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The stabilization law for transonic flows

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A MATHEMATICAL justification of the "stabilization" law of gas parameters for transonic flow past bodies, which is known from experimental aerodynamics, is given here. The quantitative formulation of this law is based on a simple asymptotic analysis of solutions of the Kármán equation. The numerical analysis reported here and concerning the velocity field about a body of revolution the meridional section of which is a Chaplyghin profile has confirmed with high accuracy the argument of the asymptotic theory.

Podano matematyczne uzasadnienie znanego z badań doświadczalnych w aerodynamice prawa ustalania parametrów gazu opływającego ciała z prędkością przydźwiękową. Ilościowe sformułowanie tego prawa jest oparte na prostej asymptotycznej analizie rozwiązań równania Kármána. Przedstawiona analiza numeryczna, dotycząca pola prędkości w pobliżu obrotowego ciała, którego południkowy przekrój jest profilem Chaplygina, potwierdziła z dużą dokładnością koncepcję teorii asymptotycznej.

Приводится математическое обоснование известного в экспериментальной аэродинамике закона „стабилизации" газовых параметров при обтекании тел потоком с околозвуковой скоростью. Количественная формулировка этого закона опирается на простой асимптотический анализ решений уравнения Кармана. Предпринятый численный расчет поля скоростей вокруг тела вращения, меридианальным сечением которого служит профиль Чаплыгина, с высокой точностью подтвердил выводы асимптотической теории.

THE understanding of many peculiarities of gas flow past bodies in the transonic velocity range is based on so-called the "stabilization" law. It is probable that this law was established experimentally at various times and in many countries. Its most complete formulation, illustrated by numerous examples, was given in 1964 by HOLDER in the second lecture held in commemoration of REYNOLDS and PRANDTL. In particular, two experimental diagrams were given in that lecture, showing the distribution of pressure and the local Mach number over the profile depending on the Mach number of the undisturbed flow. Diagrams were used to illustrate the most general features of a flow with the Mach numbers approaching unity. As an example, a symmetric profile with a relative thickness of 10% and an incidence angle of 2° has been selected in Fig. 1, and a symmetric profile of special form, the relative thickness being 4% and the incidence angle 1.5° — in Fig. 2. It has been designed in such a manner that for a small incidence angle there is no separation of flow for any Mach number. We shall be interested in the qualitative behaviour of the stream, therefore, by contrast with the diagram of Holder, the data concerning the behaviour of the flow past the upper surface of the profile are the only given. The flow past the lower surface is analogous.

Let us consider these diagrams in detail. The length of the profile is assumed to be unity and measured along the horizontal axis. In Fig. 1, the ordinates are local

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Mach numbers and the pressure is referred to its full value H_0. In Fig. 2, the pressure, which is also referred to H_0, is the only ordinate represented. Each of the experimental curves shown in the figures corresponds to a certain Mach number M_∞ of the undisturbed flow. The diagrams show first that the location of the shock on the body varies with increasing M_∞ — it moves towards the trailing edge of the profile and second that,

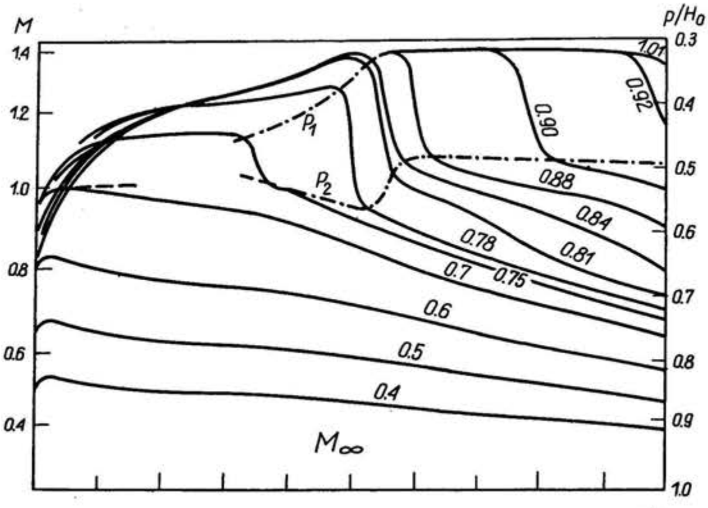

FIG. 1.

