed an improved correlation for the pressure gradient based on previous experiments. Among the studies on gas-solid flows, of interest are the ones by ROSE and BARNACLE [15], KONCHESKY *et al.* [7, 8] and PFEFFER *et al.* [14]. The findings of all the above papers are summarized in the monograph by GOVIER and AZIZ [4] and the 8th Chapter of the handbook by HETSRONI [5].

It appears that most of the scientific work on fluid solid mixtures is empirical, with correlations playing an important role on the determination of the average flow variables. Two semi-analytical approaches by JULIAN and DUKLER [6] and SHOOK and DANIEL [18] suffer from the fact that many flow quantities must be known from empirical equations or are defined in an intuitive manner. A thoroughly analytical approach (MICHAELIDES, [10]) covers only the subject of air-solid mixtures and yields good agreement between the analysis and the experimental data.

The objective of this work is to develop a model for any fluid-solid mixture based on equations applicable to a variable-density fluid. This fluid is taken to behave as a complex single-phase one with variable denisty across the duct in which it flows. The variation of the density is due to the distribution of the solid particles inside the flow domain. This distribution is in accordance with the experimental data by SOO *et al.* [19, 20] and SPENCER *et al.* [21]. The variation of the local denisty contributes a second term to the Reynolds stresses of the flow; this term is due to the instantaneous density fluctuations and is modeled in the same way as the velocity fluctuation terms. Thus two closure equations are obtained for the Reynolds stresses, both emanating from an extension of Prandtl's mixing length hypothesis. For steady pipe flow this yields the velocity profile and hence the average velocity and mass flux for the flow can be estimated. The model yields also the shear stress, the friction factor and other average quantities of interest.

The model is based on phenomenological assumptions and yields good results for the time-average variables of the solids in fluid mixtures. It does not answer questions on the behavior of individual particles, their interactions or trajectories. It must be considered as a mechanistic model predicting those average quantities which are of interest to the pipeline designers.

2. General assumptions and formulation of the problem

This study examines the adiabatic, steady state flow of solid particles in fluids. It includes the flow in slurry pipelines or in pneumatic conveying systems, in a horizontal or vertical direction.

The model pertains only to axisymmetric flows with respect to density and velocity. This is always true if the pipeline is vertical. In horizontal flows the symmetrical assumption holds if the Froude number based on the particle diameters (V^2/gd) is very high. In such flows the effects of gravity on the distribution of particles are negligible and the flow is axisymmetric to a good approximation. This condition is satisfied at high average flow velocities (always true in the pneumatic systems) or small particle sizes (fine particle flows). The velocity in actual transportation systems is always high enough for the flow to be turbulent. Otherwise, the particles would settle and no transportation would take place.

The Navier-Stokes equations are developed for this type of flow. It is known that if the boundary conditions do not vary appreciably in one direction, then the problem posesses two length scales, one much longer than the other. Changes in fluid properties will occur much more gradually in the logitudinal direction than in the radial one. Thus, with the exception of the pressure gradient, all the other longitudinal derivatives may be neglected. Any logitudinal change may be introduced later with no loss of accuracy. This technique has been successfully applied in aerodynamics and in pipe flows. Accordingly, the flow is assumed in one dimension, z, with $\partial u/\partial r \gg \partial u/\partial z$. The pressure gradient in the radial direction is zero and the flow is taken to be isothermal. Under these assumptions the momentum equation reads

$$\frac{dP}{dz} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial r}(r\tau')+gz\varrho, \tag{2.1}$$

where P is the pressure, μ the dynamic viscosity, u the longitudinal velocity of the fluid, r the radial direction measured from the pipe center, τ' the Reynolds stresses, ϱ the density and g the acceleration due to gravity. In a horizontal flow $g = 0$.

