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Theory of viscoplasticity of irradiated materials

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A MATHEMATICAL structure of a thermodynamic theory of viscoplasticity of irradiated materials is presented. In this theory use is made of an internal variable description. A set of rules of interpretation for internal state variables introduced is given. These rules are based on the analysis of dissipative mechanisms for time-dependent plastic flow of irradiated materials and on available experimental data. The thermal activation process and damping of dislocation motion by phonon viscosity are assumed as most important mechanisms for proper explanation of the strain rate and temperature sensitivity of irradiated materials.

Przedstawiono matematyczną strukturę termodynamicznej teorii lepkoplastyczności materiałów napromieniowanych. W teorii tej wykorzystano opis za pomocą parametrów wewnętrznych. Podano zbiór zasad interpretacji dla wprowadzonych parametrów wewnętrznych. Zasady te są oparte na analizie mechanizmów dysypacyjnych dla zależnego od czasu plastycznego płynięcia materiałów napromieniowanych i na dostępnych rezultatach badań eksperymentalnych. Proces termicznych aktywacji i tłumienia ruchu dyslokacji lepkością fononową przyjęto jako najważniejsze mechanizmy dla właściwego wyjaśnienia wrażliwości materiałów napromieniowanych na prędkość odkształcenia i temperaturę.

Представлена математическая структура термодинамической теории вязкопластичности облучаемых материалов. В этой теории использовано описание при помощи внутренних параметров. Дается совокупность принципов интерпретации для введенных внутренних параметров. Эти принципы опираются на анализ механизмов диссипации для зависящего от времени пластического течения облучаемых материалов и на доступные результаты экспериментальных исследований. Процесс термической активации и затухание движения дислокаций фононной вязкостью приняты как самые важные механизмы для надлежащего объяснения чувствительности облучаемых материалов к скорости деформации и к температуре.

1. Introduction

The main object of this paper is to construct a phenomenological theory describing irradiation effects during time-dependent plastic flow of metals. In this theory use is made of an internal variable description and thermodynamic approach.

Available experimental data are discussed to show changes of thermo-mechanical properties due to neutron irradiation of metals. From our standpoint, particular value attaches to results of investigating the following effects: (i) The influence of neutron irradiation on the yield limit and stress-strain curve of a material; (ii) Changes in rate sensitivity of a material due to neutron irradiation; (iii) The temperature dependence of the yield stress of neutron irradiated material; (iv) The temperature dependent annealing of neutron irradiation hardening.

The most important results of irradiation in metals is the production of defects (cf. review by LENSKY [13], SEEGER [34] and SEEGER and JAN [35]).

Mechanisms responsible for internal dissipation of an irradiated material are discussed. It has been assumed that two mechanisms — namely, the thermal activation process and the damping of dislocation by phonon viscosity — are most important for proper explanation of the strain rate and temperature sensitivity of an irradiated material. In Sec. 2 are presented simple physical models for the description of these two mechanisms. A general mathematical theory is given in Section 3. Two groups of internal state variables are introduced to describe rheological properties and time-dependent plastic flow. In Sec. 4 is given the description of the behaviour of an elastic-viscoplastic irradiated material in an entire spectrum of strain rate and temperature. This description is obtained by basing physical interpretation for internal parameters introduced on simple physical models constructed in Sec. 2.

2. Physical and experimental motivations

ROSENFIELD and HAHN [31] and CAMPBELL and FERGUSON [4] have shown that in the temperature-strain rate spectrum of plain carbon steel we can consider several regions which reflect different mechanisms of plastic deformation.

CAMPBELL and FERGUSON [4] have shown that curves for the flow stress at lower yield against the logarithm of strain-rate for the various temperatures can be divided into three regions (Fig. 1). Following ROSENFIELD and HAHN [31] these will be referred to as Regions

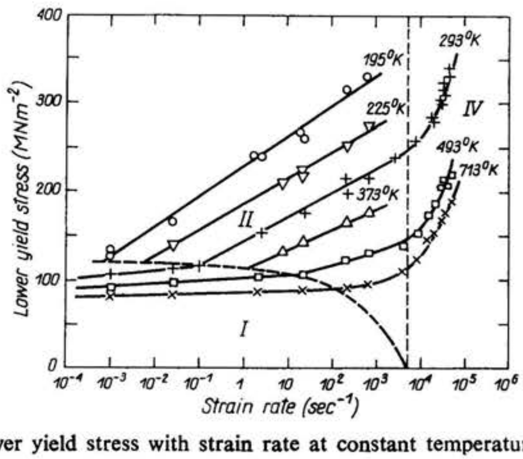

FIG. 1. Variation of lower yield stress with strain rate at constant temperature, for mild steel. After CAMPBELL and FERGUSON [4].