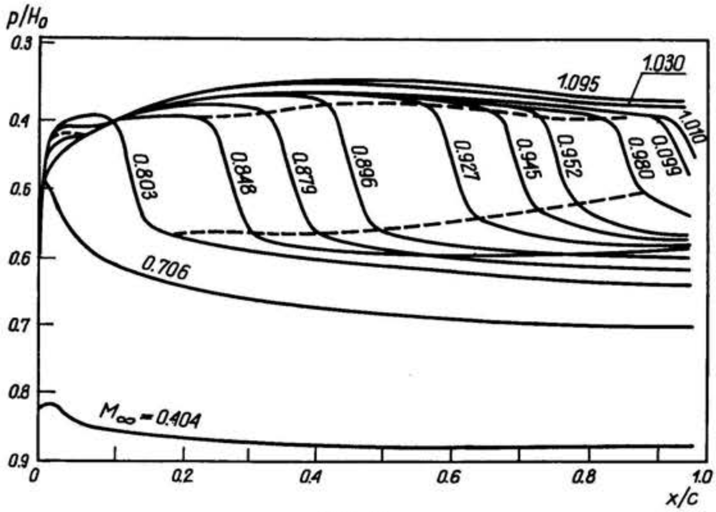

FIG. 2.

beginning from a certain value of M_∞ near unity, the pressure distribution over the body surface up to the shock deviates in an insignificant manner from the corresponding limit distribution under the conditions of critical velocity at infinity.

The phenomenon of "stabilization" of the parameters of the gas in transonic flow is, of course, of considerable practical importance. Although the consideration of this

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phenomenon means considerable simplification of the computation of the aerodynamic characteristics of bodies, its mathematical justification has not yet been attempted.

The test results quoted in Holder's lecture concern wing profiles only, but analogous results can undoubtedly be obtained for flows past bodies of more complicated form. To verify this supposition we shall consider axially symmetric flow past a body having a circular cross-section.

Let us proceed now to explain the mathematical form of the experimental relationships just mentioned. Our object will be to give not only a qualitative analysis but also a quantitative formulation. We shall confine ourselves to the study of flow past a slender body. It will be assumed that the velocity v_∞ of flow at infinity differs little from the velocity of sound a_∞. Let us denote by x, y the axes of a cylindrical system of coordinates, the x-axis being parallel to the undisturbed gas flow. Except for the regions adjacent to both ends of the body, the velocity will approach, at every point of the space, its critical value. Assuming that the stream is irrotational, our analysis will be based on the approximate equation

$$-\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}=0 \tag{1}$$

for the potential of the perturbed velocity, established by KÁRMÁN [2]. The potential φ and the lengths along the axes x and y are expressed in a dimensionless system of units. For the derivation of the Eq. (1), the most important is the potential of uniform flow having a critical velocity a_*, but not v_∞.

Let us determine the region in which the solution is considered. Let this region be sufficiently distant from the body, so that the form of the latter does not have any essential influence on the structure of the velocity field and, on the other hand, let the distance from that region to the body be sufficiently small for the values of the parameters of gas to deviate from the corresponding values at infinity in a negligible manner. More exactly speaking, we shall consider the perturbations of the velocity in the region considered to be induced, above all, by the body, their fraction depending on the difference $1-\mathsf{M}_\infty$ being small. The above explanations enable us to express the principal term of the solution of the Kármán equation in terms of the function $\varphi_0(x, y)$, which determines in regions far from the body the perturbation of uniform flow at sonic velocity. Considering the Refs. [3] to [6], we write

