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Study on the internal forces of container ships

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THIS PAPER gives the results of a study on the influence of the heterogeneity of cargo and oblique waves on such internal forces as torsional and bending moments. Fot the sake of simplicity a core of the hold cross-section was introduced (as the domain of possible positions of gravity centres of cargo) and calculated. Torsional moment acting on the ship due to yawing, vertical motion and possible positions of the centre of gravity in the core was considered in general and in the simple case of rectangular waterplane of the slip. External forces due to heterogenous cargo were also considered as sets of equimeasurable functions. Some theorems enabling to calculate extreme values of internal forces functional in these sets are established and illustrated on simple examples.

W pracy podano wyniki badań nad zagadnieniem wpływu niejednorodności ładunku oraz fali skośnej na wielkość sił wewnętrznych, takich jak momenty skręcające i zginające. Dla uproszczenia analizy wprowadzono pojęcie rdzenia przekroju ładowni, zdefiniowanego jako obszar możliwych położeń środków ciężkości ładunku. Przedyskutowano ogólnie zależność momentu skręcającego kadłub statku od jego myszkowania, kołysania pionowego oraz położenia środka ciężkości ładunku; przeanalizowano także prosty przypadek szczególny wodnicy prostokątnej. Uwzględniono także siły zewnętrzne, wynikające z niejednorodności ładunku, traktując je jako zbiory jednakowo mierzalnych funkcji. Przedstawiono szereg twierdzeń pozwalających obliczać ekstremalne wartości funkcjonału sił wewnętrznych w tych zbiorach ilustrując je prostymi przykładami.

В работе даются результаты исследований проблемы влияния неоднородности груза и косой волны на величину внутренних сил, таких как скручивающие и изгибающие моменты. Для упрощения анализа введено понятие сердцевины сечения грузового тюма, определенного как область возможных положений центров тяжести груза. Обсуждена в общем зависимость момента скручивающего корпус судна от его рыскания, вертикальной качки и положения центра тяжести груза; проанализирован тоже простой частный случай прямоугольной ватерлинии. Учтены также внешние силы вытекающие из неоднородности груза, трактуя их как множество одинаковой меры функций. Представлен ряд теорем позволяющих вычислять экстремальные значения функционала внутренних сил на этих множествах, иллюстрируя их простыми примерами.

1. Introduction

MODERN progress in automation of the process of transport of various types of loads, general cargo in particular — i.e. loads of variable shape and specific weight — resulted in the last decade in the development of special types of ships adapted for container transportation. Ships of this type are characterized by special construction. In order to ensure prompt and effective stevedoring, the hatchways must be sufficiently large, which results in shifting the centers of shear under the keel and in reduction of the overall torsional rigidity of the cross-sections of the bull. At the same time, the action of waves leads to a non-linear problem of cooperation between the hull (subject to torsion and bending) and the hatches under the assumption that all the clearances are filled by packing material.

The existing bibliography of the problem directly concerning open ships embraces, in periodicals dealing with mechanics of ship structures — or more generally, with the shipbuilding industry — about seventy references. The most important references, together with those concerning the results obtained in the Gdańsk Technical University Shipbuilding Institute, are given at the end of the paper.

In computing centers of Bureaus of Standard — e.g. in the Bureau Veritas [2] — extensive computing programs are already in use which make it possible to analyze the influence of the geometry of structure of a ship on the torsional rigidity of the hull, local phenomena, stresses etc., and to evaluate the effects of boundary conditions. The work [2] contains an extensive description of research problems concerning ships of that type, their designing and exploitation.

The most frequently used theoretical model of open ships' structures is a classical thin-walled beam with an open profile, or a closed-open model (double hull), closed by transversal deck strips between the holds [3, 4, 5], and by rigid and elastic endings of the hull (stern and fore body). A slightly different model is presented in [10]; it consists of a combination of a framework and a thin-walled, prismatic bar.

The development of digital computers has introduced also the finite element method into this field of technology. The programs used in Japan [6], in which the ship is divided into some one thousand elements, are verified by direct measurements on actual ships on calm sea and loaded by suitably filled ballast tanks. The authors found satisfactory agreement between the theoretically predicted and experimentally measured stresses and strains — except for the so-called local states. Such an approach offers the possibility of solution of the problem of simultaneous torsion and bending outlined in [7].

It seems, therefore, that the main theme which should be developed within the framework of existing solutions concerning open ships is the analysis of loadings, which are difficult to predict in the process of exploitation. A cargo consisting of containers is a typical example of a heterogeneous load of a sectionally constant generating function [8], determined by the so-called cryptonime for loaded containers and by the number of empty containers.

In this paper we shall deal with the problem of influence of nonhomogeneity of the cargo on internal forces and on the algorithm of evaluation of torques acting on container ships placed oblique waves — under the simplest assumptions eliminating the effects of side-sway and pitch from the dynamic interactions. The only dynamic factors essential from the point of view of torque appraisal will be the so-called yawing of the ship and its vertical motions caused by the less than unity block coefficient.

2. Core of the cross-section for nonhomogeneous cargo

Let an arbitrary region $\mathcal{D}$ be given on the plane x, y. Assume two materials of constant specific weights $\gamma_1 < \gamma_2$, infinitely divisible and incompressible. Let the region $\mathcal{D}$ be divided into two arbitrary, measurable and disjoint parts $\mathcal{D}_1$ and $\mathcal{D}_2$. Let the material with specific weight γ_i be distributed in a layer of constant thickness over the region $\mathcal{D}_i$, $i = 1, 2$. The loci of all positions of the mass centers in $\mathcal{D}$, within the set of subdivisions of the region into two measurable parts $\mathcal{D}_1$ and $\mathcal{D}_2$, will be called the core of the cross-section. Let us determine the contour of the core.

