VIBRATIONS OF A DOUBLE-BEAM SYSTEM WITH INTERMEDIATE ELASTIC RESTRAINTS DUE TO A MOVING FORCE

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1. Mathematical model and governing equations

We investigate the dynamic behavior of a system of two beams with arbitrary boundary conditions, connected with a number of k elastic restraints of finite stiffness s (see figure 1). Beams can have different flexural rigidity EI, mass density m, damping coefficient c and length L. One of the beams is subjected to a point force of constant magnitude P moving with constant velocity v. Equations of motion describing flexural vibrations $w_I = w(x_I, t)$ and $w_{II} = w(x_{II}, t)$ of the beam I and II have the form:

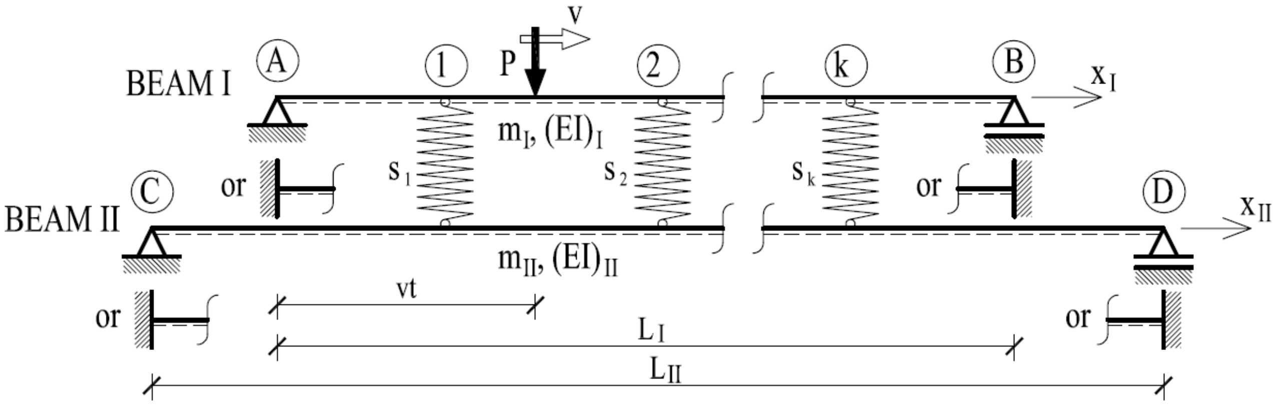

Fig 1. Double-beam system with multiple elastic restraints subjected to a moving force

(1)
$$(EI)_I w_I^{IV} + c_I \dot{w}_I + m_I \ddot{w}_I + \sum_{i=1}^{k} s_i [w_I - w_{II}] \delta(x_I - x_{I,i}) = P\delta(x_I - vt)$$

(2)
$$(EI)_{II} w_{II}^{IV} + c_{II} \dot{w}_{II} + m_{II} \ddot{w}_{II} + \sum_{i=1}^{k} s_i [w_{II} - w_I] \delta(x_{II} - x_{II,i}) = 0$$

where roman numerals denote differentiation with respect to spatial coordinates x_I and x_{II} while dots ($\dot{}$) denote differentiation with respect to time t. Symbol $\delta(.)$ denotes the Dirac delta.

In the presented method we replace analyzed structure with two single-span beams. Vibrations of the upper beam can be described as $w_I = w_I^P + \sum_{i=1}^{k} w_I^{Xi}$ while vibrations of the unloaded lower beam are equal to $w_{II} = -\sum_{i=1}^{k} w_{II}^{Xi}$. Expression w_I^P denotes vibrations of the single-span beam due to the given moving force P while expressions w_I^{Xi} and w_{II}^{Xi} are vibrations of the I and II beam resulting from the force $X_i(t)$ in the „i" elastic restraint. Forces $X_i(t)$ can be determined from a set of compatibility equations:

(3)
$$w_I^P(x_{I,i}, t) + \sum_{i=1}^{k} w^{Xi}(x_{I,i}, t) + \sum_{i=1}^{k} w^{Xi}(x_{II,i}, t) + \frac{X_i(t)}{s_i} = 0$$

If we assume that vibrations w_I^{Xi} and w_{II}^{Xi} are presented in the convolution form with use of Duhamel's integral, equation (3) will have the form of the Volterra integral equation of the second order which can be solved numerically applying methods described in [4].