$$\varphi_0 = r^{-2/7}f_0(\xi), \qquad \xi = \frac{x}{r^{4/7}}.$$

We normalize the self similar variable in such a manner that the line $\xi = 1$ is the limit characteristic of the corresponding sonic flow. In the fragment of the region considered lying between the negative x-axis and that line, we shall change the variable by η according to the formula

$$\xi = \frac{12\eta-5}{7\eta^{2/7}}.$$

Then

$$f_0 = 2^5\cdot 7^{-3}\cdot\eta^{1/7}(12\eta^2-15\eta-25).$$

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To take into consideration the boundary condition $M_\infty \neq 1$ at infinity, let us write the expression for the potential in the form of the sum

$$\varphi = \varphi_0(x, r) + \varepsilon \cdot \varphi'(x, r), \tag{2}$$

assuming that $\varepsilon \ll 1$. The correction $\varphi'(x, y)$ satisfies the homogeneous linear equation which results from the Eq. (1) if the non-linear term $\frac{\partial \varphi'}{\partial x} \frac{\partial^2 \varphi'}{\partial x^2}$ is rejected. An analysis of this linear equation was performed in Refs. [7] and [8]. According to the results given there, we have

$$\varphi' = r^{8/7} f_1(\xi) = b r^{8/7} \eta^{-4/7}(1-4\eta), \quad b = \text{const}.$$

We shall now establish the relation between the small parameter ε in the Eq. (2) and the Mach number of the gas flow. To this end let us subject the independent coordinates x, r and potential φ to the transformation

$$x \to x/x_0, \quad r \to \tau r/x_0, \quad \varphi \to \varphi/(\tau^2 x_0) \tag{3}$$

which is usually applied for the derivation of the similarity law for axially symmetric transonic flows. The symbol τ denotes the relative thickness of the body and x_0 — a constant. With such a transformation the self similar variable will be written in the form

$$\xi = x_0^{-3/7} \tau^{-4/7} \frac{x}{r^{4/7}}.$$

It was shown by KÁRMÁN [2] for the relative thickness that $\tau \sim |1-M_\infty|^{1/2}$ (for affine bodies $\tau \to 0$ for $M_\infty \to 1$). It is clear, however, that, for any value of the velocity of particles at infinity, the first term of the right-hand member of the formula (2) should remain unchanged, because it corresponds to a flow with $M_\infty = 1$ about a body with prescribed dimensions. Hence we find immediately that $x_0 \sim \tau^{-4/3} \sim |1-M_\infty|^{-2/3}$. Now transforming the potential φ according to (3), we find for the small parameter that, as far as the order of magnitude is concerned, we have $\varepsilon \sim \tau^{10/3} \sim |1-M_\infty|^{5/3}$. Finally [7, 8],

$$\varphi = r^{-2/7} f_0(\xi) + |1-M_\infty|^{5/3} r^{8/7} f_1(\xi). \tag{4}$$

Making use of the formula (4) it is easy to give a mathematical justification to the "stabilization" law. It should be observed, above all, that it may be applied to the calculation of the front part of the velocity field only, upstream of the shock. Indeed, the form of the correction term $\varphi'(x, r)$ was established in the works [7, 8] by analysing the structure of the velocity field in the neighbourhood of the negative x-axis and the limiting characteristic of flow, which is represented, for $M_\infty = 1$, by the line $\xi = \eta = 1$. The boundary of the region in which the Eq. (4) is valid has already been discussed. It lies at a sufficiently large distance from the body surface. However, if the solution of the Kármán equation is prolonged analytically from that region to the surface of the body, its dependence on the number M_∞ or, more exactly, on the difference $1-M_\infty$ should remain unchanged. It follows that if the velocity of the undisturbed flow varies about its critical value, the parameters of the gas in the external velocity field and along the surface of the body will deviate from their limit values for the sonic flow in proportion to the quantity

$$\varepsilon \sim |1-M_\infty|^{5/3}. \tag{5}$$

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The quantitative conclusion is in full agreement with the experimental data, which were discussed in Holder's lecture [1]. However, these data should not be used for quantitative confrontation. It is not because the quantities measured characterize the flow past wings of infinite span. The order of magnitude of the variation of various gas parameters can easily be estimated in this case also. However, the accuracy of the data given in the lecture [1] is insufficient for quantitative confrontation with the theory.