Let us assume for a moment that regions $\mathscr{D}_1, \mathscr{D}_2$ are obtained by dividing the region by means of a line perpendicular to x and at point x. Let $x = x_{\min}$ and $x = x_{\max}$ be tangent to the region $\mathscr{D}$ and have the property that the entire region is contained between these straight lines. Let $b_x(\xi)$ denote the global length of segments lying on the line $x = \xi$ and belonging to $\mathscr{D}$, and let $\mathscr{D}_1, \mathscr{D}_2$ denote areas of these regions into which $\mathscr{D}$ has been subdivided. Then the coordinate $x = x_0$ of the mass center is equal to

$$x_0 = \frac{\gamma_1 \int\limits_{x_{\min}}^{x} b_x(\xi)\xi d\xi + \gamma_2 \int\limits_{x}^{x_{\max}} b_x(\xi)\xi d\xi}{\gamma_1 \mathscr{D}_1 + \gamma_2 \mathscr{D}_2}. \tag{2.1}$$

Let us determine when the mass center position reaches an extremum — i.e., when $x_0 = (x_0)_{\max}$. Differentiation of the Eq. (2.1) leads, under the condition that $x_0' = 0$, to the equation

$$x = \frac{\gamma_1 \int\limits_{x_{\min}}^{x} b_x(\xi)\xi d\xi + \gamma_2 \int\limits_{x}^{x_{\max}} b_x(\xi)\xi d\xi}{\gamma_1 \mathscr{D}_1 + \gamma_2 \mathscr{D}_2}, \tag{2.2}$$

whence the conclusion may be drawn [by comparison with the Eq. (2.1)] that in the extremal position the center of mass is located at the line dividing the region $\mathscr{D}$ into $\mathscr{D}_1$ and $\mathscr{D}_2$.

It will be proved that the Eq. (2.2) determines the coordinate of the boundary point of the core. In fact, let $\mathscr{D}_1^0$ and $\mathscr{D}_2^0$ denote an arbitrary subdivision of $\mathscr{D}$ by a line perpendicular to the x-axis (load γ_i is placed at $\mathscr{D}_i^0$, $i = 1, 2$). Let now $\check{\mathscr{D}}_i^0$, $i = 1, 2$, denote an arbitrary different subdivision into regions which are measurable but such that $\operatorname{mes}(\check{\mathscr{D}}_i^0) = \operatorname{mes}(\mathscr{D}_i^0)$, $i = 1, 2$.

Let us denote

$$\mathscr{D}_i' \equiv \check{\mathscr{D}}_i^0 \cap \mathscr{D}_i^0, \quad i = 1, 2, \tag{2.3}$$

and

$$\mathscr{D}_i'' \equiv \mathscr{D}_i^0 \setminus \mathscr{D}_i', \quad i = 1, 2. \tag{2.4}$$

Obviously, $\operatorname{mes}(\mathscr{D}_1'') = \operatorname{mes}(\mathscr{D}_2'')$ and

$$\bigwedge_{(x', y') \in \mathscr{D}_1''} \bigwedge_{(x'', y'') \in \mathscr{D}_2''} x' < x''. \tag{2.6}$$

In the region $\mathscr{D}_1''$ (at subdivision $\mathscr{D}_i^0$) is placed the heavier material of specific weight γ_2, and in the region $\mathscr{D}_2''$ — the material with specific weight γ_1. By means of the Eq. (2.5) we may, be means of simple exchange of materials, obtain the subdivision $\mathscr{D}_i^0$ by a line perpendicular to the x-axis, though, in view of $\gamma_1 < \gamma_2$, the mass center will be displaced in the positive direction of x. A division by the line perpendicular to x is hence extremal in the set of all subdivisions of $\mathscr{D}$ into the parts $\mathscr{D}_i$ of fixed areas. The solution (2.2) is thus also an extremal solution in the set of all perpendicular subdivisions, and determines the core of the cross-section.

It is obvious that, in order to determine the boundary point of the core on the line tangent to an arbitrary axis directed, say by the unit vector $\bar{k}$, a similar method may be applied, and the core boundary is easily obtained by assuming the set of all unit vectors as a parameter. A further conclusion is the convexity of the core, since only one boundary point for each tangent may exist. It is valid for any measurable sets $\mathscr{D}$ on the plane (i.e. not only connected ones, as in most applications). Generalization to a spatial case is trivial.

Also obvious is the solution of a slightly different problem of determination of the cross-section of the core. A sequence of specific weights $\gamma_1 < \gamma_2 < \ldots < \gamma_n$ of materials distributed over the region $\mathscr{D}$ is given. Let us determine the core of that region.

The form and dimensions of the core depend exclusively on γ_1 and γ_n. This is by the method indicated for two materials. If, however, the amounts of individual materials are fixed, then the maximal coordinate of the core in the direction of a fixed axis t with unit vector $\bar{k}$ is obtained by dividing $\mathscr{D}$ by means of straight lines perpendicular to t within the subregions $\mathscr{D}_i$, $i = 1, 2, \ldots, n$; materials with specific weights γ_i are consecutively placed in these regions, coordinate t for the subdivision is found and, as a result, we obtain the boundary point of the core.

3. Examples and applications

The core of a circle is also a circle. If the circle (region $\mathscr{D}$) has radius r, and the core — radius ϱ, we introduce the parameter $\gamma_2\gamma_1^{-1} = n$ and obtain the following system of transcendental equations for ϱ:

(3.1)
$$\frac{\varrho}{r} = -\cos\alpha_1,$$

and

(3.2)
$$n = \frac{4\sin^3\alpha_1 + 3\sin 2\alpha_1 + 6\alpha_1\cos\alpha_1}{4\sin^3\alpha_1 - 3\sin 2\alpha_1 - 6(\pi-\alpha_1)\cos\alpha_1},$$

where $\alpha_1 \in \langle \pi/2, \pi\rangle$.

