

## NUMERICAL SIMULATIONS OF HARDENING AND COOLING OF THE EARLY-AGE CONCRETE

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**Abstract:** The paper presents a new method of control, modeling and identification of thermal fields in concrete structures. The proposed methodology boils down to four interdependent tasks: 1) the determination of a time-dependent intensity of the heat of hardening and other thermophysical properties of concrete by means of the numerical inverse problem solution, which is based on experimental measurements of temperature, 2) numerical modeling of the evolution of the thermal field in the maturing concrete structure, using material properties determined in the first point, 3) experimental verification of a thermal field modeling (point 2) using 2D and 3D concrete samples, 4) the formulation of the problem of optimum design of the cooling system to mitigate thermal stress concentrations during maturing of massive concrete structures.

**Key words:** concrete structures, inverse heat transfer problem, pipe cooling system.

### 1. Introduction

Early-age concrete behaviour is a problem of great concern. Especially important are thermal issues in a fresh concrete because of exothermic nature of hydration reactions and possible negative effects of high temperature on mechanical concrete properties. To calculate temperature evolution in concrete structures accurate values of the thermo-physical coefficients are needed together with the precise numerical model. In proposed method a standard heat conduction equation is used to compute temperature distribution in concrete elements. To develop individual parameters appearing in the heat equation for a particular concrete mixture a method based on the inverse heat transfer problem [1–2] combined with point temperature measurements in the one-dimensional mold (Fig. 1a) is adapted. The identification is formulated as an optimization problem and it is solved by means of mesh adaptive direct search (MADS) algorithm [3]. The solution of the inverse problem allows to calculate temperature fields in any concrete structure. In this paper the so-called 2D and 3D cases are presented (see Fig. 1b and 1c). However, in case of very massive concrete structures like bridge pylons or dams, there is a problem with heat removal due to relatively big size of the structure and low thermal

conductivity of cement-based materials. In such cases, to improve heat dissipation, often a cooling pipe system is introduced to the construction. In this paper a simple model of equispaced pipe network [4] is used to propose the optimal cooling system. To perform this task two cost functions, maximum temperature and maximum temperature gradient are minimized simultaneously. It leads to the bi-objective optimization problem, which is solved by means of BiMADS algorithm [3]. The result is a set of undominated points which are Pareto optimal.