I, II and IV. In Region I, curves are characterized by a small, almost constant slope, and the flow stress shows little temperature dependence. The curves in Region II are also straight, but the rate and temperature dependence are considerably greater. In Region IV, the flow stress shows a further increase in rate dependence, while the temperature dependence is unaffected.

It is generally accepted that in Region I the plastic flow is governed by a thermal mechanism, in Region II by the thermal activation rate theory, and in Region IV by the theory of the damping of dislocation motion by phonon viscosity.

In the case of irradiated materials, the question arises as to what extent the irradiation changes the picture as presented in Fig. 1. In other words, we ask what is the influence of neutron irradiation on the mechanisms of plastic flow in an entire spectrum of strain rate and temperature.

It is well known that thermo-mechanical properties of structural materials may be considerably altered by irradiation. The effects of radiation on the properties of materials are due principally to defects produced by radiation.

From the theoretical point of view, radiation can harden a metal in two basically different ways, by making the nucleation of slip difficult (source hardening) or by making the propagation of slip difficult (lattice hardening). MAKIN and MINTER [14] showed that in well annealed metals such as copper and nickel both types of hardening occur on irradiation.

SEEGER [32] proposed a theory for lattice hardening based upon cutting a dislocation through a forest of obstacles under the action of both stress and thermal activation. This theory is valid in Region II and the inelastic strain rate is given by the equation:

$$\dot{P} = (Nb/N_z)\nu_0 \exp[-U(T-Y)/k\vartheta], \tag{2.1}$$

where $\dot{P}$ is the inelastic deformation rate, N is the number of dislocations per unit volume held up against obstacles, N_z is the number of obstacles per unit area of glide plane, b is the Burgers vector, ν_0 is the frequency of oscillation of a dislocation, $U(T-Y)$ is the activation energy for cutting through the obstacles at stress $T^* = T-Y$, and ϑ is the absolute temperature.

In Seeger's theory, it is assumed that the activation energy U_0 required by a dislocation to cut through an obstacle is a constant, and that N_z, the number of obstacles per unit area of glide plane, is directly proportional to the dose φ, where φ is determined by the radiation flux ϕ by the formula:

$$\varphi(t) = \int_0^t \phi(z)\,dz. \tag{2.2}$$

MAKIN and MINTER [14] compared experimental data obtained for irradiation hardening in copper and nickel with Seeger's theory. This comparison showed that Seeger's assumption of a constant U_0 is an over-simplification(1).

We shall modify Seeger's theory by introducing additional parameters responsible for a description of radiation effects observed by experiments.

First we assume

$$Nb\nu_0/N_z = \gamma_1^*(\vartheta, \xi), \tag{2.3}$$

where ξ is the concentration of defects. Similarly, the yield stress in static loading Y is assumed to be the function of temperature ϑ, the concentration of defects ξ and the inelastic deformation P. Thus the inelastic deformation rate in the modified theory is as follows (cf. PERZYNA [22, 29]):

$$\dot{P} = \gamma_1^*(\vartheta, \xi)\Phi\left[\frac{T}{Y(\vartheta, \xi, P)} - 1\right], \tag{2.4}$$

where Φ is a new function of excess of stress over the static yield point. This function can be chosen according to experimental data for irradiated materials.

(1) Further comments on Seeger's theory can be found in KOPPENAAL [9, 10], MAKIN [18, 21], MAKIN and SHARP [19], ARSENAULT [1] and KOPPENAAL and ARSENAULT [11]; cf. also SEEGER [33].

To determine the rate of the concentration of defects $\dot{\xi}$, we shall consider two main sources of generation of defects (cf. SEEGER [34]) — namely irradiation and plastic deformation.

The initial, direct observation of the lattice defects created by neutron irradiation was made by SILCOX and HIRSCH [38]. They used the thin film transmission electron microscope technique to make a quantitative study of neutron damage in copper(²). It has been observed that dislocation loops are formed in copper irradiated with neutrons at a temperature slightly above room temperature. For small doses (10^{18} neutron/cm^2), many of the defects observed can only be resolved as small regions of strain. The diameter of those defects which can be recognized as loops is 100Å or less. With increasing dose of radiation, the average diameter of these loops increases. For large doses of irradiation (10^{20} neutron/cm^2), dense dislocation networks are observed in some areas. Radiation hardening is interpreted by SILCOX and HIRSCH in terms of a forest of dislocation loops. They observed that radiation hardening anneals out at a temperature at which the loops disappear by climb(³).