A rectangular cross-section important for applications is somewhat more complicated. Let us assume the rectangular coordinate system (z, y) at the center of the rectangle, with axes parallel to its sides; coordinates of the points of intersection of the core boundary with the axes y and z, for positive coordinate values, are

(3.3)
$$y_0 = \frac{B}{2}\,\frac{\sqrt{n}-1}{\sqrt{n}+1},$$

and similarly for the z-axis

(3.4)
$$z_0 = \frac{H}{2}\,\frac{\sqrt{n}-1}{\sqrt{n}+1}.$$

Here $n = \gamma_2\gamma_1^{-1}$.

Since the core is known to be convex, its lower bound may be estimated by inscribing a quadrangle with vertices placed at the points of intersection of its contour with the coordinate axes. Let us estimate these differences for a square with sides a by determining the position of the boundary point of the core at the diagonal. If $\check{\xi}_0$ denotes the distance of the core contour from the point (0, 0) measured along the diagonal, then

$$\check{\xi}_0 = \frac{\sqrt{2}}{4}(n-1)\frac{u^3-\frac{2}{3}u^2}{u^2\frac{n-1}{2}+1}a, \tag{3.5}$$

where u satisfies the equation

$$\frac{1}{3}u^3\frac{n-1}{2}+u-1 = 0. \tag{3.6}$$

Comparison of the values $\xi_0 a^{-1}$ for different n with those for the simplified core $(\check{\xi}_0 a^{-1})$ is given in Table 1.

Table 1

n	1	2	4	16
$\xi_0 a^{-1}$	0	0.060	0.166	0.300
$\check{\xi}_0 a^{-1}$	0	0.064	0.172	0.313

In other cases, for instance when the cross-section consists of two rectangles (a case of great practical importance), the formulae are more complicated and will not be analyzed here.

Knowledge of the cross-section core enables us to solve the following problem. A rigid, cylindrical hull with vertical sides is given, the x-axis being directed from the stern to the bow. The hull floates on calm sea, with small vertical motions and side-swaying. A continuous, linear load $\Gamma(x)$ of the hull is given, as also the cross-section cores $\mathcal{R}(x)$ for each x. If $y_1(x)$ is the y-coordinate of the line of gravity centers, then the torque at cross-section $x = x_0$ is obtained in the form:

$$M_s(x_0) = \int_{I_L} \Gamma(x)\omega(x, x_0)y_1(x)dx, \tag{3.7}$$

where

$$\omega(x, x_0) = \begin{cases} 1-I_{y,x_0}/I_y, & x \in \langle L_1, x_0\rangle, \\ -I_{y,x_0}/I_y, & x \in \langle x_0, L_2\rangle. \end{cases} \tag{3.8}$$

Here I_y is the moment of inertia of the waterplane with respect to its axis of symmetry, I_{y,x_0} is the moment of inertia with respect to the axis of symmetry of that part of the waterplane which satisfies the condition $x \leqslant x_0$, and $I_L \equiv \langle L_1, L_2\rangle$.

Assuming that the line of mass centers in each cross-section is contained within the tube generated by the core $\mathcal{R}(x)$ for $x \in I_L$, we may easily determine the extremal values of the torque by selecting y_1 on the corresponding wall of the tube, as depending on the sign of $\omega(x, x_0)$. The solution yields both the upper and lower bounds of the torque. In the next section, in discussing another one-dimensional problem, a different formulation and solution of the problem of extremum internal forces will be given.

4. Extrema mechanical parameters in the sets of distributions of one-dimensional, non-homogeneous loads

Let us assume that load $f(x)$ of the hull represents one of all the possible configurations of a non-homogeneous load, provided f is a measurable function characterized by the measure function

$$m(y) = \operatorname{mes}(\Omega_y) \equiv \operatorname{mes}\{x, f(x) > y\} \equiv \varphi(f, y). \tag{4.1}$$

The function $f^0(x)$ defined for $x \in I_L$ and non-increasing in that interval is called the generating function of a nonhomogeneous load provided it is equimeasurable with $f(x)$. The set of all functions equimeasurable with f^0 constitutes an ordered set of the same nonhomogeneous load. The generating functions most frequently encountered in practice is the sectionally constant generating function which models the load consisting of portions of a homogeneous load [8].

Let the set Ψ of equimeasurable functions be determined by the generating function $f^0(x)$, which is continuous and decreasing in the interval $\langle L_1, L_2\rangle$. Let the functional J defined on Ψ by an integral, possess a continuous and decreasing influence function $j(\xi)$ in $\langle L_1, L_2\rangle$. The arguments of the functional J extrema are then:

for the maximum

$$\varphi^+(x) \equiv f^0(x), \tag{4.2.1}$$

and for the minimum

$$\varphi^-(x) \equiv f^0(L_1+L_2-x). \tag{4.2.2}$$

The proof of the Eq. (4.2.1): we should construct a decreasing function equimeasurable with the generating function f^0. Such a function is unique, which concludes the proof. In the case of the Eq. (4.2.2), an increasing function belonging to the set Ψ should be constructed in the interval $\langle L_1, L_2\rangle$ (it is also unique). From the relation

$$\varphi^-(x) \equiv f(x_1), \tag{4.3.1}$$

where

$$L_2 - x = x_1 - L_1,$$

we obtain the original thesis.

Now, the more general case may be considered in which the function $j(x)$ determined in $I_L \equiv \langle L_1, L_2\rangle$ has the *property* 1: The interval I_L is represented in the form:

$$\bigcup_{s=1}^{k} I_s = I_L, \quad I_j \cap I_k = \Phi, \quad j \neq k \tag{4.4}$$

and in each interval I_s the influence function $j(x)$ is continuous, monotonic and bounded.

