Archives of Mechanics • Archiwum Mechaniki Stosowanej • 26, 3, pp. 383–389, Warszawa 1974

Aerodynamic interference in a system of two harmonically oscillating airfoils in an incompressible flow

J. GRZĘDZIŃSKI (WARSZAWA)

CALCULATION was made of the aerodynamic derivatives due to interaction of two harmonically oscillating thin airfoils with chords situated on the straight lines parallel to the direction of undisturbed flow. It was assumed that the fluid is inviscid and incompressible. Under certain simplifying assumptions leading to the linearization of the problem, a system of two integral equations for the pressure distribution on the airfoils was arrived at and solved by the method of least squares. Numerical results concerning a biplane are presented.

Praca dotyczy obliczania współczynników aerodynamicznych wynikających ze wzajemnego oddziaływania dwóch harmonicznie oscylujących cienkich profili, których cięciwy leżą na prostych równoległych do kierunku przepływu niezaburzonego. Założono, że przepływ jest nielepki i nieściśliwy. Przy pewnych założeniach upraszczających, prowadzących do linearyzacji zagadnienia, otrzymano układ dwóch równań całkowych dla rozkładu ciśnień na profilach. Układ ten rozwiązano metodą najmniejszych kwadratów. Przedstawione wyniki obliczeń numerycznych dotyczą układu dwupłata.

Работа касается расчета аэродинамических коэффициентов, которые вытекают из взаимодействия двух гармонически осциллирующих тонких профилей, хорды которых находятся на прямых параллельных направлению невозмущенного течения. Предполагается, что течение является невязким и несжимаемым. При некоторых упрощающих предположениях, приводящих к линеаризации задачи, получена система двух интегральных уравнений для распределения давлений на профилях. Эта система решена методом наименьших квадратов. Представленные результаты численных расчетов касаются системы биплана.

1. Introduction

THE AIM of this paper is to investigate the influence of the vortex wake due to unsteady flow on the aerodynamic interference without effect of finite wing aspect ratio. It is assumed that all parameters are harmonic functions of time characterized by the reduced frequency coefficient. The problem has been formulated from the numerical point of view. The advantage of the method is that calculations can be performed in a wide range of frequency coefficients without decrease their accuracy.

2. Formulation of the problem

The problem consists of calculation of forces and pitching moments acting on two harmonically oscillating thin airfoils in a plane inviscid incompressible fluid flow which has a uniform velocity U at infinity (Fig. 1).

The chords of the airfoils are situated parallel to the direction of undisturbed flow. The position of the airfoils is described by two coordinates L and H of the chord center of the second airfoil with respect to an orthogonal axis system $x_1 - z_1$ fixed to the center of the first airfoil. The semi-chords of airfoils are denoted by b_1 and b_2, respectively.

In order to solve the problem, all assumptions are the same as in the case of computing forces acting on a single isolated airfoil [2]. Under these assumptions, the equations of flow and the boundary conditions are linearized.

1. The flow velocity at any point of the plane is the sum of undisturbed velocity U and of small perturbation velocity components u and w along the x_1 and z_1 axes, respectively.

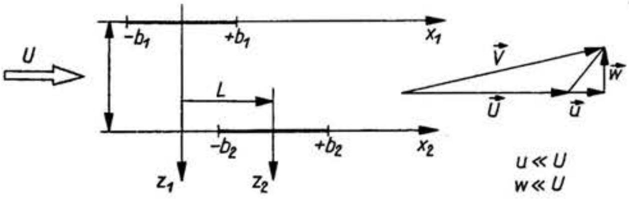

FIG. 1.

The flow is irrotational — there exists acceleration potential of the perturbation $\psi(x, z, t)$ satisfying the Laplace equation.