To verify the above formulation of the "stabilization" law, accurate computation was carried out for the velocity field about a body of revolution with a smooth tail part; its meridional section is a symmetric Chaplyghin profile. In a parametric form the equation of this profile is

$$x = 1+z_1\cos z_2, \quad y = z_1 \sin z_2,$$

$$z_1 = (1+\delta)^{1-k}\left|\sin\frac{t}{2}\right|^{2-k}(1-2\delta\cos t+\delta^2)^{\frac{k-1}{2}}, \tag{6}$$

$$z_2 = \pi-\frac{k}{2}(\pi+t)+(1-k)\operatorname{arctg}\frac{\delta\sin t}{1-\delta\cos t}.$$

The values of the parameter t are contained within the interval $-\pi \leqslant t \leqslant 0$. It was assumed for the computation that $k = 0.1$ and $\delta = 0.05$. The length of the profile was unity, its maximum thickness — about 0.144. The computation was carried out by the relaxation difference scheme, very similar to that used in Ref. [9] for the solution of two-dimensional problems of gas dynamics. A special modification of the scheme enabled the use of data for axially symmetric flows. The results of computation are represented in Figs. 3 and 4.

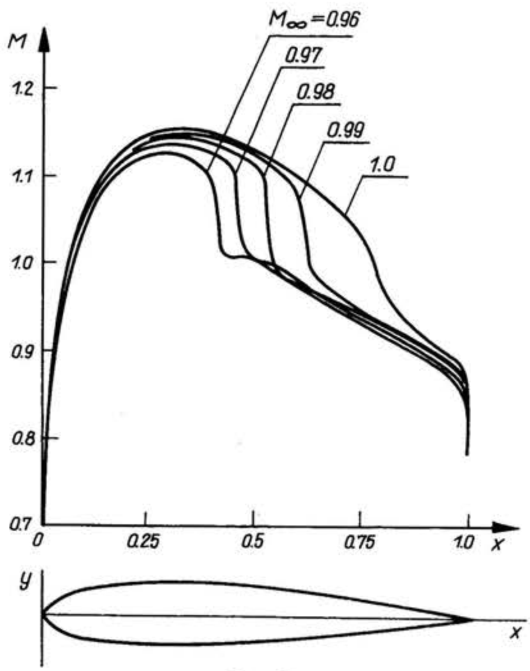

FIG. 3.

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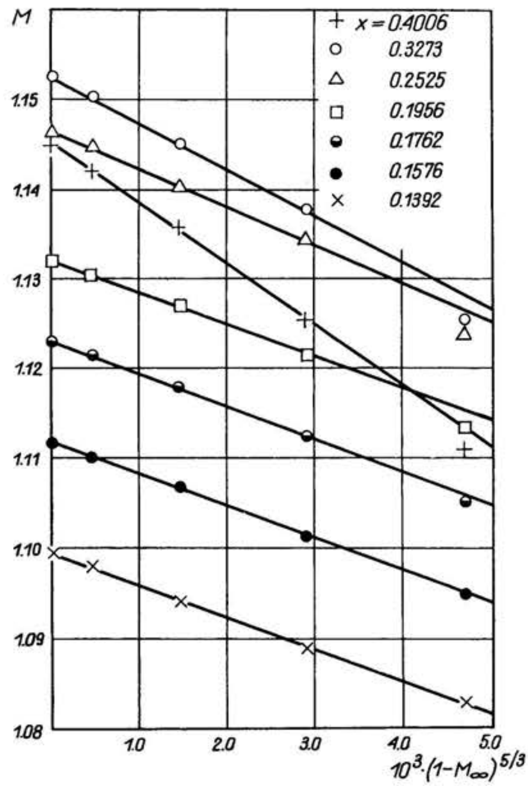

FIG. 4.

