ELLIPTICITY OF LARGE STRAIN THERMO-PLASTICITY: THEORY AND NUMERICAL ANALYSIS

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1. Introduction

When instability in a material occurs, for example due to material or thermal softening, the boundary-value problem loses its ellipticity and, as a consequence, becomes ill-posed. The theoretical solution of such problem leads to strain localization in a set of measure zero (for three-dimensional problem it is a surface), whereas the results of numerical simulations pathologically depend on discretization [4]: the size and the mesh orientation govern the width and the direction of the localized deformation band. Thus, if ellipticity is lost the problem requires some regularization.

The ellipticity conditions are well described for isothermal problems involving large strains or inelastic behaviour, see e.g. [6] or [5] However, in the case of thermo-mechanical coupling researchers usually assume some limitations, for example small strains, elasticity [1], or internal adiabaticity (lack of heat conduction) [3]. The aim of this paper is the analysis of the ellipticity condition for large strain thermo-plasticity.

2. Material model

The considered material model involves hyperelasticity and plasticity with the von Mises yield criterion and associative flow rule. Young's modulus and initial yield strength may depend on the change of temperature. The thermomechanical coupling includes thermal expansion and heat production in a plastic process. Following [7], the multiplicative split of the deformation gradient into reversible and plastic components is used, $\mathbf{F} = \mathbf{F}^r\mathbf{F}^p$, and the free energy function has the form

$$(1)\quad \psi(\mathbf{b}^r,\gamma,T) = \frac{K}{2}\left[\frac{1}{4}\left[\ln(J^{br})\right]^2 - 3\alpha_T(T-T_0)\ln(J^{br})\right] + \frac{G}{2}\left[\mathrm{tr}([J^{br}]^{-1/3}\mathbf{b}^r) - 3\right] + \psi^p(\gamma) + \psi^T(T)$$

where $\mathbf{b}^r = \mathbf{F}^r(\mathbf{F}^r)^{\mathrm{T}}$, γ is a measure of plastic strain, T is absolute temperature, K and G denote material parameters, $J^{br} = \det(\mathbf{b}^r)$, α_T is a thermal expansion coefficient and T_0 – referential temperature. The implementation of a similar large strain thermoplasticity model within *AceGen/FEM* packages is presented in [8].

3. Ellipticity verification

The condition of ellipticity loss can be derived in two ways. The first one is based on the examination of equilibrium on a discontinuity surface [3]. It is assumed that a jump of traction (and its rate) across discontinuity surface is zero

$$(2)\qquad [\![\boldsymbol{\sigma}]\!]\mathbf{n} = \mathbf{0}, \qquad [\![\dot{\boldsymbol{\sigma}}]\!]\mathbf{n} = \mathbf{0}$$

where $\boldsymbol{\sigma}$ is Cauchy stress tensor and $\mathbf{n}$ is a normal to the discontinuity surface in the current configuration. For the thermomechanical coupling a zero jump of heat flux $\mathbf{q}$ and of its rate is also required

$$(3)\qquad [\![\mathbf{q}]\!]\cdot\mathbf{n} = 0, \qquad [\![\dot{\mathbf{q}}]\!]\cdot\mathbf{n} = 0$$

The second approach involves the analysis of the perturbation of a base state [1]. On the initially homogeneously deformed specimen with constant temperature distribution the following perturbations are imposed

$$\mathbf{u}^{pert}(\mathbf{x},t) = \exp(ik\,(\mathbf{n}\cdot\mathbf{x}+vt))\hat{\mathbf{u}}, \qquad T^{pert}(\mathbf{x},t) = \exp(ik\,(\mathbf{n}\cdot\mathbf{x}+vt))\hat{T} \tag{4}$$

where $\mathbf{x}$ is the particle current position, t is time, $\hat{\mathbf{u}}$ and $\hat{T}$ are constant amplitudes, v is wave speed, k – wave number and i – imaginary unit. Inserting definitions (4) into balance equations for linear momentum and energy, the set of equations for $\hat{\mathbf{u}}$ and $\hat{T}$ is obtained.

Both approaches might be formulated in the referential or current configuration. It can be noted that for the coupled thermomechanical problem both methods lead to a set of two equations. There are two special limit cases which are analysed: isothermal and adiabatic, for which singularity of isothermal and adiabatic acoustic tensors, respectively, indicates the loss of ellipticity. It is worth mentioning that an alternative approach involving the analysis of generalized eigenvectors is considered in [2].

The derived conditions are numerically tested for samples in tension, simulated within the finite element method using *AceGen/FEM* package. The crucial feature of the package is automatic differentiation which is efficiently applied for the calculation of material tangents required to obtain acoustic tensors. The ellipticity conditions are verified at selected Gauss points after converged load steps. The exemplary output of the ellipticity analysis is presented in Figure 1.

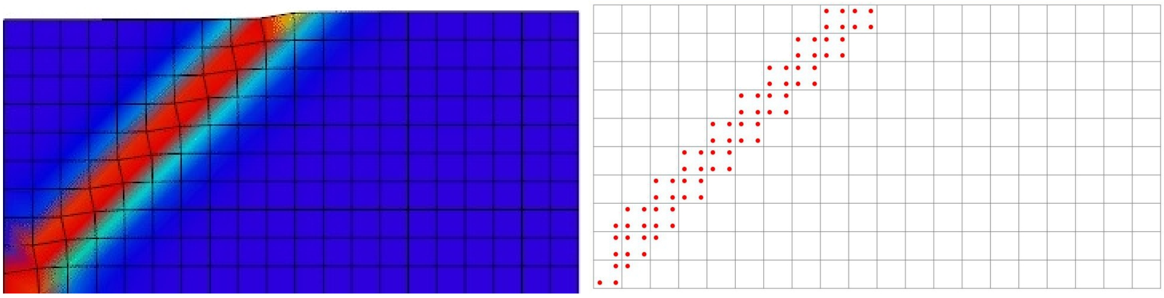

Figure 1: Deformed sample with plastic strain measure distribution (on the left) and Gauss points at which ellipticity is lost (on the right) – results for large strain elastoplasticity

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