3. The linearized boundary condition may be assumed on each airfoil.

4. The Kutta condition is satisfied at the trailing edges.

That means that the reduced difference of pressure $\gamma(x, t)$ on each airfoil defined by the relation

$$\gamma(x, t) = \Delta\psi(x, t)/U^2,$$

where $\Delta\psi(x, t)$ denotes the difference of acceleration potential between uper and lower surface of the airfoil, must drop to zero at the trailing edges.

5. Both airfoils oscillate harmonically with circular frequency

$$w(x, t) = w(x)e^{i\omega t}.$$

Hence the acceleration potential and reduced difference of pressure have the same form.

The complex component of velocity $w(x)$ on the airfoil is determined taking two degrees of freedom for each airfoil.

The angle $\alpha(t)$ between velocity vector U and chord line of the airfoil and non-dimensional displacement $h(t)$ of the chord center are chosen as the generalized coordinates (Fig. 2).

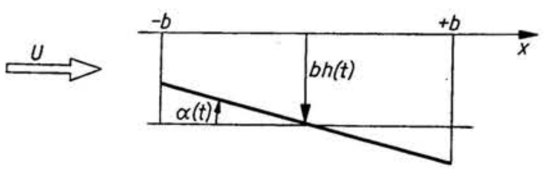

FIG. 2.

Under these assumptions and for harmonic motion of airfoils

$$\alpha_j(t) = \alpha_j e^{i\omega t},$$
$$h_j(t) = h_j e^{i\omega t}, \quad j = 1, 2,$$

the non-dimensional velocity is given by

$$w_j(x) = -ik_j h_j - (1+ik_j x)\alpha_j, \quad j = 1, 2,$$

where k denotes frequency coefficients

$$k_j = \omega b_j / U.$$

3. Equations of the boundary-value problem

Usually, the solution of such a boundary-value problem is obtained using an equation of the Possios type [3]. The unknown reduced difference of pressure is given in this equation in an explicit form. The main inconvenience of this equation is the necessity of isolation of all singularities of the kernel before performing computations. Furthermore, it is necessary to integrate numerically the functions of fast oscillations which leads to a decrease in accuracy for large values of frequency coefficient.

To avoid these inconveniences, a different method of formulating the problem has been chosen. Under the assumptions already indicated, the boundary-value problem for the acceleration potential may be formulated as follows:

$$\text{(3.1)} \qquad \left.\frac{\partial \psi}{\partial z}\right|_{\text{on } c_j} = -\frac{1}{2\pi}\int\limits_{c_j} \gamma_j(\xi) K_{\psi j}(x_j, \xi) d\xi, \quad j = 1, 2,$$

where the integration is carried out along the chords of airfoils c_1 and c_2, respectively. The functions $K_{\psi 1}$ and $K_{\psi 2}$ are singular kernels.

Next Eqs. (3.1) were transformed by the operator which defines the integrated acceleration potential $\Psi(x, z)$ as follows:

$$\Psi(x, z) = \int\limits_{-\infty}^{x} \psi(\xi, z) d\xi.$$

Thus the integral equations have the form:

$$\text{(3.2)} \qquad \sum_{l=1}^{2} \Phi_{jl}(x_j) = w_j(x_j) + ik_j \int\limits_{-1}^{x_j} w_j(\xi) d\xi + ik_j e^{ik_j} \int\limits_{-\infty}^{-1} \sum_{l=1}^{2} \Phi_{jl}(x_j) e^{ik_j x_j} dx_j, \quad j = 1, 2,$$

where $\Phi_{jl}(x_j)$ are given by

$$\Phi_{jl}(x_j) = \frac{1}{2\pi} \int\limits_{-1}^{1} \gamma_l(\xi) K_{jl}(x_j, \xi) d\xi,$$

and the kernels are given as follows:

$$\text{(3.3)} \qquad \begin{aligned} K_{jl}(x_j, \xi) &= \frac{\dfrac{(-1)^l L - b_j x_j}{b_l} + \xi}{\left[\dfrac{(-1)^l L - b_j x_j}{b_l} + \xi\right]^2 + \left(\dfrac{H}{b_l}\right)^2}, \quad j \neq l, \\ K_{jj}(x_j, \xi) &= \frac{1}{\xi - x_j}. \end{aligned}$$

In these equations, the velocity components of airfoils are referred to the velocity U of undisturbed flow. The last integrals in the Eqs. (3.2) depend on reduced differences of pressure $\gamma_j(x_j)$ and must be found as a part of the solution.