Figure 3 shows diagrams of local Mach numbers for various velocities of undisturbed flow. These diagrams have been drawn in the same form as usually applied in experimental aerodynamics [1]. Confrontation of curves shows that, if the quantity M_∞ is varied, the distribution of local Mach numbers along the body surface up to the shock front differs little from the corresponding limit distribution in the case of critical velocity at infinity. However, the shock itself moves relatively quickly with increasing flow velocity towards the trailing edge of the body. Thus, from the qualitative point of view the results of computation are in full agreement with the data for wing profiles obtained by wind tunnel tests.

For quantitative verification of the fundamental argument of the asymptotic theory let us consider Fig. 4. The curves represented there show how the local Mach number varies at a given point on the body with increasing M_∞. It is seen at a glance that if the general flow velocity varies about its critical value, the Mach numbers at points of the surface of the body deviate from their limit values for the sonic flow in direct proportion to the small parameter ε, given by the estimate (5). An analogous estimate is valid for the relative excess pressure on the leading part of the body bounded by the shock. This is the most accurate formulation of the "stabilization" law for transonic flows.

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It would be a great mistake, however, to think that the total drag of a body of revolution will vary, for transonic flow at infinity, in the same manner as the gas parameters upstream the shock wave. Behind its front the correction term $\varphi'(x, r)$ in the right-hand member of the Eq. (2) may have a different structure.

Investigation of the flow at $\mathsf{M}_\infty = 1$ showed [10 to 13] that the following term appears in that region

$$\varphi_2' = r^{-4/7} f_2(\xi) = cr^{-4/7}\zeta^{2/7}(1+\zeta)^{2/5},$$

$$\xi = (2-\sqrt{3})^{1/7}\frac{12\zeta+5}{7\zeta^{2/7}}, \quad c = \text{const}.$$

If we consider the region upstream the shock, the function $f_2(\xi)$ has a singularity either for $\xi \to -\infty$ or for $\xi = 1$. In the first case, the velocity field is irregular if we approach the negative x-axis, in the second — on the limit characteristic. It follows immediately that $\varphi_2'(x, r) \equiv 0$ must be set upstream the shoek. The first term in the Eq. (2) determines the law of asymptotic behaviour of the parameters of flow. The drag computed by its means is equal zero. The term $\varphi_2'(x, r)$ may be interpreted as a source of perturbations of the sonic flow in the region downstream the shock. The diagram of flow thus constructed, has the following drawbacks, however. The rate of flow across a closed surface surrounding the body as obtained from the Eq. (2) is not zero. In addition, the entropy of such a flow, produced by the shock tends to infinity at the symmetry axis. On the other hand, a close body cannot be a source of mass, therefore to construct a physically consistent diagram of flow within the frames of Euler's equations in the region adjacent to the x-axis it is necessary to introduce an entropy wake. This region of flow will be referred to as internal and the region in which the flow distribution is described by (2) — external. The solution in the wake was constructed by TOURNEMINE [12] and matched to the solution (2). The principal term of the expansion of that solution for $x \to +\infty$ represents a slip flow