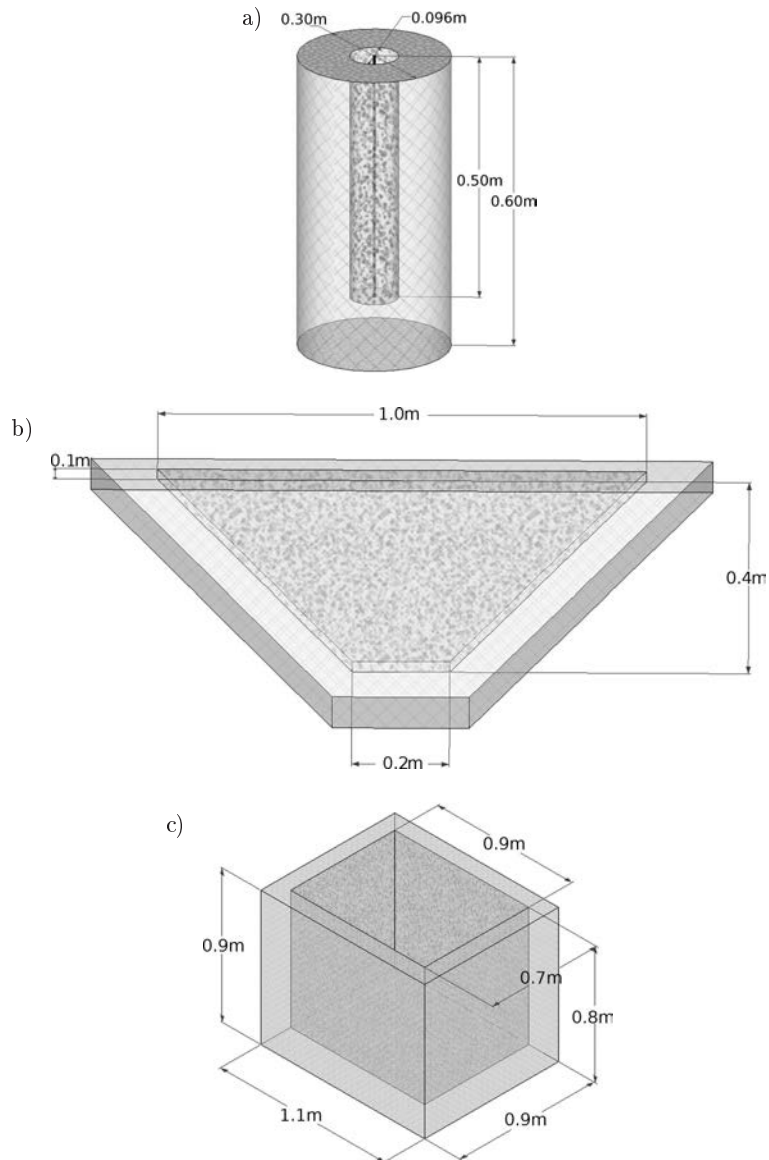


FIG. 1. Measurement molds: a) 1D, b) 2D, c) 3D.

## 2. Mathematical model

In general case heat conduction equation for concrete can be written as follows:

$$(1) \quad \rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + S,$$

where  $T$  is the temperature,  $S$  is an internal heat source,  $\rho$  is density,  $c$  is specific heat capacity and  $k$  is a thermal conductivity. In proposed model Eq. (1) is solved simultaneously with the ordinary differential equation for equivalent age of concrete  $t_e$  (which replaces time in case of transient thermal properties of concrete):

$$(2) \quad \frac{dt_e}{dt} = \beta_T = \exp \left[ \frac{E}{R} \left( \frac{1}{293} - \frac{1}{T + 273} \right) \right],$$

where in Eq. (2)  $E$  is an activation energy,  $R$  is the universal gas constant and  $\beta_T$  is maturity function. Standard Dirichlet and Neumann boundary conditions are used. The problem is solved by means of the method of lines [5] in one-dimensional case, and using in-house finite element method software in other cases [6]. To determine thermophysical parameters the objective function  $E$ , which is based on a numerical calculation  $T^n$  and experimental temperature measurement  $T^m$ , is introduced. It is important to bear in mind that all values of thermal properties are changing during hydration process, which depends on temperature, chemical composition of concrete and mixing ratio. Therefore the objective function depends on a set of unknown parameters  $\alpha$  which describe three unknown functions: heat source, specific heat capacity and thermal conductivity:

$$(3) \quad E(\alpha) = \|T^n - T^m\| + \gamma \sum_{p=1}^P \alpha_p^2.$$

The term  $\gamma$  in Eq. (3) is a Tikhonov regularization parameter introduced to the model to improve stability of the objective function. Each problem was solved at least 20 times with direct search optimization algorithm with random initial values for unknown  $\alpha$  parameters. Additionally, the information about the heat loss through wall was introduced to the model.

In order to calculate the influence of cooling pipe network on temperature distribution in massive concrete structure, the heat conduction equation for concrete (Eq. (1)) has to be coupled with the heat equation for water. To solve this task a series of simplifications were applied. Firstly, the heat flow is considered only in a representative cylindrical element conceptually extracted from the whole structure with centrally located cooling pipe [4]. It is assumed that concrete cylinder is thermally insulated on the boundaries. This assumption makes the conduction in  $z$  direction much smaller than in  $r$  direction, thus the derivative with respect to  $z$  can be omitted in Eq. (1) (the system of equations is written in cylindrical coordinates). Due to relative fast velocity of liquid through the pipe the Reynolds number is approximately equal to 13 500 for typical values in described problem and as a consequence the water flow is turbulent. It means that mean velocity in radial and angular direction is equal to zero. Also it is assumed that temperature of water  $\theta$  only minimally depends on the radial position and instead

the mean temperature  $\bar{\theta}$  is considered. The next assumption is based on the fact that the Nusselt number for the problem is approximately equal to 100, it means that the conductive component is much smaller than the convective one and can be neglected in heat equation for water. Last simplification leads to neglect the time derivative, because of different time scale for concrete and water [4]. The final set of equation can be written as