4. Numerical solution

The unknown reduced difference of pressure $\gamma_j(x_j)$ on each airfoil is represented by a series:

$$\gamma_j(x_j) \approx \gamma_j^*(x_j) = 2\sqrt{\frac{1-x_j}{1+x_j}} \sum_{l=0}^{n} a_l^{(j)} P_l^{\left(\frac{1}{2}, -\frac{1}{2}\right)}(x_j), \tag{4.1}$$

where $P_l^{\left(\frac{1}{2}, -\frac{1}{2}\right)}(x_j)$ are the Jacobi-polynomials which are orthogonal in the interval $(-1, 1)$ with weighting function $\varrho(x_j)$

$$\varrho(x_j) = \sqrt{\frac{1-x_j}{1+x_j}}.$$

Only two terms of the series (4.1) are needed to calculate the forces and moments acting on each airfoil.

The resulting integral equations can be written in operator form:

$$\mathbf{K}\boldsymbol{\gamma}(x) = \mathbf{w}(x),$$

where $\boldsymbol{\gamma}(x)$ and $\mathbf{w}(x)$ are two-dimensional vector functions and $\mathbf{K}$ is a matrix operator defined by the kernels (3.3).

The unknown coefficients $a_l^{(j)}$ of the series expansion (4.1) are calculated from the system of linear equations obtained by using the method of least squares to minimalize the square of the norm

$$||\mathbf{K}\boldsymbol{\gamma}^*(x) - \mathbf{w}(x)||^2 = \int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\, |\mathbf{K}\boldsymbol{\gamma}^*(x) - \mathbf{w}(x)|^2 dx.$$

Most of the integrals deal with during computations, arising from the operator $\mathbf{K}$, can be solved analytically and the result is given in a closed form.

For this problem, the application of the method of least squares seems to be more effective than the well-known collocation method. The method of least squares gives satisfactionary results using four-terms series expansion for difference of pressure on each airfoil. The collocation method needed far more terms to achieve the same accuracy.

5. Results and remarks

The aerodynamic properties of the system of airfoils analyzed are described by aerodynamic derivatives for each airfoil and, after GARRICK [3], defined as:

$$A_{1m}^{jl} = \frac{1}{\pi} \int_{-1}^{1} \gamma_j(\xi)\, d\xi \qquad \text{— lift coefficient,}$$

$$A_{2m}^{jl} = \frac{1}{\pi} \int_{-1}^{1} \xi\gamma_j(\xi)\, d\xi \qquad \text{— moment coefficient about the center of a chord,}$$

where subscript l indicates which of the airfoils is oscillating and subscript m is equal to one or two for bending and torsional oscillations, respectively. These coefficients are functions of the geometry of the model and reduced frequency. The numerical calculations have been performed for various configurations of airfoils in a wide range of the frequency coefficient.

As an example, we presented results of computations for the following values of parameters: chord's ratio $b_2/b_1 = 1$, the distance between chord's centers along x-axis $L = 0$, the ratio $H/b_1 = 2$. These data represent a biplane. In this case, the forces and

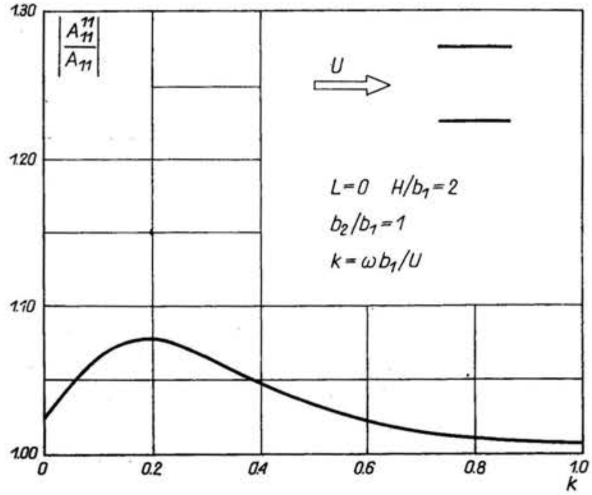

FIG. 3.