In addition, the above assumptions yield

$$\frac{\partial P}{\partial r} = 0, \tag{2.2}$$

and

$$u = u(r). \tag{2.3}$$

3. The Reynolds stresses and closure equations

In a compressible (variable denisty fluid) it is well known (SCHLICHTING [17], PAI [13]) that

$$\tau' = -\overline{(u+u')(v+v')(\varrho+\varrho')}, \tag{3.1}$$

where u, v, and ϱ are the time-average values of the velocity and density and the primed quantities are the instantaneous fluctuations. The bar denotes time averaging. Given that the flow is one-dimensional, (hence, $v = 0$) and that the product $\overline{\varrho'v'u'}$ can be neglected in comparison to the other terms, the above equation becomes

$$\tau' = -\varrho\overline{u'v'}-u\overline{\varrho'v'}. \tag{3.2}$$

Thus a second term involving density and velocity fluctuations is added to the usual velocity fluctuation term.

The choice of the closure equation for the Reynolds stresses must satisfy the dissipation inequality

$$\tau' \frac{du}{dy} \geqslant 0, \tag{3.3}$$

where $y = r_0 - r$ and r_0 is the radius of the pipe.

One may apply the mixing length hypothesis to obtain the usual form for the time average product of the velocity fluctuations:

$$\overline{u'v'} = -l_u^2 \left(\frac{du}{dy}\right)^2, \tag{3.4}$$

where l_u is the mixing length for the velocity.

Similarly one may define a density mixing length l_ϱ, and employing the same arguments that led to the hypothesis expressed by Eq. (3.4), the following closure equation is obtained:

$$\overline{\varrho'v'} = -l_u l_\varrho \left(\frac{du}{dy}\right)\left(\frac{d\varrho}{dy}\right). \tag{3.5}$$

The inequality (3.3) is satisfied in general if the combination of Eqs. (3.2), (3.4), and (3.5) is written as

$$\tau' = \left|\varrho l_u^2 \frac{du}{dy} + u l_u l_\varrho \frac{d\varrho}{dy}\right| \frac{du}{dy}. \tag{3.6}$$

In the axisymmetric flows considered here all the quantities l_u, l_ϱ, du/dy and $d\varrho/dy$ are positive. Therefore the use of the absolute values is not required.

In the absence of any experimental data on the flow of solid particles the two mixing lengths l_ϱ and l_u will be assumed equal. The spatial variation of the mixing length will also be taken according to the original assumption by Prandtl:

$$l_\varrho = l_u = \varkappa y. \tag{3.7}$$

where $\varkappa$ is a constant.

While one may choose other functions for the mixing length such as van DRIEST'S [22] or NIKURADSE'S [12] functions, it is thought that in view of the other assumptions made, this refinement will not improve drastically the accuracy of the model's predictions but will complicate the computations. For this reason $\varkappa$ is taken to be 0.4 for air-solid flows and a known function given by Eq. (8.2) for the liquid-solid flows.

4. The density distribution

This study examines symmetric flows of solid particles. The Froude number in all these flows is high enough for the density of the flow to exhibit an axisymmetric distribution as observed by Soo and coworkers [19, 20] in gas-solid flows or NEWIT *et al.* [11]. For slurry flows (symmetric regime) under such circumstances the spatial variation of the density is given as follows:

$$\varrho = \varrho_f (1 + \alpha y / r_0)^m, \tag{4.1}$$

where ϱ_f is the denisty of the fluid and α and m are dimensionless constants. The maximum of the density occurs at the center of the pipe and it must be lower than the solids density:

$$\varrho_s > \varrho_f(1+\alpha)^m. \tag{4.2}$$

The last inequality yields an upper limit for α:

$$0 \leqslant \alpha < (\varrho_s/\varrho_f - 1)^{1/m}. \tag{4.3}$$

The space-average denisty $\bar{\varrho}$ is given by a spatial integration of Eq. (4.1):

$$\bar{\varrho} = \frac{2}{\alpha^2(m+1)(m+2)}\,[(1+\alpha)^{m+2}-1], \tag{4.4}$$

and the average area occupied by the solid particles $\bar{\phi}$ can be deduced form $\bar{\varrho}$ as follows:

$$\bar{\phi} = \frac{\bar{\varrho}-\bar{\varrho}_f}{\varrho_s \varrho_f}. \tag{4.5}$$

Experimental data (Soo *et al.*) show that m is a weak function of the solids concentration and its value is between 0.4 and 0.6. In this study m is assumed to be a constant for al the density profiles equal to 0.5. Thus the density distribution depends primarily on α which will be deduced from the solids concentration, as will be explained in Sect. 6.