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + S,$$

$$(4) \quad \frac{dt_e}{dt} = \beta_T,$$

$$\bar{\theta} = T - (T - \theta_0)e^{-\tilde{H}z},$$

where  $\theta_0$  denotes initial temperature of water and  $\tilde{H}$  is heat exchange coefficient between cooling pipe and concrete. The set of Eqs. (4) is solved iteratively for subsequent concrete layers in the  $z$ -direction and as a result one can obtain temperature evolution in concrete sleeve and in piped water. Based on calculated temperature it is possible to perform a multi-objective optimization using bi-objective BiMADS algorithm [3] with the following cost function:

$$(5) \quad \min_{\Omega, t} F = \min_{\Omega, t} (\max_{\Omega, t} T, \max_{\Omega, t} \nabla T).$$

The BiMADS launches successive runs of MADS algorithm on a single-objective reformulations of the problem to construct Pareto front of undominated points.

### 3. Results

Laboratory tests were performed on two kinds of experimental concrete mixture: with high calcium fly ash addition (mixtures A) and mixtures where cement content was elevated over standard levels (mixtures B). The results of temperature evolution during hardening of concrete in 1D molds are presented in Fig. 2. In each case measurements were performed with two sensor configurations. It is clearly seen that fly ash addition decreases maximum temperature. The results of the inverse problem solution are presented in Fig. 3. Similarly as in the case of temperature the intensity of the heat source is much higher for mixture B. Based on the analytical approximations of the heat source functions an optimization problem for cooling system was solved in respect to the inlet water temperature ( $\theta_0$ ), pipe wall thickness ( $s$ ), water flow ( $Q$ ), concrete cylinder radius ( $R$ ) and pipe radius ( $a$ ). Figure 4 shows the Pareto front for two concrete mixtures (in each case three examples of solutions are presented in detail). Figure 4a shows Pareto front for mixture A, and Fig. 4b shows it for mixture B. As mentioned earlier in this approach there is no possibility to choose one optimal solution because of the multi-objective optimization nature. Finally, results of the comparison between model and measurements are presented in Fig. 5. The temperature measurements obtained in the experiment have been compared with calculated values in 2D mold for mixture B

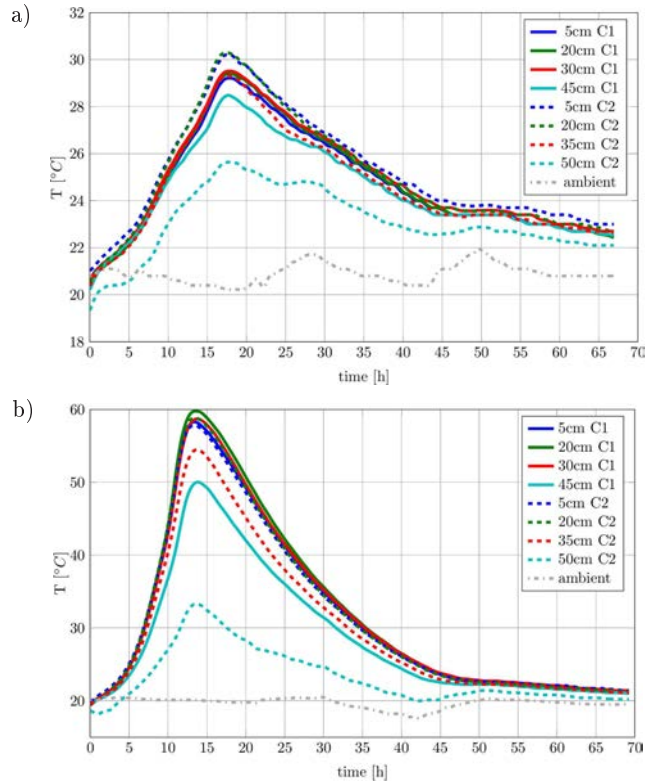


FIG. 2. Temperature evolution during hardening of concrete in 1D molds: a) mixture A, b) mixture B. Solid lines denote measurement in configuration C1, and dashed lines in configuration C2. Sensor positions in each configuration are listed in the legend.