The same experimental technique was used by MAKIN, WHAPHAM and MINTER [15] to measure the density of observable defect clusters as a function of diameter after various neutron doses between $0.61 \cdot 10^{18}$ and $3.79 \cdot 10^{18}$ fast neutrons/cm^2. They found that the

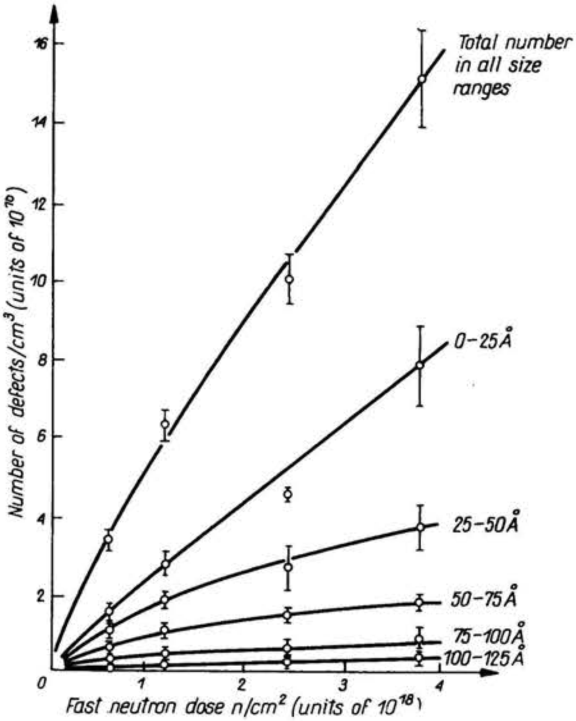

FIG. 2. The density of defects of various diameters as a function of the neutron dose. After MAKIN, WHAPHAM and MINTER [15].

(²) SHARP [36], using the same technique, investigated the slip process in neutron irradiated single crystals of copper.

(³) Cf. MAKIN and MANTHORPE [17].

total number of clusters and point defects in the visible damage is approximately linear with the dose([4]). The data are presented in Fig. 2, where the number of defects of a given diameter is plotted as a function of the neutron dose.

MAKIN and MANTHORPE [16] used the transmission electron microscopy to measure the density of defect clusters of annealing. Typical curves of the density as a function of diameter after various annealing times at 275°C are shown in Fig. 3.

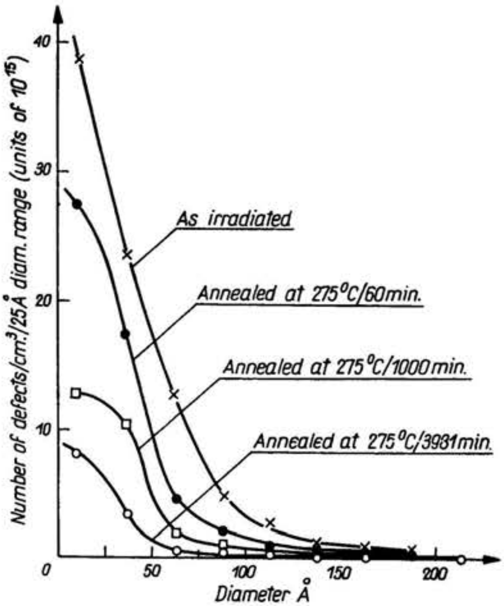

FIG. 3. The density of defects as a function of diameter after annealing at 275°C for various times. After MAKIN and MANTHORPE [16].

Equation (2.4) yields the dynamical relation for stress

$$T = Y(\vartheta, \xi, P)\{1+\Phi^{-1}[P/\gamma_1^*(\vartheta, \xi)]\}. \tag{2.5}$$

In the limit case, when $\gamma_1^* \to \infty$ this relation gives

$$T = Y(\vartheta, \xi, P) \tag{2.6}$$

— i.e., the static stress-strain curve for irradiated materials([5]).

Equation (2.5) describes the dependence of the flow stress upon the rate at which plastic deformation occurs. The strain rate dependence of the flow stress has been measured in neutron irradiated single crystals of copper by a number of workers([6]) — cf., for instance, a review paper by KOPPENAAL and ARSENAULT [11].

([4]) Cf. also MAKIN, MINTER and MANTHORPE [20].