5. The spatial variation of velocity

In horizontal flows the shear stress distribution is given from an integration of Eq (2.1) as follows:

$$\tau = \frac{r}{2}\cdot\frac{dP}{dz} = \frac{r}{r_0}\tau_w, \tag{5.1}$$

where τ includes both the viscous and turbulent components of the stress and τ_w is it value at the wall. According to Eqs. (2.1), (3.6) and (3.7), the above equation yields

$$\frac{r}{r_0}\tau_w = \mu_f\frac{du}{dy} + \varkappa^2 y^2\left(\varrho\frac{du}{dy} + u\frac{d\varrho}{dy}\right)\frac{du}{dy}. \tag{5.2}$$

The last equation is made dimensionless by using the shear velocity $V^* = \sqrt{\tau_w/\varrho_f}$ and the usual boundary layer coordinate $\eta = \ln(y/y^*)$, where y^* is a measure of the laminar boundary layer. Thus Eq. (5.2) becomes

$$\tau_f V^{*2}[1-e^{\eta-\eta^*}] = \frac{\mu_f}{r_0 e^{\eta-\eta^*}}\frac{du}{d\eta} + \varkappa^2\left[\varrho\frac{du}{d\eta} + u\frac{d\varrho}{d\eta}\right]\frac{du}{d\eta}, \tag{5.3}$$

where $\eta^* = \ln(r_0/y_0^*)$ and $y = r_0 e^{\eta-\eta^*}$. The measure of the laminar sublayer is given by the following equation (MICHAELIDES [10]):

$$y^* = 0.111k\left(\frac{\nu_f}{kV^*} + 0.3\right), \tag{5.4}$$

where ν_f is the kinematic viscosity of the fluid and k is the roughness of the pipe. Thus η^* becomes

$$\eta^* = \ln\frac{r_0}{y^*} = 2.198+\ln\frac{r_0}{k}-\ln\left(\frac{\nu_f}{kV^*}+0.3\right). \tag{5.5}$$

Equation (5.3) when rearranged yields a nonlinear differential equation for the velocity profile:

$$\varkappa^2\varrho^*\left(\frac{du^+}{d\eta}\right)^2+\left(\varkappa^2\varrho^+u^+\frac{d\varrho^*}{d\eta}+\frac{1}{\mathrm{Re}^*e^{\eta-\eta^*}}\right)\frac{du^*}{d\eta}-(1-e^{\eta-\eta^*}) = 0, \tag{5.6}$$

where

$$u^* = \frac{u}{V^*},$$

$$\varrho^* = \frac{\varrho}{\varrho_f}, \tag{5.6'}$$

$$\mathrm{Re}^* = \frac{V^*r_0\varrho_f}{\mu_f}.$$

The boundary condition for this differential equation is $u^*(-\infty) = 0$. Given that the width of the laminar sublayer is very small, the usual assumption is made here, choosing $u^*(0) = 0$. Actually, the choice of the η^* expression Eq. (5.5) allows this simplification (MICHAELIDES [10]).

Equation (5.6) may be solved as a quadratic equation for $du^*/d\eta$. Since it is known that the velocity increases in the η direction, the positive root of this equation needs to be taken and numerically integrated from $\eta = 0$ to $\eta = \eta_0^*$. This yields the time-average velocity profile $u^*(\eta)$ or $u^*(y)$.