To construct the extremum arguments of the functional J in the set Ψ, let us introduce the intervals

$$E_{s;y} = \{x; x \in I_s, \quad j(x) > y\}. \tag{4.5}$$

Consider now the generating function f^0 of the set Ψ, and assume it to be monotonic decreasing. Then

$$\bigwedge_{y} \bigvee_{x_0 \in I_L} x_0 - L_1 = \sum_{s=1}^{k} m_s(E_{s,y}), \tag{4.6}$$

where the right-hand number represents the global length of all these intervals for which $j(x) > y$. Observe, moreover, that for $E_{s;y} \neq 0$ there exists a point $x_0(y) \in I_s$ at which $j(x_s) = y$.

In order to construct a function equimeasurable with $f^0(x)$ and yielding a maximum of J, we should assume

$$\varphi^+(x_s(y)) = f^0(x_0), \tag{4.7}$$

where the relation between x_s and x_0 is found from the composition of the functions $x_0 = A(y)$ [according to the Eq. (4.6)] and $y = j(x_s)$. Disregarding the index s after the composition and substitution into the Eq. (4.7) — since the possible values of x_s fill up the interval I_L — we obtain the argument of the functional maximum with the influence function $f(x)$ having the property 1:

$$\varphi^+(x) = f^0[A(j(x))]. \tag{4.8}$$

A similar reasoning yields the formula for the argument of minimum φ^-,

$$\varphi^-(x) = f^0[B(j(x))], \tag{4.9}$$

where the function $x_0 = B(y)$ is determined from the relation

$$L_2 - x_0 = \sum_{s=1}^{k} \operatorname{mes}(E_{s,y}). \tag{4.10}$$

In practical applications, the generating function is often sectionally constant, the influence function preserving the property 1. Arguments of the extremum are then also sectionally constant — and it remains to determine these intervals (their ends).

Let x_i, $i = 1, 2, \ldots, R$, be the internal (within the interval I_L) discontinuity points of the first kind for the generating function f^0. The requirement of equimeasurability of φ^+ and f^0 implies the inequality to be satisfied:

$$x_i - L_1 = \sum_{s=1}^{k} \operatorname{mes}(E_{s,y_i}). \tag{4.11}$$

The function $f(x)$ is sectionally monotonic, and hence

$$y_i = A^{-1}(x_i). \tag{4.12}$$

Let $x_{s,i}$ denote the discontinuity points of extremum arguments within the interval I_s. Then

$$y_i = j(x_{s,i}) \Rightarrow x_{s,i} = j_s^{-1}(y_i), \tag{4.13}$$

where the index s denotes the inversion of the influence function in I_s.

Combining the relations (4.12) and (4.13) we obtain a transformation of the points of discontinuity of the generating function into the discontinuity points of the argument of maximum

$$x^{+}_{s,i} = j_s^{-1}[A^{-1}(x_i)], \quad i = 1, 2, \ldots, R. \tag{4.14}$$

In an analogous manner, the solution for the argument of minimum may be obtained:

$$x^{-}_{s,i} = j_s^{-1}[B^{-1}(x_i)], \quad i = 1, 2, \ldots, R, \tag{4.15}$$

where B^{-1} is found from the Eq. (4.10).

The criteria for selecting the values of φ^+ and φ^- in the neighbourhood of points $x_{s,i}$ are:

If

$$\lim_{x \to x_i^{\vartheta}} f^0(x) \equiv \Delta_i^{\vartheta}, \quad \vartheta \equiv +, - \tag{4.16}$$

and, as has been assumed, $\Delta_i^- > \Delta_i^+$, then for $j(x) \nearrow$ in I_s

$$\lim_{x \to x^{+}_{s,i}+} \varphi^+(x) = \Delta_i^-; \quad \lim_{x \to x^{-}_{s,i}+} \varphi^-(x) = \Delta_i^+, \tag{4.17.1}$$

while for $f(x) \searrow$ in I_s

$$\lim_{x \to x^{+}_{s,i}-} \varphi^+(x) = \Delta_i^-; \quad \lim_{x \to x^{-}_{s,i}-} \varphi^-(x) \pm = \Delta_i^+. \tag{4.17.2}$$

5. Examples and applications

Example 1. Let $I_L \equiv \langle -L, L \rangle$, $j(x) = x^2 - L^2/4$, and the generating function $f^0(x) = 2e^{-x}$. This case of the influence function possessing the property 1.

The function $A(y)$ determined from the Eq. (4.6) is equal to

$$A(y) = L - 2\sqrt{y + L^2/4}, \tag{5.1}$$

while the arguments of the extremum, according to the Eqs. (4.8), (4.9), are equal to

$$\varphi^+(x) = 2e^{-(L-2|x|)}, \tag{5.2.1}$$

$$\varphi^-(x) = 2e^{-(2|x|-L)}. \tag{5.2.2}$$

Example 2. Changing the generating function in Example 1 and assuming

$$f^0(x) = \begin{cases} f_1^0 & x \in \langle -L, 0 \rangle \\ f_2^0 & x \in \langle 0, L \rangle \end{cases} \quad f_1^0 > f_2^0, \tag{5.3}$$

we consequently obtain

$$A^{-1}(x) = \left(\frac{L-x}{2}\right)^2 - \frac{L^2}{4} \tag{5.4.1}$$

$$j_1^{-1}(y) = \sqrt{y + \frac{L^2}{4}} = -j_2^{-1}(y), \tag{5.4.2}$$

and in view of $x_i = 0$ (discontinuity point of the generating function), also $A^{-1}(0) = 0$ and the discontinuity points of the maximum arguments are

$$x_{1,1} = j_1^{-1}(0) = -\frac{L}{2}, \qquad x_{2,1} = j_2^{-1}(0) = \frac{L}{2}. \tag{5.5}$$

The methods of the preceding section may easily be applied to the case of bending and torsion of a container ship. Let, for example, $\omega(\xi)$, $\xi \in I_L$, denote the cross-sectional areas of a hold filled with cargo; $\gamma_0(x)$ is the generating function of a three-dimensional load given in the form of specific weight. $\gamma_0(x)$ is defined in what is called the equivalent hold, in the interval of length L_{red} [8].