(7)
$$v_x = a_* U(\psi), \quad v_r = 0,$$
$$S = S_* + \frac{\varkappa}{\varkappa+1}\ln\left[\frac{\varkappa+1}{2}\left(1-\frac{\varkappa-1}{\varkappa+1}U^2\right)\right],$$

where ψ is the stream function and $\varkappa$ — Poisson's adiabatic coefficient. The subsequent terms in Ref. [12] were obtained in the coordinates (x, r) not in the Mises coordinates (x, ψ), which are more convenient for this purpose. It is seen that flow in the region of the wake is completely determined by the variation in entropy of particles $S(\psi)$ at the crossing of the shock. The deficiency in flow rate through the region of the wake due to the entropy distribution in it is compensated by the excess of flow rate through the external region. The drag obtained as a result of integration of the rate of momentum in the wake is equal to

$$F_x = 2\pi p_* a_*^2 \int_0^\infty \frac{1-U^2}{U^2(1-\mu^2U^2)^{\frac{\varkappa}{\varkappa-1}}}\, d\Psi.$$

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We have $\mu^2 = (\varkappa-1)/(\varkappa+1)$ and the modified stream function replacing the stream function ψ is

$$\Psi = \int_0^{\psi} \left(\frac{\varkappa+1}{2}\right)^{-\frac{1}{\varkappa+1}} e^{S-S_*} d\psi .$$

The occurrence of exactly the same situation should be expected for $\mathsf{M}_\infty \neq 1$. Only a brief discussion of this problem will be given here. More detailed information will be contained in further publications of the present authors.

The perturbation of the sonic flow, resulting from the variation of the boundary conditions at infinity will be sought for in the form

$$\varphi = r^{-2/7} f_0(\xi) + r^{-4/7} f_2(\xi) + \varepsilon \sum_{k=0} r^{-2(m+k)/7} \chi_{m+k}(\xi), \tag{8}$$

where $m < 1$ is an unknown parameter to be determined. The function $\chi_m(\xi)$ satisfies the equation

$$\left(\frac{df_0}{d\xi} - \frac{16}{49}\xi^2\right)\frac{d^2\chi_m}{d\xi^2} + \left[\frac{d^2 f_0}{d\xi^2} - \frac{16}{49}(m+1)\xi\right]\frac{d\chi_m}{d\xi} - \frac{4}{49}m^2\chi_m = 0. \tag{9}$$

The remaining functions χ_{m+k} satisfy nonhomogeneous equations. Their right-hand members will not be quoted.

Since we consider the perturbation of a flow which is sonic at infinity due to a deviation of M_∞ from unity, the quantity ε may be related in an equivocal manner to the difference $(1-\mathsf{M}_\infty)$ and the parameter m. By performing operations exactly the same as those used for the derivation of the Eq. (5), we obtain the relation

$$\varepsilon = |1-\mathsf{M}_\infty|^{-\frac{m-1}{3}}. \tag{10}$$

The maximum value of m for which the solution is analytic at the limit characteristic $\xi = 1$ and satisfies the condition of symmetry at the crossing of the negative x-axis is, as observed above, equal to -4. For $m = -4$, the relation (10) becomes (5). The condition of $m < 1$ shows that there are distances for which the term of the expansion (8) connected with χ_m becomes comparable to the first term. In this case the expansion (8) in the small parameter ε is no more valid. The order of magnitude of these distances obtained by setting equal the quantities added in (8), with constant ξ, leads to the following estimates for the validity limits of all the expansions obtained in the present paper

$$r \sim |1-\mathsf{M}_\infty|^{-7/6}, \quad x \sim |1-\mathsf{M}_\infty|^{-2/3}. \tag{11}$$

These estimates tell that, within the range considered, the flow parameters differ little from the corresponding values for the sonic flow. Thus, for instance, neither the end of the local supersonic region nor the point of occurrence of the shock belong to that region for $\mathsf{M}_\infty < 1$.

With increasing M_∞ the shock moves rapidly towards the trailing edge [1]. This fact can be explained only by the existence in the region downstream the density jump of a solution of the Eq. (9), the value of m, lying within the interval $-4 < m < 1$.