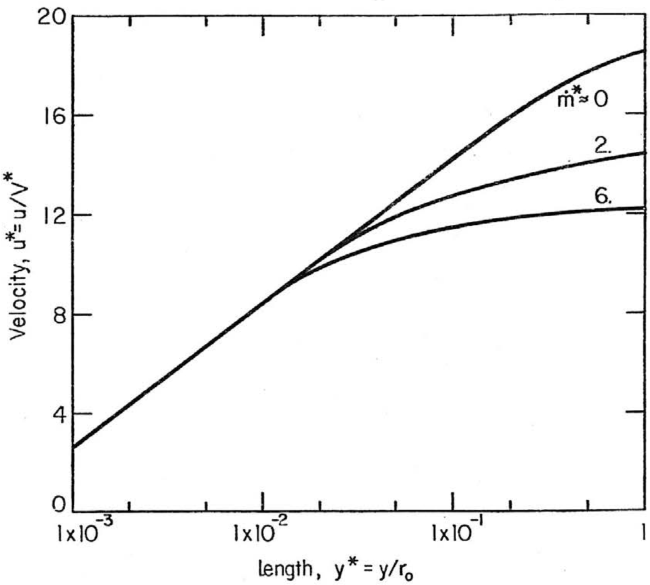

FIG. 1. Velocity distributions for various loadings.

The velocity profiles for dilute air-solid mixtures are computed and depicted in Fig. 1 for various loadings ($\dot{m}$ is the ratio of the solids to gas mass fluxes). It is observed that the average velocity profile in the complex mixture is different (flatter) than when the gas flows alone. This result is confirmed by experimental observations (Soo and coworkers [19, 20]) and other semianalytical methods (JULIAN and DUKLER, [6]).

6. The volumetric flux and the mass flux

The volumetirc mass flux (or space-average velocity) and the total mass flux (space-averaged also) are always known to the engineer in the flow situations considered here (or can be deduced from given data as explained by Govier and Aziz or Hetsroni). In terms of the time-average quantities u and ϱ, the two fluxes are defined as follows:

$$V = \frac{2}{r_0^2} \int_0^{r_0} ur\,dr = V^* I_1, \tag{6.1}$$

and

$$G = \frac{2}{r_0^2} \int_0^{r_0} u\varrho r\,dr = V^* \varrho_f I_2. \tag{6.2}$$

The two integrals I_1 and I_2 may be written in terms of the boundary layer η (and after integration by parts is performed) as follows:

$$I_1 = \int_0^{\eta^*} (1-e^{\eta-\eta^*})^2 \frac{du^*}{d\eta}\,d\eta, \tag{6.3}$$

and

$$I_2 = \int_0^{\eta^*} \left[u^* \frac{d\varrho^*}{d\eta} + \varrho^* \frac{du^*}{d\varrho}\right] (1-e^{\eta-\eta^*})^2 d\eta. \tag{6.4}$$

Thus the integrals I_1 and I_2 can be evaluated numerically. Given that V and G are known quantities, Eqs. (6.1) and (6.2) can be perceived as two integral equations with two unknowns V^* and α. V^* is an explicit unknown while α enters the two integrals implicitly through the denisty function $\varrho(y)$ or $\varrho(\eta)$. The two equations may be solved by iteration for the correct values of α and V^*. In practice, a computer program is developed, which assumes initial values for α and V^* and a Runge–Kutta subroutine is used to integrate the velocity profile and yield the corresponding values of V and G. Then the initial values of α and V^* are corrected appropriately and new results for V and G are generated. After a number of such iterations the resulting V and G become approximately equal to their actual values and the iteration stops. The last V^* and α values computed are taken as the correct values of these two variables and are subsequently used for the calculation of the other flow parameters or for comparisons with experimental data.

7. The pressure loss and the friction factor

An average friction factor f_m is defined for the flowing mixture according to the following equation:

$$f_m = \frac{\tau_w}{\frac{1}{2}\varrho_f V_f^{s2}}. \tag{7.1}$$

Here V_f^s is the superficial velocity of the fluid (GOVIER and AZIZ, [4], HETSRONI, [5]) defined as the mass flux of the fluid divided by its denisty $[V_f^s = \dot{m}_f/(\varrho_f A)]$. Given that the shear stress τ_w is known in terms of the shear velocity V^* (which can be calculated by the method explained in Sect. 6), the above equation may be rewritten as

$$f_m = \frac{2V^{*2}}{V_f^{s2}}. \tag{7.2}$$

The pressure loss per unit length accordingly is given by the following equation:

$$-\frac{dP}{dz} = \frac{2}{r_0}\tau_w = \frac{f_m \varrho_f V_f^{s2}}{r_0}. \tag{7.3}$$