Let the influence function $j(x)$ possess the property 1. The extremal solution is constructed in the following manner:

Let us construct the sets

$$E_{y;s} = \{x, j(x) > y \quad \text{and} \quad x \in I_s\}. \tag{5.6}$$

The hold contained in the interval $E_{y;s}$ should be filled with cargo showing the greatest specific weight, its volume being preserved. The latter requirement is written in the form

$$\omega_0 x_0 = \int\limits_{\bigcup_{s=1}^{k} E_{y;s}} \omega(\xi)d\xi \equiv \omega_0 A(y). \tag{5.7}$$

For each y we must determine such points $x_s \in I_s$ (provided they exist) that $y = j(x_s)$. As a result, we obtain the extremal density distribution as an argument of the extremum (e.g. in the case of bending) in the form:

$$\gamma^+(x_s) = \gamma_0(x_0) = \gamma_0[A(j(x))]. \tag{5.8}$$

x_s is an arbitrary point taken from I_s, and hence the index s may be disregarded in the expression for the extremum argument:

$$\gamma^+(x) = \gamma_0[A(j(x))]. \tag{5.9}$$

The argument of minimum is easily found to have the form

$$\gamma^-(x) = \gamma_0[B(j(x))], \tag{5.10}$$

where the function

$$B(y) \equiv L_{red} - A(y). \tag{5.11}$$

Example 3. Assuming the influence function as in Example 1, and the distribution of cross-sections of the hold as

$$\omega(\xi) = a(L^2 - x^2) \qquad x \in \langle -L, L \rangle, \tag{5.12}$$

dimensions of the equivalent hold being

$$\omega_0 = \frac{2}{3} a L^2, \tag{5.13.1}$$

$$L_{red} = 2L, \tag{5.13.2}$$

we obtain from the Eq. (5.7):

(5.14) $$A(y) = 2L-\frac{3}{L}\sqrt{y+\frac{L^2}{4}}\left[L^2-\frac{1}{3}\left(y+\frac{L^2}{4}\right)\right].$$

If the generating function is also taken from the Example 1, the argument of maximum may be calculated from the Eq. (5.9)

(5.15) $$\gamma^+(x) = 2\exp\left[-2L+3|x|-\frac{x^2|x|}{L^2}\right].$$

It will be demonstrated that the problem of static twisting due to a nonhomogeneous load may also be reduced to the problem described by the relations (5.9)–(5.11).

Indeed, let us consider a hull with vertical sides, called a linear hull (e.g. [8]). Let $\mathscr{D}$ denote the floating waterplane with the x-axis as the axis of symmetry of the waterplane and y-axis intersecting the x-axis at the center of gravity of $\mathscr{D}$. Denote

(5.16) $$\mathscr{D}_{x_0} = \{(x, y), x, y \in \mathscr{D}, x \leqq x_0\}.$$

The torque produced by the two-dimensional load $\varphi(x, y)$ at the waterplane $\mathscr{D}$ may be written, at small (blok), in the form

(5.17.1) $$M_s(x_0) = \iint_{\mathscr{D}} m_s(\xi, \eta; x_0)\varphi(\xi, \eta)d\xi d\eta,$$

where

(6.17.2) $$m_s(\xi, \eta;\, x_0) = \begin{cases} \left(1-\dfrac{I_y(x_0)}{I_y(L_2)}\right)\eta, & \xi, \eta \notin \mathscr{D}_{x_0}, \\ -\dfrac{I_y(x_0)}{I_y(L_2)}\eta, & \xi, \eta \notin \mathscr{D}_{x_0}; \end{cases}$$

$I_y(x_0)$ is the moment of inertia of a part $\mathscr{D}_{x_0}$ of the waterplane $\mathscr{D}$ with respect to y, L_2 is the bow coordinate. Due to $I_y(x_0)/I_y(L_2)$ for $x_0 \neq L_2$ it is easily seen that in order to find e.g. the extremal distribution of load leading to the maximal torque in the cross-section, the load should be distributed only over the regions:

(5.18) $$\mathscr{D}_{1,x_0} = \{(x, y), x, y \in \mathscr{D}_{x_0}, y > 0\}$$

and

(5.18.2) $$\mathscr{D}_{2,x_0} = \{(x, y), x, y \in \mathscr{D}\setminus\mathscr{D}_{x_0}, 0 > y\}.$$

Let us introduce the notations:

(5.19) $$u(x_0) \equiv \frac{I_y(x_0)}{I_y(L_2)},$$

$a^+(y, x_0)$ is length of the segment from the line $x = x_0$ to the waterline, measured along the line perpendicular to y-axis, for positive $y > 0$ in $\mathscr{D}_{1,x_0}$, and for negative $y < 0$ — in $\mathscr{D}_{2,x_0}$.

The maximum torque may be obtained from the nonhomogeneous load by so distributing it that the loads at the lines $y = \text{const}$ remain constant. The integral (5.17.1) should then be written in the form:

$$M_s(x_0) = \int_{-B/2}^{B/2} \left[\frac{1}{2}(1+\operatorname{sgn}\eta) - u(x_0)\right] \eta a^+(y, x_0)\varphi(\eta)\,d\eta, \tag{5.20}$$

where B denotes the waterplane width. Alternatively, introducing the height h of the holds at sections a^+, and denoting by $\omega(\eta, x_0)$ the area of cross-section of the hold space made by a plane perpendicular to the y-axis in the part above $\mathscr{D}_{1,x}$ or $\mathscr{D}_{2,x}$, we obtain

$$M_s(x_0) = \int_{-B/2}^{B/2} \left[\frac{1}{2}(1+\operatorname{sgn}\eta) - u(x_0)\right] \eta\omega(\eta, x_0)\gamma(\eta)\,d\eta; \tag{5.21}$$

$\gamma(\eta)$ is obviously the specific weight of the load.