([5]) Cf. OLSZAK and PERZYNA [23].

([6]) The effect of strain rate on the yield stress of irradiated stainless steel was investigated by WILSON and BERGGREN [39]. Their results indicated that the yield stress of the irradiated steel is markedly dependent on strain rate. Similar results were obtained by CAMPBELL and HARDING [3].

In Region II, a linear relationship between dT and $d(\ln \dot{P})$ has been reported by several authors.

Equation (2.6) represents the dependence of the flow stress upon temperature, upon concentration of defects due to irradiation, and upon plastic deformation in the static test. To determine this relation, we can use experimental data reported by BLEWITT, COLTMAN, JAMISON and REDMAN [2]. They investigated the effect of neutron irradiation on the mechanical properties of copper single crystals. In Fig. 4 is shown the effect of

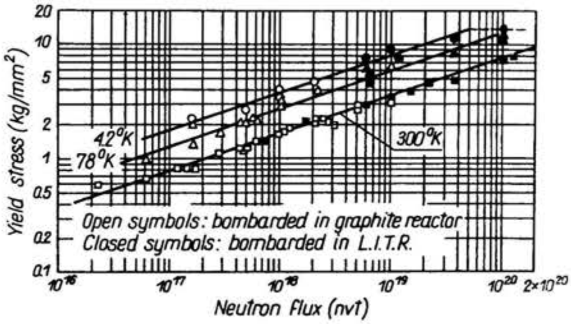

FIG. 4. Effect of neutron irradiation on the yield stress of copper at various temperatures. After BLEWITT, COLTMAN, JAMISON and REDMAN [2].

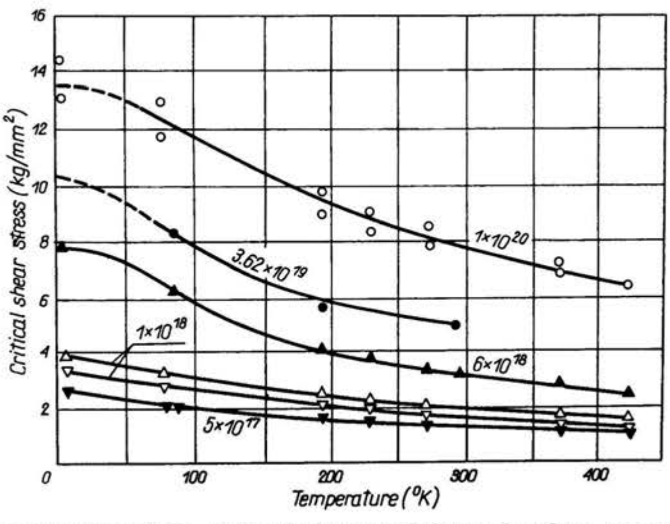

FIG. 5. The temperature dependence of the yield stress of reactor irradiated copper single crystals at various neutron doses. After BLEWITT, COLTMAN, JAMISON and REDMAN [2].

neutron irradiation on the yield stress of copper at various temperature. The logarithm of the yield stress is plotted against the logarithm of the dose. The fact that a straight line of slope 1/3 can be drawn through the data, clearly shows that the yield stress is proportional to the cube root of the dose from $5 \cdot 10^{16}$ neutron/cm² through $1 \cdot 10^{20}$ neutron/cm². The yield stress has also been determined as a function of temperature following bombardment at reactor ambient temperatures. This shows a strong dependence on the temperature, increasing significantly as the temperature decreases (Fig. 4). Details of the temperature dependence of the critical shear stress are shown in Fig. 5. The critical shear stress is plotted as a function of temperature for samples irradiated from $5 \cdot 10^{17}$ to $1 \cdot 10^{20}$ neutron/cm^2. Each of the curves of different doses is approximately parallel, which shows that the influence of irradiation dose is independent of the influence of temperature.

The one-third power-dependence has been reported by several authors (see comments by KOPPENAAL and ARSENAULT [11]). The interpretation of this relationship remains unclear.

The non-linear relation (2.6) can describe such effects as radiation annealing or saturation, which have been observed by experiment.