8. Comparison with experimental data and correlation

The determination of the pressure loss or, equivalently, of the friction factor is of great importance to engineering applications. Because of this, several experimental projects have been undertaken in the past to provide data on these two quantities. The experiments span a wide range of fluids used as carriers, several solid species differing in size, shape, density and concentration (or loadings). It was widely recognized that the amount of solids expressed in terms of loading or concentration and the ratio of densities ϱ_s/ϱ_f are the most important factors in the determination of the pressure loss (PFEFFER *et al.* [14]. KONCHESKI *et al.* [8]). These two parameters are the ones entering explicitly the equations of the present model. Other parameters such as the ratio of particle diameters to that of the pipe (d/D), particle shape factors or distribution of sizes do not enter the model explicitly, but may be accounted for by using an appropriate form of the mixing lengths l_ϱ and l_u or of the denisty gradient.

Some of the predictions of the present model are given here in Fig. 2 where the predicted friction factor f_m is plotted against the superficial Reynolds number Re_G^s for an air-solids flow. This type of Reynolds number is defined as

$$\mathrm{Re}_G^s = \frac{V_G^s \varrho_G D}{\mu_G} \tag{8.1}$$

and is widely used in the literature. The subscript G refers to the gas phase and D is the pipe diameter. The parameters in the figure are the pipe roughness and the loading $\dot{m}^*$. For the calculations the solids density is taken as $\varrho_s = 1000$ kg/m³ (similar to the density of some polymers). The figure is the equivalent of a Moody diagram for air-solid flows with an added parameter, $\dot{m}^*$.

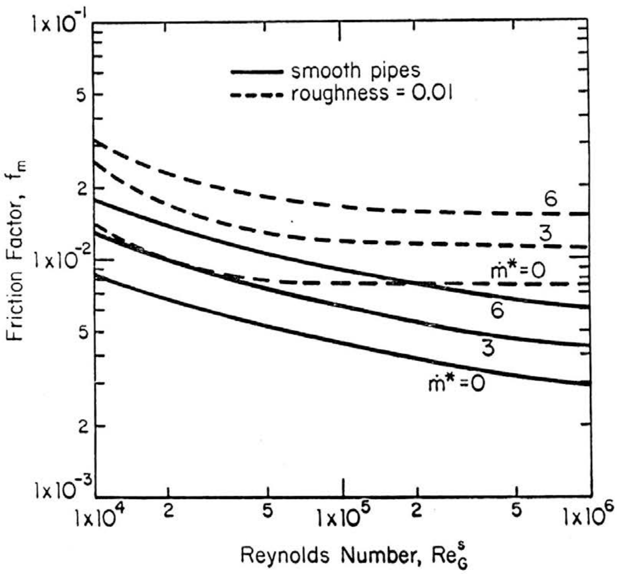

FIG. 2. The friction factor for different loadings and pipe roughnesses.

The results emanating from the present model are compared with experimental data or correlations under the same flow conditions (same pipe diameter, solids density, mass flow rates and average velocity). Figure 3 shows such a comparison with Pfeffer's correlation [14] for air-solids mixtures. It is seen that there is a good agreement between the results and the data.

The present model's results were compared also with data for air-solids systems from the projects of ROSE and BARNACLE [15] and DOGIN and LEBEDEV [1]. The comparison is