The influence function $j(\eta)$ in the functional (5.21) is:

$$j(\eta) = \left[\frac{1}{2}(1+\operatorname{sgn}\eta) - u(x_0)\right]\eta. \tag{5.22}$$

The problem of torsion is thus reduced to the one-dimensional problem identical, in the general approach, with the problem of bending. If the generating function is prescribed in the equivalent hold of cross-section ω_{red} and length L_{red}, then in order to determine the maximum $M_s(x_0)$, we should use the heaviest load of the volume of holds placed above the region $\mathscr{D}_{1,x_0} \cup \mathscr{D}_{2,x_0}$ and apply the method described by the relations (5.6)–(5.11).

6. A simplified method of determination of dynamic torque

Let us consider a linear rigid hull on an oblique plane wave. The method is a generalization of that presented in [3] to the case of nonhomogeneous loads, vertical motions and dynamic interactions due to yawing. An extensive numerical discussion for an actual ship, based on results of the present paper, may be found in [10].

Let x, y, z denote the Cartesian coordinate system connected with the waterplane (x-axis from stern to bow, z-axis perpendicular to the waterplane). The Cartesian reference frame ξ, η will be twisted by angle α with respect to x, y. The plane wave is given by the equation $\zeta = h(\xi)$. Due to the fact that coordinates x, y are transformed to ξ, η by means of the formulae

$$\eta = x\sin\alpha + y\cos\alpha, \tag{6.1.1}$$

$$\xi = x\cos\alpha - y\sin\alpha, \tag{6.1.2}$$

the wave ordinates at the sides of the hull are

$$\zeta_1 = h(x\cos\alpha - F(x)\sin\alpha), \tag{6.2.1}$$

$$\zeta_2 = h(x\cos\alpha - F(x)\sin\alpha). \tag{6.2.2}$$

The inequality $F(x) \geqslant 0$ yields an even contour of the floating waterplane for $x \in I_L \equiv \equiv \langle -L/2, L/2 \rangle$. Side-sways and vertical motions of the ship are assumed to be absent, the draught of the wave is equal to that at calm sea.

It is additionally assumed that the extra pressure produced by the waves is calculated according to hydrostatic laws, with additional reduction coefficients k_B for the bottom and k_S for the sides. The methods of calculations of k_S and k_B are outlined, e.g., in [3].

It may be shown by elementary methods that only two of all hydrodynamic interactions do not vanish. The vertical buoyancy force applied to the center of the floating waterline:

$$P_v = 2k_B \gamma_w \int_0^{L/2} F(x)(\zeta_1 + \zeta_2) dx \tag{6.3}$$

and the yawing moment:

$$M_z = 2\gamma_w k_B \int_0^{L/2} x \left[\zeta_2 \left(d + \frac{\zeta_2 - |\zeta_2|}{2} \right) - \zeta_1 \left(d + \frac{\zeta_1 - |\zeta_1|}{2} \right) \right] dx + \gamma_w k_B \int_0^{L/2} x(\zeta_2 |\zeta_2| - \zeta_1 |\zeta_1|) dx; \tag{6.4}$$

L is ship's length, γ_w — specific weight of water, d — calm sea draught of the ship.

The forces are equilibrated by instantaneous inertia forces of the masses of the hull and of the associated water. Omitting the detailed considerations of all the load components, let us list only those which contribute to the torque.

Continuous vertical load due to cargo

$$P_v = q_2(x) \tag{6.5}$$

acting along the line given by the parametric equations:

$$z = \bar{z}_2(x), \tag{6.6.1}$$

$$y = \bar{y}_2(x). \tag{6.6.2}$$

Continuous vertical load due to hydrodynamic forces acting on the bottom:

$$\check{P}_v = k_B \gamma_w (\zeta_1 + \zeta_2) F(x). \tag{6.7}$$

Coordinate $\check{y}(x)$ of the line of action of that load satisfies the equation:

$$\check{y}(x)(\zeta_1 + \zeta_2) = \frac{1}{2}(\zeta_1 - \zeta_2) F(x). \tag{6.8}$$

Continuous horizontal load due to hydrodynamic pressure acting on the sides

$$P_h = \gamma_w k_B \left[\zeta_2 \left(d + \frac{\zeta_2 - |\zeta_2|}{2} \right) - \zeta_1 \left(d + \frac{\zeta_1 - |\zeta_1|}{2} \right) \right] + \frac{1}{2} \gamma_w k_B (\zeta_2 |\zeta_2| - \zeta_1 |\zeta_1|); \tag{6.9}$$

$P_h > 0$ when its direction agrees with that of the y-axis. Coordinate $\tilde{z}(x)$ of the line of action of the load P_n satisfies the equation

$$(6.10)\quad \frac{1}{2}\tilde{P}_{h,1}\left(d+\frac{\zeta_1-|\zeta_1|}{2}\right)+\frac{1}{2}\tilde{P}_{h,2}\left(d+\frac{\zeta_2-|\zeta_2|}{2}\right)+\check{P}_{h,1}\left(d+\frac{1}{2}\zeta_1-\frac{1}{6}|\zeta_1|\right)+\check{P}_{h,2}\left(d+\frac{1}{2}\zeta_2-\frac{1}{6}|\zeta_2|\right)=P_h\left(d-\tilde{z}(x)\right),$$

where

$$(6.11.1)\quad \tilde{P}_{h,i} = -\gamma_w k_B \zeta_i\left(d+\frac{\zeta_i-|\zeta_i|}{2}\right), \quad i = 1, 2,$$

$$(6.11.2)\quad \check{P}_{h,i} = \frac{1}{2}\gamma_w k_B \zeta_i|\zeta_i|, \quad i = 1, 2.$$

Continuous vertical load due to inertia forces of vertical accelerations:

$$(6.12)\quad P_{v,2} = -\ddot{z}q_2(x)\frac{1}{g},$$

where

$$(6.13)\quad \ddot{z} = \frac{2k_B\gamma_w\int\limits_0^{L/2} F(x)(\zeta_1+\zeta_2)dx}{2\int\limits_0^{L/2} m_v(x)dx+\int\limits_{-L/2}^{L/2}\frac{1}{g}q(x)dx}.$$

In the formula, $m_v(x)$ is the linear density of mass of the associated water during the vertical motion of the hull, $q(x)$ — the total linear weight of the hull (cargo and structure).