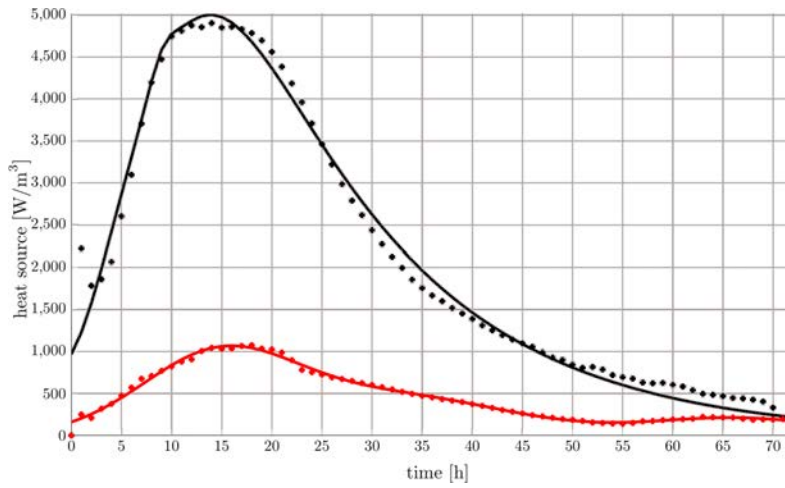


FIG. 3. Heat source determined by means of inverse heat transfer problem solution for mixture A (red points) and mixture B (black points). Solid lines denote fitted analytical approximations of these functions.

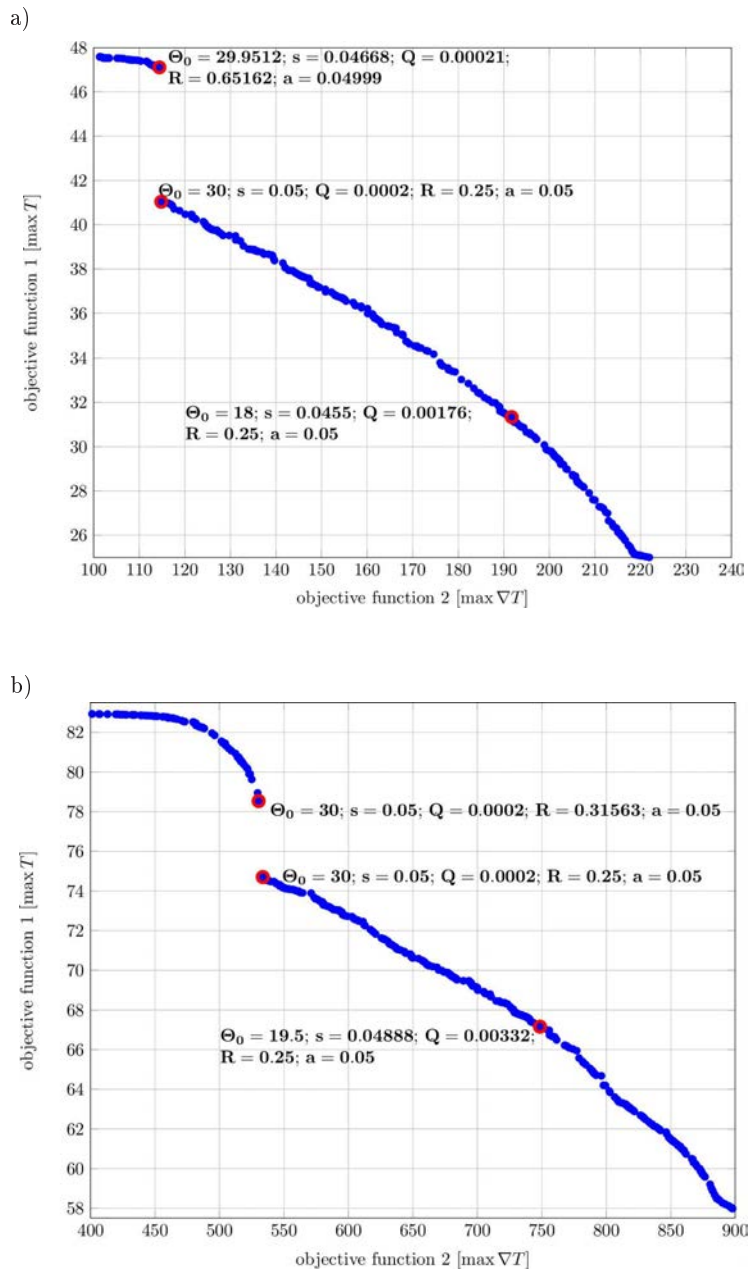


FIG. 4. Pareto front: a) for mixture A, b) for mixture B.