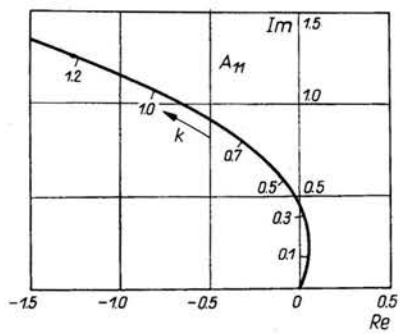

FIG. 4.

moments acting on the first airfoil are the same as those acting on the second airfoil due to the symmetry of the biplane.

Disregarding for the moment the presence of the first airfoil, the aerodynamic forces acting on the second one are characterized by four standard lift and moment coefficients connected with bending and torsional oscillations. These standard coefficients are modified when the first motionless airfoil is present in the fluid flow. The effect of this modification is shown is Fig. 3 and Fig. 5.

The first curve (Fig. 3) represents a modulus of a ratio of a lift coefficient connected with bending oscillations of the second airfoil to the respective standard coefficient, i.e., to the lift coefficient A_{11} (Fig. 4) connected with bending oscillations of single isolated airfoil. On the x-axis there are values of the frequency coefficient. The one maximum of

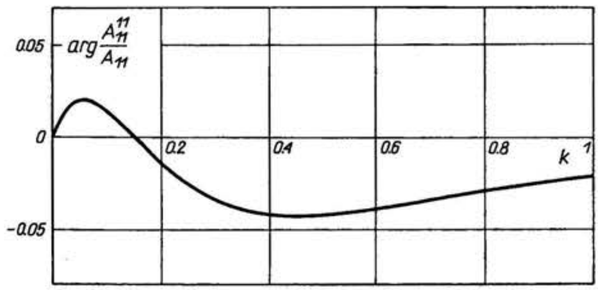

FIG. 5.

this curve is characteristic for a biplane. For the other configurations, this curve has an oscillatory form.

The phase shift of this ratio is shown in Fig. 4.

When the first airfoil is oscillating, the aerodynamic coefficients on the second one are more complicated. It is convenient to show these coefficients on the complex plane (Fig. 6). The values of the frequency coefficient corresponding to the points of the curve

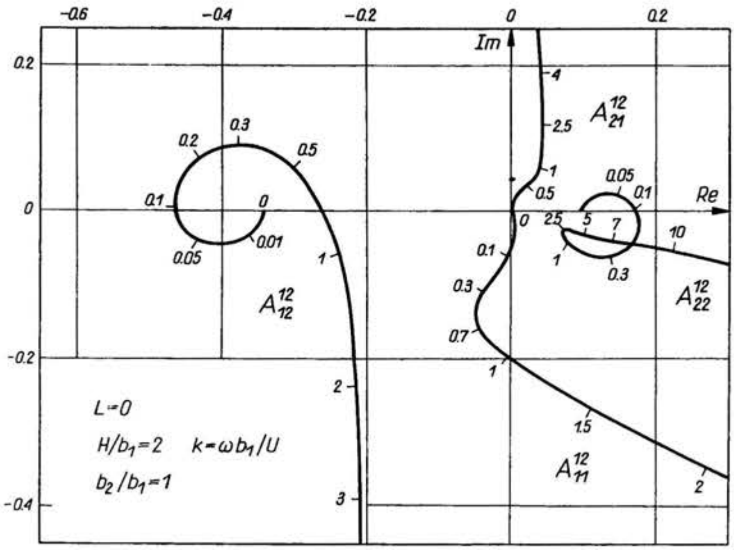

FIG. 6.

are written alongside. The curves show successively: the lift coefficient connected with bending (A_{11}^{12}) and torsional (A_{12}^{12}) oscillations, the moment coefficient connected with bending (A_{21}^{12}) and torsional (A_{22}^{12}) oscillations. The modulus of the coefficients connected with bending oscillations increases monotonically with increase in the frequency coefficient.