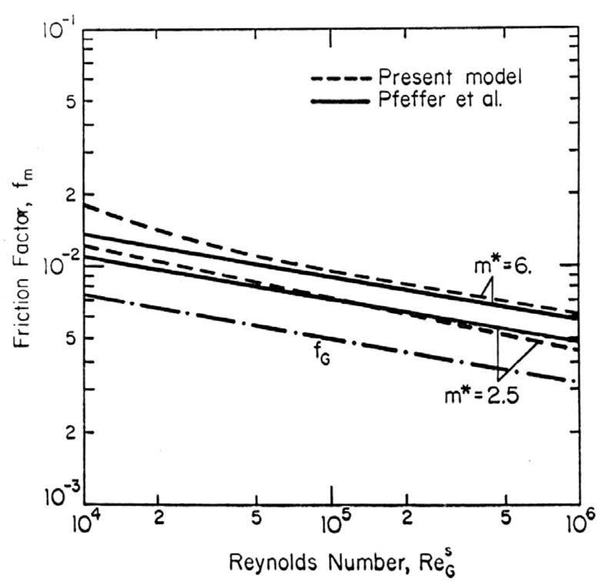

FIG. 3. Comparison of the results from present model with Pfeffer's correlation.

depicted graphically in Fig. 4 where the friction factor is plotted against the superficial gas Reynolds number. All the data and the results are for loading $\dot{m}^* = 6$ and ratio of densities equal to 1000. It is observed that the results from this study agree well with the Dogin and Lebedev data and the Pfeffer *et al.* data. The agreement with the Rose and

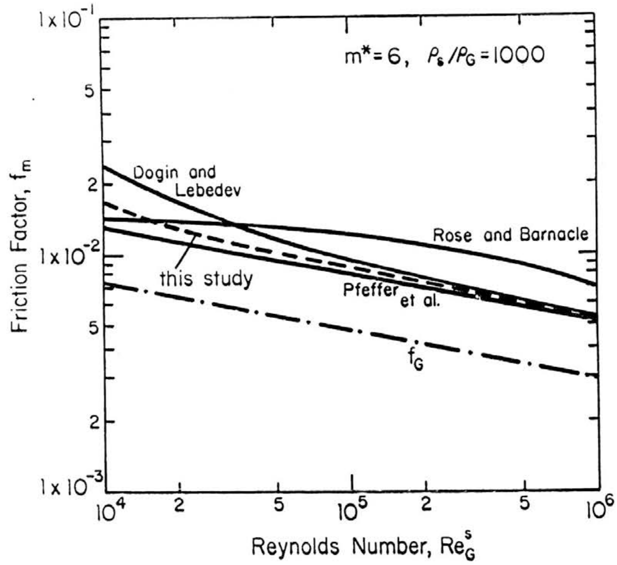

FIG. 4. Comparison of the results from the present model with results emanating from empirical correlations.

Barnacle data, although not so good, is within the uncertainty of the experimental correlation.

The last comparison with results from pneumatic systems is with the semiempirical model proposed by Julian and Dukler. This is shown in Fig. 5 where good agreement between the two models is observed.

As regards the slurry flows, comparisons were made with experimental data gathered by NEWIT *et al.* [11] and WASP *et al.* [23]. The most common representation of pressure loss data in slurries is a graph of $-dP/dz$ (or $\Delta P/\Delta L$) versus the mean flow velocity V_m (same as the space average velocity V). The same coordinates, although dimensional, were chosen to be used here because they are familiar to the engineer working with slurry systems. Thus the data of Newit *et al.* are plotted in Fig. 6 together with results from this model for a pipe carrying water and fine sand of 10% concentration. Similarly, in Fig. 7 the data are from an experiment with coarse sand and 25% concentration. In both figures there is good agreement between this model and the experiment at high mean velocities. However, at lower velocities the results diverge because the flow is not axisymmetric any more and some of the basic assumptions of the model are not met.

Another comparison with data from coal slurries was made. Here the data of Wasp *et al.* were compared with the results of the present model. The results are depicted in Figs. 8 and 9 for 5 and 15% concentrations respectively. The agreement is very good at