Coordinate y of the line of action of the load is

$$(6.14)\quad y = \bar{y}_2(x)$$

as for the statical load due to cargo.

Dynamic loads due to yawing. Load due to inertia of water (horizontal)

$$(6.15)\quad P_{h,1} = -\ddot{\Psi}xm_\varphi(x),$$

where

$$(6.16)\quad \ddot{\psi} = \frac{M_z}{2\int\limits_0^{L/2} m_\varphi(x)x^2dx+\int\limits_{-L/2}^{L/2}\frac{1}{g}q(x)x^2dx}$$

M_z is the yawing moment, $\ddot{\psi}$ — yawing acceleration, $m_\varphi(x)$ — mass density of associated water at yawing.

Such a problem has not so far been solved in hydrodynamics ,for heavy sea conditions in particular. The point of application of the resultant force remaining unknown, let us assume for an approximation, that $z = z_h$ = const and equals one half of the draught d. This assumption may be justified by certain solutions obtained in the literature (by a-nalogue method) in which the distributions of masses of the associated water at the sides of rectangular cross-sections were determined (almost constant).

The last load is the horizontal load due to the inertia of the hull when yawing:

(6.17)
$$P_{h,2} = -\ddot{\psi}x\frac{q(x)}{g}.$$

coordinate of its line of application being $z = \bar{z}(x)$, where $\bar{z}$ is coordinate z of the line of mass centers for the entire ship, together with the cargo.

In calculating the torque intensity due to the vertical loads, e.g. P_h, applied to the line $z(x)$, it should be noted that the line of shear centers $c(x)$ constitutes a "crankshaft", and hence

(6.18)
$$M_s(x_l) = \int_{L_1}^{x_0} P_h[c(x_0)-z(x)]\,dx.$$

The torque intensity, after differentiation of both sides of Eq. (6.18) with respect to x_0, yields:

(6.19)
$$m_s(x_0) = P_h[c(x_0)-z(x_0)]+\sum\delta(x-x_i)\Delta c_{x_i},$$

where $\delta(x-x_1)$ denotes the Dirac delta applied to the point of discontinuity of the line of shear centers, the corresponding jump being equal to

(6.20)
$$\Delta c_{x_i} = c(x_i^+)-c(x_i^-).$$

In the example it is assumed that $C(x_0) = \text{const}$ and then the torque intensity is calculated from the formula

(6.21)
$$m_s(x) = q_2(x)\bar{y}_2(x)-\check{P}_v(x)\check{y}(x)+P_h(x)[c-\tilde{z}(x)]-P_{v,2}\bar{y}_2(x) +P_{h,1}(x)(c-z_h)+P_{h,2}[c-\bar{z}(x)].$$

The criterion for uncoupling with the side-sway is reduced to the requirement that the horizontal load $P_{h,2}$, possessing a non-vanishing horizontal component, should not give rise to a non-vanishing moment with respect to an arbitrary straight line parallel to x. This requirement is easily reduced to the condition:

(6.22)
$$\int_{I_L} q(x)\bar{z}(x)x\,dx = 0.$$

The method of selection of initial parameters in control computations which yielded the order of magnitude of the torque and which constituted a control test for a program elaborated in [10] was as follows:

Case A. Assuming the load nonhomogeneity and calculating the point of intersection of the core with the axes $\bar{z}_2^{(1)}$ and $\bar{z}_2^{(2)}$, the states of loading are assumed to be symmetric, i.e. such that $\bar{y}_2(x) \equiv 0$. The ship is then not twisted in calm sea.

The condition of vanishing of heels is now satisfied identically. The condition of uncoupling with the side-sways (6.22) is now written in a particular form, by assuming that the line of gravity centers of cargo coincides with the boundary points of the core and has only one discontinuity point of the first kind for a certain value of $x = x_i$.

Equation (6.22) then takes the form:

(6.23)
$$\int_{I_L} q_1(x)\bar{z}_1(x)\,dx+\bar{z}_2^{(i)}\int_{-L/2}^{x_l} q_2(x)x\,dx+\bar{z}_2^{(j)}\int_{x_l}^{L/2} q_2(x)x\,dx = 0,$$

containing the unknown x_i to be determined. The limits of integration in the Eq. (6.23) describe the intervals in which $\bar{z}_2(x)$ are equal to the coefficient standing before the integrals. $\bar{z}_1(x)$ is the coordinate of the centers of gravity of the structure. Two separate cases are distinguished here, A_i, $i = 1, 2$, according to the choice of the index i in the Eq. (6.23).