The behaviour of the remaining coefficients is more complicated. There are a few ranges of frequency coefficient where aerodynamic coefficients increase or decrease.

The form of these curves is also characteristic for a biplane, because a small change of the distance between the centers of the chord of the airfoils along x-axis imply a qualitative change of it.

It is useful to divide values of these aerodynamic coefficients by a square of the frequency coefficient. Such aerodynamic coefficients (shown in Fig. 7) are used in a flutter analysis.

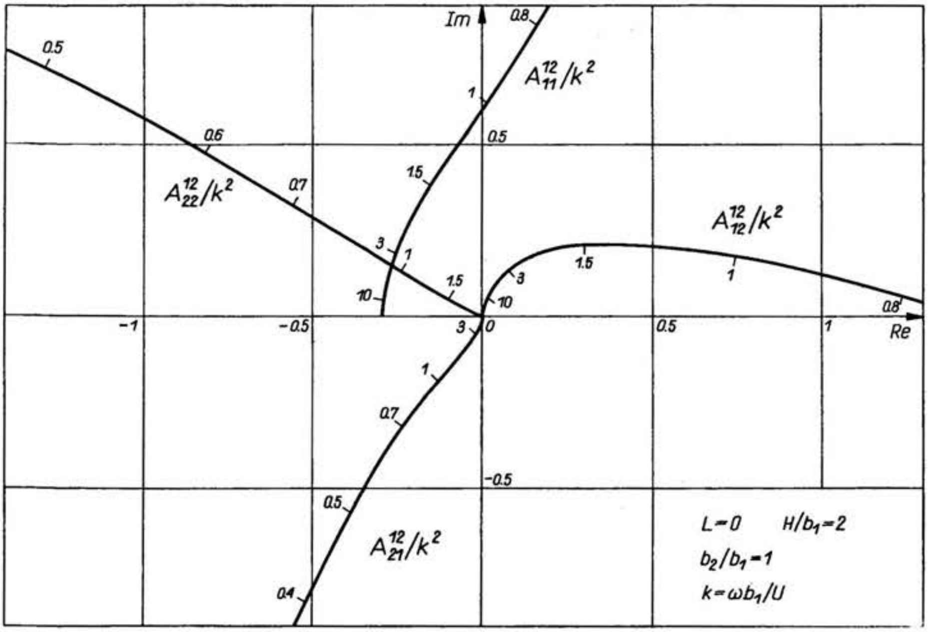

FIG. 7

It can be seen that only the lift coefficient connected with bending oscillations tends to infinity in the same manner as a square of frequency coefficient. The other coefficients tend asymptotically to linear functions of the frequency coefficient.

The numerical results show that the aerodynamic forces arising from the aerodynamic interference have the same order of magnitude as forces acting on a single isolated airfoil. Thus, when we calculate forces acting on a system of airfoils, the aerodynamic interference must not be disregarded.

References

1. M. T. LANDAHL, and V. J. E. STARK, *Numerical lifling-surface. Theory. Problems and progress*, AIAA Journal, 6, 11, 2049–2060, 1968.
2. T. THEODORSEN, *General theory of aerodynamic instability and the mechanism of flutter*, NACA Rept. 496, 1935.
3. I. E. GARRICK, *Nonsteady wing characteristics*, Section F, Vol. VII, High Speed Aerodynamics and Jet Propulsion, Princeton University Press, 1957.

POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.