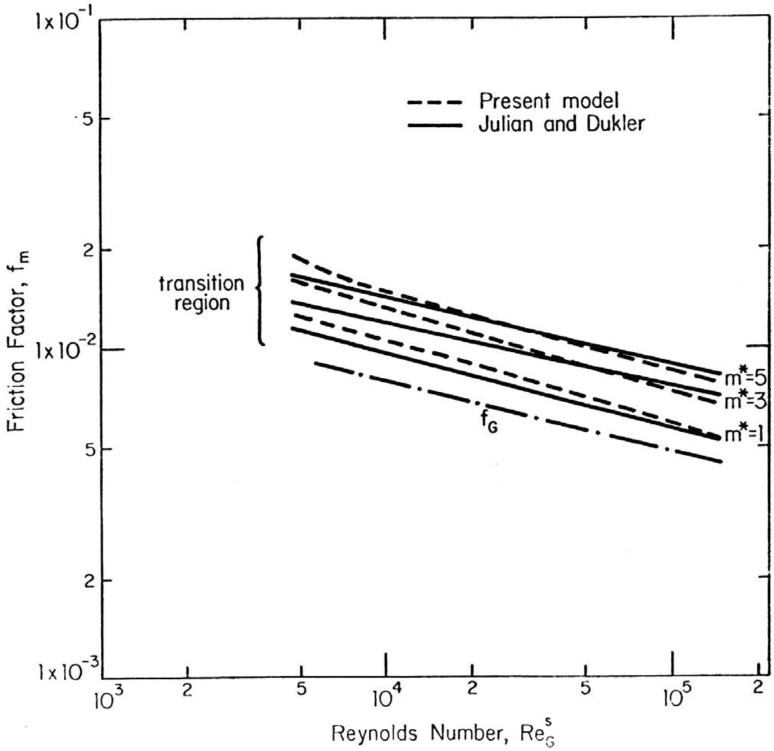

FIG. 5. Comparison of the results from the present model with the ones from Julian and Dukler's model.

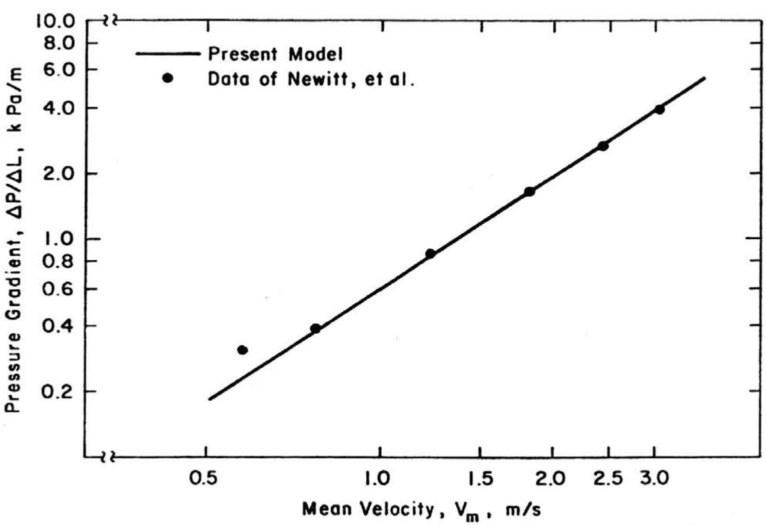

FIG. 6. Comparison with data of Newit et al. Water and fine sand, 10% concentration.

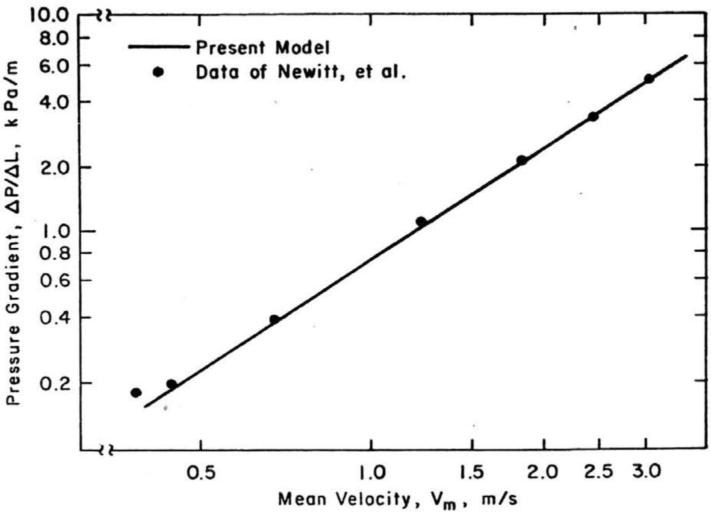

FIG. 7. Comparison with the Newit et al. data: Water and coarse sand 25% concentration.