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For the solution $\chi_m(\xi)$ the conditions of continuity of the potential and the equation of shock polar should be satisfied at the shock. This leads to a Cauchy problem at the shock for the Eq. (9).

Let us consider now the flow in the region of the vortex wake. The additional functions v_x', v_r', s' and Ψ' satisfy the equations of continuity, Crocco, and those of an adiabatic process and for Ψ', all of them being linearized with reference to the solution (7). These functions should satisfy the boundary conditions for $\Psi \to \infty$ which are obtained from the principle of asymptotic matching with the solution in the external region. Three of them are used for the determination of the perturbations of velocity and entropy. The fourth matching condition, which is responsible for the continuity of flow rate at the passage from the external to the internal region, enables the obtainment of the value of m. As a result of the solution of the problem in the wake flow we find that the vertical velocity $v_r = \varepsilon v_r'$ tends to infinity in the same manner as r^{-1} if the x-axis is approached and $\Psi'(x, 0) \neq 0$.

This result makes us consider one more region, lying in close neighbourhood of the x-axis. It will be referred to as the sub-wake. Its transverse dimension and also the transverse velocity of particles in it is of the same order of magnitude as $\varepsilon^{1/2}$ and the horizontal velocity is $v_x = 0(1)$. The principal expansion term in ε of the flow parameters in that region represents a uniform flow with

$$v_x = a_* U(0), \quad v_r = 0.$$

The value of the stream function in the sub-wake varies from $\Psi'(x, 0)$ to zero. This ensures its continuity.

Thus, the asymptotic flow diagram satisfies the conditions at the density jump and is symmetric about the x-axis. It remains to find the value of m for which the flow rate of gas across a closed surface surrounding the body is zero. As a result of a relatively complicated calculation it is found that this condition can be satisfied for $m = 0$ only.

The result obtained signifies, in particular, that the drag can be expanded in series of integer powers of $\varepsilon = |1-\mathsf{M}_\infty|^{1/3}$. It may happen for a flow past a body of revolution that certain coefficients powers of ε in this expansion become zero. Thus, for instance, first coefficient becoming zero depends on whether the term independent of x in the expansion of S' in powers of x is or is not zero. Its value is determined by the particular form of the body. Thus, for the axially symmetric body obtained by revolution of a Chaplyghin profile (6) it was found to be zero. It follows that the variation of the drag coefficient C_x for that body, which is shown in Fig. 5, is proportional to $|1-\mathsf{M}_\infty|^{2/3}$. In other words the relation

$$C_x(1) - C_x(\mathsf{M}_\infty) = d|1-\mathsf{M}_\infty|^{2/3}, \quad d = \text{const}$$

is valid.

The difference between the coordinates $x_{sh}(1)$ and $x_{sh}(\mathsf{M}_\infty)$ which determine the location of the shock on the body for a given M_∞ and for $\mathsf{M}_\infty = 1$, is estimated in a similar manner. It is necessary to note that, if the drag is computed by integrating the pressure coefficient along body surface, a major part of the variation of that integral depends on the displacement of the shock. This circumstance was not taken into con-

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sideration by GUDERLEY [7], which led to an inaccurate estimation of the rate of growth of C_x for $M_\infty \to 1$.

Let us analyse now the fundamental limitation imposed on the relative thickness of the body. In the theory of small perturbations the body is assumed to be thin. Let us now consider the flow past a body the relative thickness of which approaches unity. Since the region in which the solution has been constructed is at a sufficient distance from the body, the assumptions of the theory of small perturbations remain valid. It follows that a solution can be sought for as before, in the form (2). We subject it to a similarity transformation (3), and consider τ to be a small parameter not identified with the relative thickness of the body. The relation between this parameter and the difference $(1-M_\infty)$