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2. Numerical example

Presented example is of a three-span double-beam system (see figure 1). Upper beam is simply supported while the lower beam is clamped on both ends. Beams have the same length $L = 12\ m$, flexural rigidity $EI = 4 \cdot 10^6\ Nm^2$and mass density $m = 25\ kg/m$ and are connected with two elastic restraints of stiffness $s_1 = s_2 = 1 \cdot 10^6\ N/m$. System is subjected to a point force of constant magnitude $P = 1000\ N$ moving on the upper beam with constant velocity $v = 30\ m/s$. In further calculations we analyze dynamic deflections of the middle points at the upper and lower beam (sections “a” and “b”). Results for the both sections are presented on the figure 3.

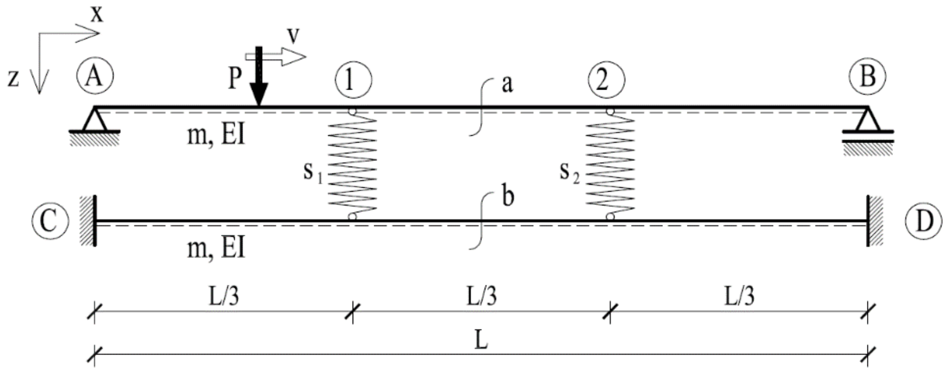

Fig 2. Double-beam system with two elastic restraints

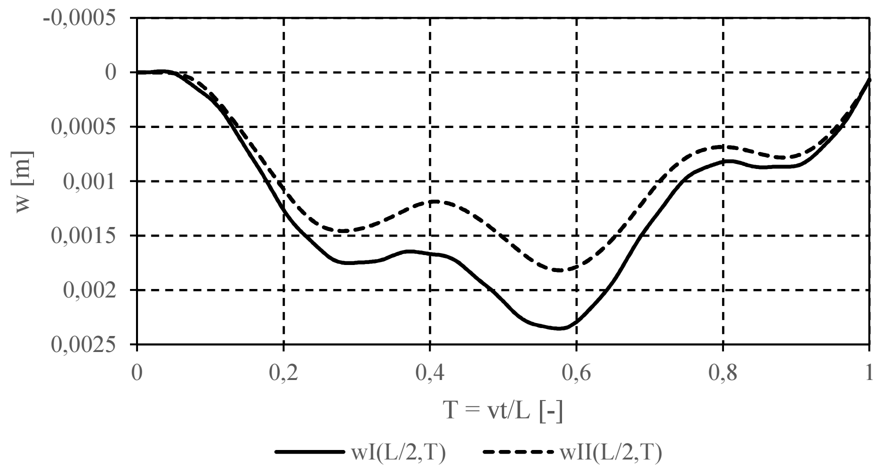

Fig 3. Dynamic deflections of the middle points at the upper and lower beam

3. References

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