MICROMORPHIC MODEL FOR SIMULATION OF LÜDERS-LIKEBANDS IN SMAs

M. Rezaee Hajidehi and S. Stupkiewicz

Institute of Fundamental Technological Research, Polish Academy of Sciences, Warsaw, Poland

e-mail: mrezaee@ippt.pan.pl, sstupkie@ippt.pan.pl

1. Introduction

Based on experimental observations, the stress-induced pseudoelastic response of shape memory alloys (SMAs), e.g. the uniaxial response of NiTi wire in tension, is often accompanied by strain localization and macroscopic phase front propagation. To simulate this phenomenon appropriately, a gradient-enhanced model and its micromorphic counterpart are developed [2]. The latter, which facilitates the computational treatment, is performed by adding an extra variable, the micromorphic peer of the volume fraction of martensite, into the model, see Maziére and Forest [1]. The resulting micromorphic model is then implemented into a finite-element framework based on an incremental energy approach combined with the augmented Lagrangian treatment, see Stupkiewicz and Petryk [3].

Finally, to study the effect of loading rate on the localization and fronts propagation, the micromorphic model is extended to a thermomechanically coupled version.

2. The thermomechanically coupled micromorphic model

In this study, the Helmholtz free energy function of SMA is adopted in the following form,

$$\phi(\varepsilon,\eta,T)=\phi_{chem}(\eta,T)+\phi_{el}(\varepsilon,\eta)+\phi_{int}(\eta) \qquad \text{with} \qquad \phi_{int}=\frac{1}{2}H\eta^2, \tag{1}$$

where $\phi_{chem}(\eta,T)$, $\phi_{el}(\varepsilon,\eta)$ and $\phi_{int}(\eta)$ are, respectively, the chemical, elastic and interaction energy contributions, η is the volume fraction of martensite, T is the temperature and H is the hardening/softening parameter.

Equation (1) is only applicable when $H > 0$, i.e. for hardening behaviour. For $H < 0$, a softening response is obtained and the loss of ellipticity of the governing differential equations occurs that makes the numerical results inadmissible.

One way to regularize the problem is to enhance the model with the gradient term $(1/2)\,G|\nabla\eta|^2$, where G is a positive parameter. However, the direct implementation of the gradient-enhanced model is not straightforward, because of the presence of the Laplacian of η. Thus, a micromorphic regularization is performed in which a new degree of freedom $\bar{\eta}$ is inserted into the local model (1), i.e. the free energy function is modified as,

$$\phi_\mu(\varepsilon,\eta,\bar{\eta},\nabla\bar{\eta},T)=\phi(\varepsilon,\eta,T)+\frac{1}{2}\chi(\eta-\bar{\eta})^2+\frac{1}{2}G|\nabla\bar{\eta}|^2, \qquad \text{with} \qquad \chi=\frac{G}{\ell^2}. \tag{2}$$

where ℓ is an internal length parameter.

In order to make the model thermomechanically coupled, first, the chemical part of the free energy $\phi_{chem}(\eta,T)$ is taken as a linear function of T and η to relate the pseudoelastic response of SMA to the temperature. Then, the latent heat of transformation together with the dissipated energy are considered as the local heat sources in the heat equation.

3. Numerical application

To demonstrate the suitability of the proposed micromorphic model, the uniaxial response of a NiTi wire in tension is studied [2]. To this end, the effect of loading rate on the force-displacement curve, the hysteresis