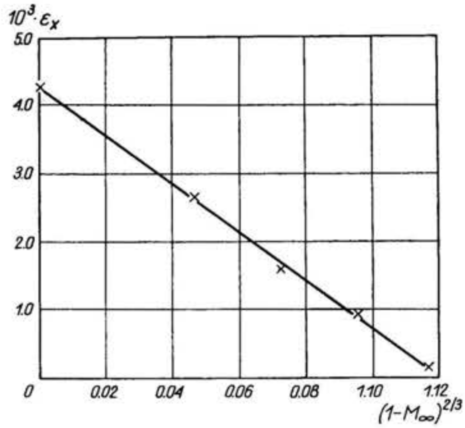

FIG. 5.

is found by satisfying the boundary condition for $x^2+r^2 \to \infty$, for which use can be made of the assumptions on which the Kármán equation is based. It follows that $\tau \sim |1-M_\infty|^{1/2}$ and the perturbation potential is prescribed by the Eq. (4). Thus, at a sufficient distance from the thick body, the velocity field has the property of "stabilization", that is its parameters vary much more slowly than the flow velocity. If the pressure and the velocity of gas particles at the surface of the body are concerned, their values are determined, of course, by the character of the analytic continuation of the solution into the region adjacent to the body. For thick bodies such a eontinuation should be based on the accurate Euler equations not the Kármán equation, which is approximate. In the selected computation example the relative thickness of the body was 0.144, therefore it may be assumed that the "stabilization" law of gas parameters at the body surface remains valid for bodies of moderate thickness at least, if the flow velocity approaches its critical value.

References

1. D. W. HOLDER, *The transonic flow past two-dimensional aerofoils*, J. Roy. Aeronaut. Soc., **68**, 644, 1964.
2. TH. VON KÁRMÁN, *The similarity law of transonic flow*, J. Math. and Phys., **26**, 3, 1947.
3. G. GUDERLEY, H. JOSHIHARA, *An axial-symmetric transonic flow pattern*, Quart. Appl. Math., **8**, 2, 1951.

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4. С. В. Фалькович, И. А. Чернов, *Обтекание тела вращения звуковым потоком газа* [Sonic gas flow pasta body of revolution], Прикл. мат. мех., **28**, 2, 1964.
5. E. A. Müller, K. Matschat, *Ähnlichkeitslösungen der transsonischer Gleichunger bei der Anström-Machzahl* 1. Proc. 11-th Int. Congr. Appl. Mech., Munich 1964. Springer, 1966.
6. D. Randall, *Some results in the theory of almost axisymmetric flow at transonic speed*, AIAA J., **3**, 12, 1965.
7. К. Г. Гудерлей, *Теория околозвуковых течений* [Theory of transonic flows], М., Изд-во ин. лит., 1960.
8. В. Н. Диесперов, О. С. Рыжов, *Об обтекании конечных тел равномерным потоком в околозвуковом диапазоне скоростей* [Uniform transonic flow past finite bodies], Ж. вычислит. мат. и мат. физ., **11**, 1, 1971.
9. P. R. Garbedian, J. Corn, *Analysis of transonic airfoils*, Comm. Pure Appl. Math., **24**, 6, 1971.
10. В. Н. Диесперов, О. С. Рыжов, *О пространственном обтекании тел звуковым потоком идеального газа* [Space sonic flow past finite bodies], Прикл. мат. мех., **32**, 2, 1968.
11. G. Tournemine, *Sur un schéma découlement sonique, tridimensionel, en aval de l'onde de choc, en fluide parfait*, C.r. Acad, Sci., Série A, **267**, 1968.
12. G. Tournemine, *Comportement asymptotique de l'écoulement sonique autour d'un corps de révolution de dimensions finies, en aval de l'onde de choc*, J. de Mécan., **7**, 3, 1968.
13. В. Н. Диесперов, О. С. Рыжов, *О телах вращения в звуковом потоке идеального газа* [The problem of sonic flow of a perfect gas past bodies of revolution], Ж. вычислит. мат. и мат. физ., **9**, 1, 1969.

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