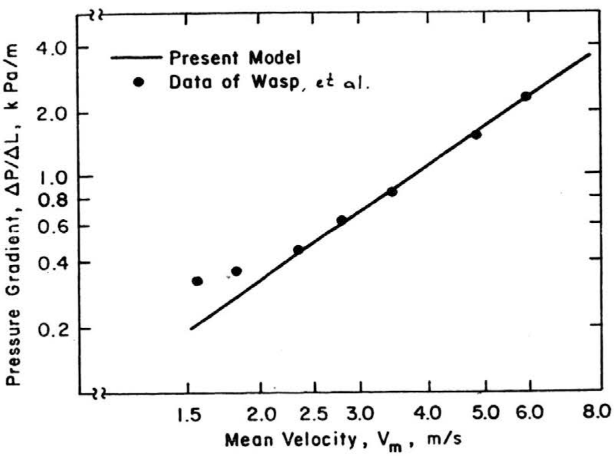

FIG. 8. Coal slurry data from Wasp et al. Concentration of solids 5%.

high velocities, but again the flow becomes asymmetric at lower mean velocities and the results disagree as expected.

It must be pointed out that for the air-solid results the mixing length constant $\varkappa$ is taken to be equal to 0.4 (as suggested by Prandtl). However, in the computation of the slurry data, $\varkappa$ is made a function of α and V^* to yield better agreement with the experimental data. An explicit form of this function is (FARMER [2])

$$\varkappa = 0.4\left[1.2-\frac{V^*}{\sqrt{gr_0}}+0.417\,\frac{V^{*2}}{gr_0}\right]. \tag{8.2}$$

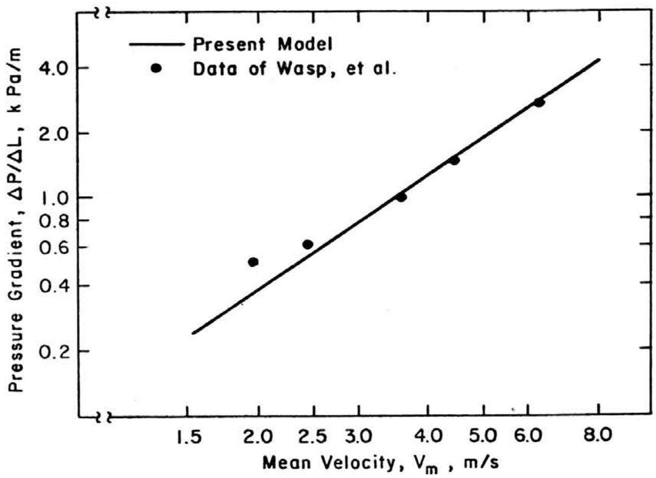

FIG. 9. Comparison with the data from Wasp et al. Coal slurry of 15% concentration.

9. Conclusions

This work outlines a model for the flow of solid particles in suspensions. The flowing mixture is taken to be as a nonhomogeneous single phase fluid with spatially variable density. For such a fluid in a circular pipe, the Navier-Stokes equation is written and developed according to assumptions and techniques applicable to turbulent flows. Thus the Reynolds tresses would contain two terms instead of one and the mixing length hypothesis would be used for them as a closure equation.

The results of this model may predict the velocity and density variations, space average mass and volume fluxes and the frictional pressure loss. These results are in good agreement with the experimental data from pneumatic and slurry flows as shown in Figs. 3 to 9.

It must be emphasized that the model is a mechanistic one and yields only time-average results. It does not answer any questions of flow transients or instantaneous variations. Furthermore, the averaging procedure adopted disregards the effects of particle sizes and shapes. However, the flows of symmetric suspensions are very well predicted and possible improvements on the closure equations guarantee even better agreement.

Acknowledgements

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