and in 3D mold for mixture A (in the latter case presented temperatures were measured in the cross-section through the center of cuboid mold). Calculated temperature fields show fair agreement in each case, which proves that implemented model can be successfully used to calculate temperature in concrete structures.

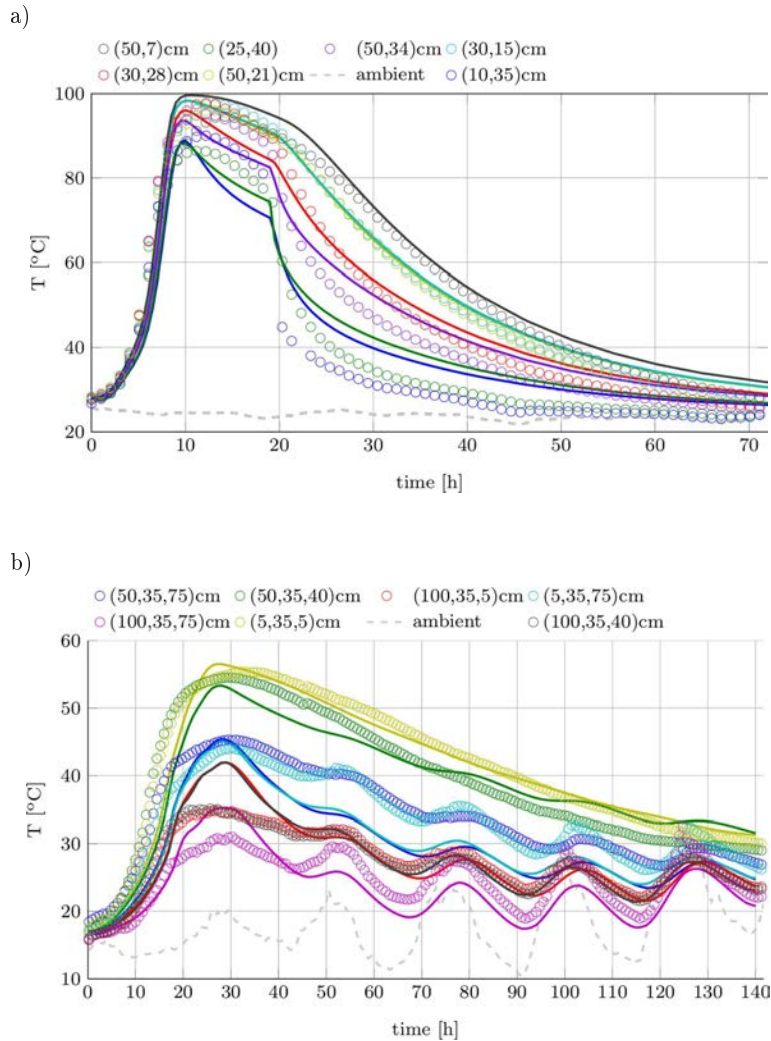


FIG. 5. Model validation: a) 2D case – mixture B, b) 3D case (cross-section) mixture A. Lines denote numerical solution, circles denote measurements.

#### 4. Conclusions

Thermal properties of hardening concrete were effectively determined using an unconventional approach. The obtained simulation results are consistent with experimental data, which indicates that this procedure can be adequate in real-size structure. Therefore the proposed procedure can replace the costly and time-consuming experimental studies designed to determine thermal properties of any concrete mixture by point temperature measurements and numerical solution of the inverse heat transfer problem. The solution gives also hints at how to design an optimal cooling system to avoid thermal cracking.

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