Case B. In the case in which the static torque is influenced by nonhomogeneity, the core is approximated by a rectangle inscribed within the actual core, it being assumed that the line of mass centers of the load runs through the vertices of the core, and that each of the coordinates in the parametric equation of the mass center line possesses at most one point of discontinuity. The condition of uncoupling with side-sways remains identical with the Eq. (6.23), while the condition of absence of heels assumes the form:

$$\int_{-L/2}^{x_0} q_2(x)dx - \int_{x_0}^{L/2} q_2(x)dx = 0. \tag{6.24}$$

This equation serves to determine the point of discontinuity of the coordinate $\bar{y}_2(x)$ which may assume two values $\pm\bar{y}_2$ obtained from the estimation of the core. Thus we have to deal here with the case C1, when $\bar{y}_2(x) = \bar{y}_2$ for $x \in \langle -L/2, x_0\rangle$ and $-y_2$ in the remaining part of the interval, and with the case C2 when the signs of $\bar{y}_2(x)$ are changed. In numerical calculations the computer evaluated the torques in four cases B_i, $i = 1, 2, 3, 4$, shown in Table 2 according to the combinations of A and C.

Table 2.

	A1	A2
C1	B1	B3
C2	B2	B4

The mass of associated water is determined by means of the generally known method of TAYLOR or DOREFEJUK. That is, however, the weakest point of the present paper. In the author's opinion, studies should be initiated on the problem of detrmination of the associated water mass for curved free surfaces. This problem will be dealt with in a separate paper.

An extensive program written in Algol 1204 for the Odra 1204 digital computer was applied to an actual structure and contained not only the determination of internal forces but also other structural problems.

Results of the test calculations of the torque for square waterplanes are shown in Figs. 1, 2, together with the corresponding input data.

The next important step in the investigation of internal forces should be an exact formulation of the hydrodynamic problem, and examination of internal forces treated as stochastic functionals of wave processes and cargo loading. Within the framework of strength problems involving container ship structures, other problems also seem to be of interest, as for instance, non-linear torsion of a hull with hatches in elastic-plastic range.

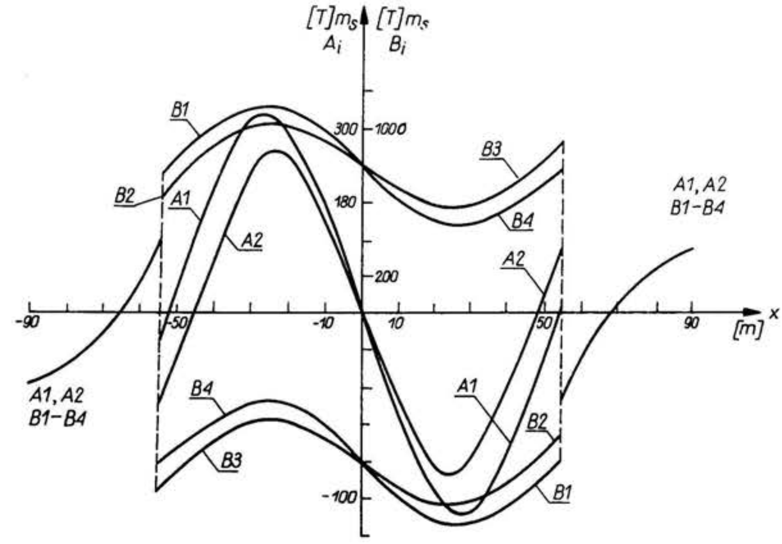

FIG. 1. Torque intensity

$L = 180$ m, $\alpha = \pi/3$, $\gamma_w = 1$ Tm^{-3}, $k_B = k_s = 1$, $c = 11$ m,

$\bar{y}_2 = 5$ m, $\bar{z}_2^{(1)} = -2$m, $\bar{z}_2^{(2)} = 4$ m, $F = 10$ m, $\bar{z}_1 = 3$ m, $q_2 = 50$ Tm^{-1},

$$q = \begin{cases} 160 \text{ Tm}^{-1} \times \varepsilon \langle -50.50, \rangle \\ 0 \qquad\qquad \times \varepsilon I_L \diagdown \langle -50.50 \rangle. \end{cases}$$

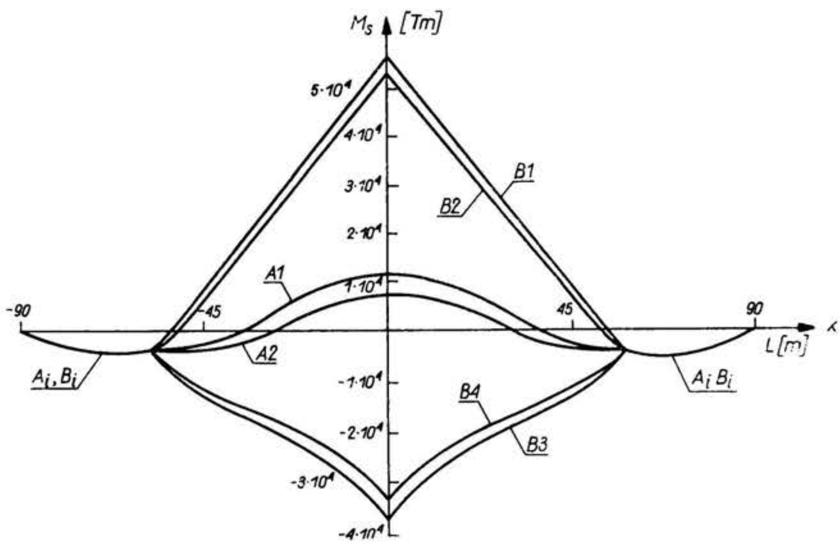

FIG. 2. Torques

$L = 180$ m, $\alpha = \pi/3$, $\gamma_w = 1$ Tm^{-3}, $k_B = k_s = 1$, $c = 11$ m,

$\bar{y}_2 = 5$ m, $\bar{z}_2^{(1)} = 2$ m, $\bar{z}_2^{(2)} = 4$ m, $F = 10$ m, $\bar{z}_1 = 3$ m, $q_2 = 50$ Tm^{-1}

$$q = \begin{cases} 160 \text{ Tm}^{-1} \times \varepsilon \langle -50.50 \rangle \\ 0 \qquad\qquad \times \varepsilon I_L \diagdown \langle -50.50 \rangle. \end{cases}$$

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