MESH-FREE AND MESH BASED FINITE VOLUME METHODS FOR THE SOLID MECHANICS ANALYSIS

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1. General

Although the Finite Volume Method (FVM) is a common numerical technique in the computational fluid dynamics, in the last two decades investigations and development of FVM have shown that the FVM can be a promising candidate for solid mechanics analysis. In FVM, the computational domain is discretized into a number of sub-domains, known as finite volumes, control volumes (CVs) or cells. Each sub-domain or control volume contains one computational point which is located in the control volume domain. The governing partial differential equation is forced to be satisfied corresponding to each CV in the integral sense over each finite volume. However, the distinctive feature of FVM is using boundary integral instead of domain integral for satisfying the governing partial differential equation.

According to the recent developments of the FVM, two main categories of FVM exist: mesh based FVM and Mesh-free Finite Volume (MFV) which are explained as follows. Depending on how the control volumes are generated, the mesh based FVM is divided in two groups: cell vertex and cell centred finite volume approaches. In the Cell Vertex Finite Volume (CV-FV), the domain is discretized by a mesh of isoparametric elements. Then, the sub-control volumes are constructed in the finite element mesh by connecting the element's centres to the midpoint of the element faces. Combining all the subcontrol volumes having common grid point, creates a polyhedra type cell that surrounds the common node of the subcontrol volumes. These cells are called as Control Volumes (CV). Figure 1 shows a two dimensional mesh consisting of triangular and quadrilateral elements with associated constructed control volume. In CV-FV, the grid point located in each CV is considered as the computational point where the unkown variables are associated with. The main feature of the CV-FV is that the unknown variables are approximated on faces of cells using the shape functions of the background isoparametric elements [1]. In the cell centred finite volume (CC-FV), a mesh of elements are used for the discretization of the domain and the unknown variables are associated with the computational points which are coincided on the centres of the elements. Different from the CV-FV, unknown variables and their derivatives are approximated based on the finite difference utilization [2]. Therefore, the accuracy of the CC-FV is independent of the order of the background elements mesh. Recently the finite volume capability has been enhanced by developing mesh-free finite volume methods where two mesh-free finite volume approaches are proposed. In both of the MFV approaches, a set of nodes with arbitrary numbers and places are distributed in the domain. In the first approach, non-overlapping CVs are formed around the filed nodes by applying Delaunay triangulation scheme or Voronoi tessellation method. In Delaunay triangulation, similar to the CV-FV scheme the subcontrol volumes are constructed in each triangle by connecting the triangle centre to the midpoint of the triangle faces. Combining all the subcontrol volumes having common domain node, creates a polyhedra type cell that surrounds that domain node. These cells are called as control volumes (CV) which are non-overlapped. It should be emphasized that the field vaiables and their derivatives on cell faces are approximated by applying MLS technique which is independent of the background triangles geometry but depends to the node distribution [4,5]. It is clear that the distribution of nodes can be wisely performed according to the analyst's interest and based on the behavior of the unknown variable in the field domain. In the Voronoy approach, the nonoverlapping CVs are formed around the field nodes by using the Voronoi diagram technique. Again the field variables and their derivatives on cell faces are approximated by applying the MLS technique [5]. In the second type of MFV, overlapping CVs are formed around the field nodes and techniqes like MLS is used for the approximation of the field variables and their derivatives on cell faces. This type of construction of CVs provides a truly meshless formulation of FV which is also named as MLPG5 by Atluri and Shen [6]. This paper discusses the recent advancement achieved in the development of finite volume method.

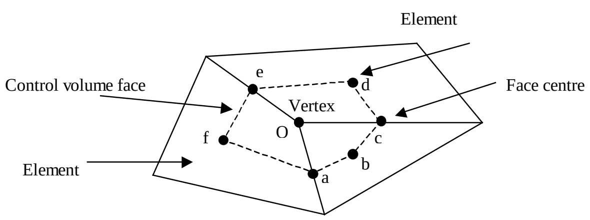

Figure 1. Construction of CVs in CV-FV method.

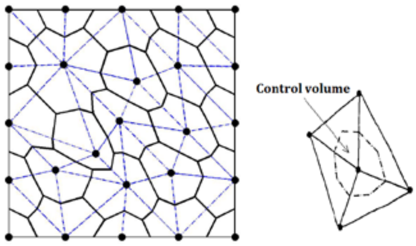

Figure 2. Delaunay triangulation around the distributed nodes and forming CVs.

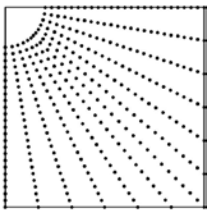

Figure 3. Wisely node distribution in meshless finite volume method.

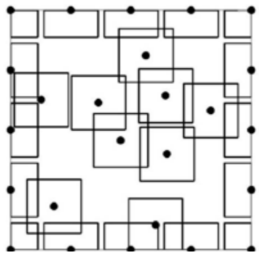

Figure 4. Overlapping CVs in meshless finite volume method.

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