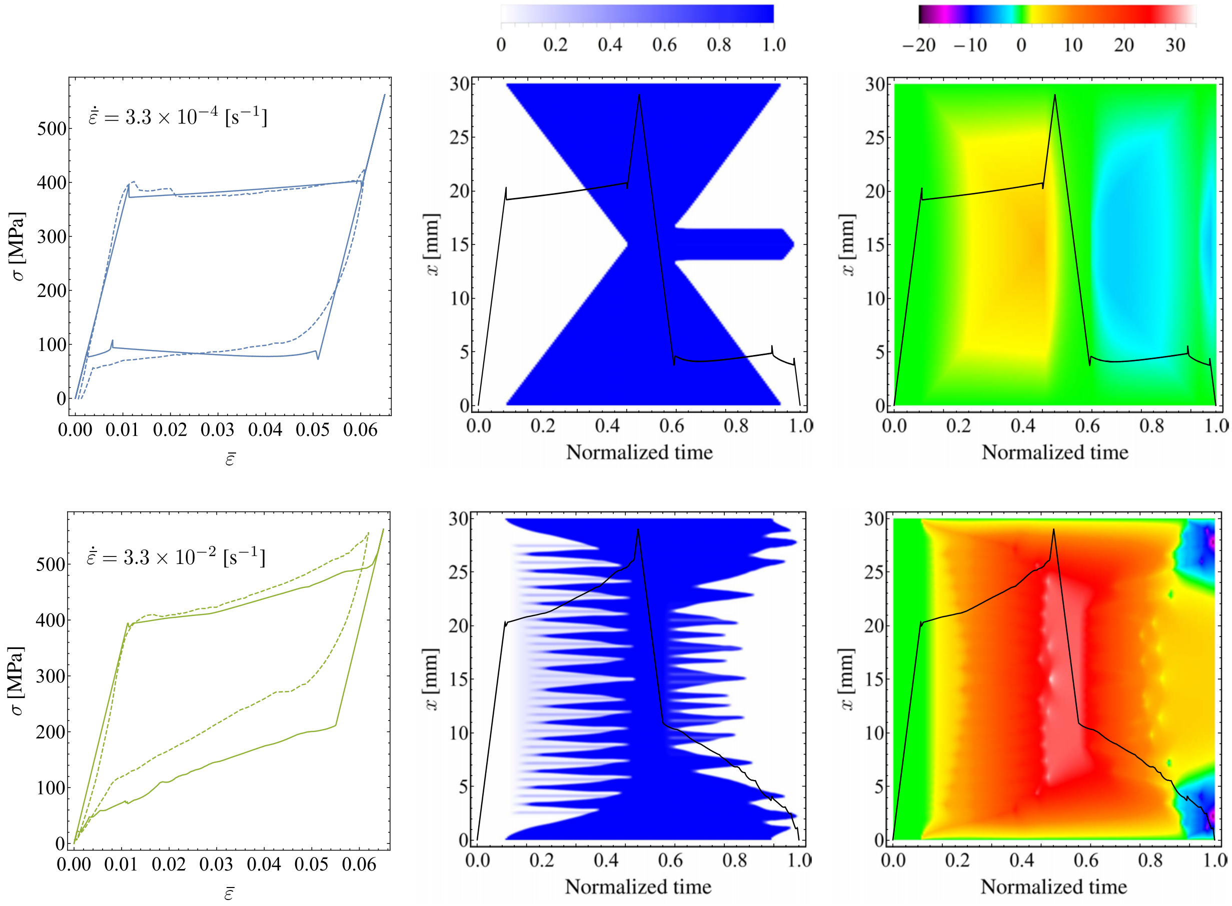

Figure 1: Detailed results corresponding to two loading rates $\dot{\bar{\varepsilon}}$: force-displacement curve (left), transformation pattern (middle), relative temperature field (right). The dashed lines in the force-displacement curves represent the experimental results of Zhang et al. [4]

area, the transformation pattern and the temperature field is studied. The numerical results are then compared with the experimental results obtained by Zhang et al. [4].

Figure 1 depicts the force-displacement curves, the transformation pattern, and the temperature field of the NiTi wire corresponding to two loading rates. The results demonstrate that for the low loading rate ($\dot{\bar{\varepsilon}} = 3.3 \times 10^{-4}$ s^{-1}) the effect of thermal hardening is insignificant. On the other hand, for the high loading rate ($\dot{\bar{\varepsilon}} = 3.3 \times 10^{-2}$ s^{-1}), a dominant thermal hardening is observable in the force-displacement curve.

Overall, despite the simplicity of the model, it is able to simulate the nucleation and transformation patterns induced by softening and its results show a good agreement with the experiment of Zhang et al. [4].

References

[1] M. Mazière and S. Forest. Strain gradient plasticity modeling and finite element simulation of Lüders band formation and propagation. *Continuum Mech. Thermodyn.*, 27:83–104, 2015.

[2] M. Rezaee Hajidehi and S. Stupkiewicz. Gradient-enhanced model and its micromorphic regularization for simulation of lüders-like bands in shape memory alloys. *Int. J. Solids Struct.*, 135(1):208–218, 2018.

[3] S. Stupkiewicz and H. Petryk. A robust model of pseudoelasticity in shape memory alloys. *Int. J. Numer. Meth. Eng.*, 93(7):747–769, 2013.

[4] X. Zhang, P. Feng, Y. He, T. Yu, and Q. P. Sun. Experimental study on rate dependence of macroscopic domain and stress hysteresis in NiTi shape memory alloy strips. *Int. J. Mech. Sci.*, 52(12):1660–1670, 2010.