Annealing experiments conducted by BLEWITT, COLTMAN, JAMISON and REDMAN [2] in the region above room temperature show that the remaining radiation hardness is

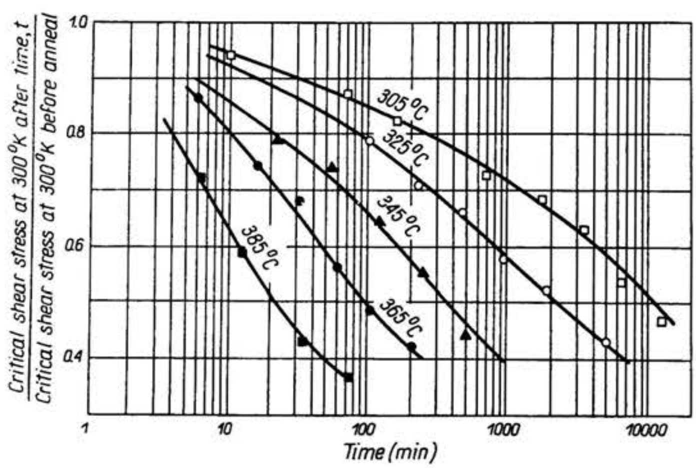

FIG. 6. Isothermal annealing of the yield stress of copper crystals irradiated to a dose of $1 \cdot 10^{19}$ neutron/cm^2. After BLEWITT, COLTMAN, JAMISON and REDMAN [2].

annealed in the region from 300°C to 400°C. Figure 6 shows isothermal annealing of the yield stress of copper crystals irradiated to a dose of $1 \cdot 10^{19}$ neutron/cm^2.

Saturation effect has been investigated by MAKIN and MINTER [14] for polycrystalline copper and nickel.

For high strain rate (Region IV), SHWINK and GRIESHAMMER [37] and DIEHL, SEIDEL and NIEMANN [6] observed that $dT/d(\ln \dot{P})$ was not constant but rapidly increased as the strain rate increased.

This experimental observation suggests that the mechanism of plastic flow in Region IV is different from that in Region II. The yield stress of irradiated metals in Region IV seems to become extremely strain-rate sensitive. We assume that the increase of the rate sensitivity in Region IV is due to viscous resistance to dislocation motion, and the plastic flow mechanism is consistent with that of the damping of dislocation motion by phonon viscosity.

We shall generalize the phonon viscosity theory as developed by MASON [22] to make it valid for irradiated materials.

In the viscous damping region, a linear dependence of stress on strain-rate is observed and can be represented by the relation

$$\dot{P} = \gamma_2^*(\vartheta, \xi)\left[\frac{T}{T_B(\vartheta, \xi, P)} - 1\right], \tag{2.7}$$

where T_B is attributed to the stress needed to overcome the forest dislocation barriers to the dislocation motion and is called the back stress; $\gamma_2^*(\vartheta, \xi)$ is called the viscosity coefficient.

Dependence of the back stress T_B on temperature ϑ and the plastic strain P has been proved by experiment (cf. KUMAR and KUMBLE [12] for copper and FERGUSON, KUMAR and DORN [8] for aluminium). Additional dependence of the back stress T_B on the concentration of defects for irradiated materials is assumed.

The viscosity coefficient is given by the relation

$$\gamma_2^*(\vartheta, \xi) = \frac{\varrho_M b^2}{B}, \tag{2.8}$$

where ϱ_M is the density of the mobile dislocation, b denotes the Burgers vector and B is called the dislocation drag coefficient.

Equation (2.7) yields the dynamical relation for stress

$$T = T_B(\vartheta, \xi, P)\,[1 + \dot{P}/\gamma_2^*(\vartheta, \xi)]. \tag{2.9}$$

3. General theory

Let us consider a body $\mathscr{B}$ with particles X and assume that this body can deform inelastically, conduct heat and is bathed in the neutron radiation field.

In order to describe deformation, temperature distribution, and the influence of radiation in a body $\mathscr{B}$, it is necessary to specify the deformation-temperature-radiation configuration $\Lambda(X, t)$ at a particle X. This configuration is given by

$$\Lambda(X, t) = \{C(X, t), \vartheta(X, t), \nabla\vartheta(X, t), \phi(X, t)\}, \tag{3.1}$$

where $C(X, t)$ denotes the right Cauchy-Green deformation tensor, $\vartheta(X, t)$ is temperature, $\nabla\vartheta(X, t)$ — temperature gradient, and $\phi(X, t)$ represents the radiation flux at the instant t.

The main purpose of this paper is to describe thermo-mechanical coupling with radiation influence in dissipative materials.

It has been pointed out by PERZYNA and SAWCZUK [30] that the most natural way to handle these phenomena is to base considerations on rational thermodynamics.

To do this, let us define a local thermodynamic process at a material point X of a body $\mathscr{B}$ in the motion χ as a collection of functions:

$$\mathscr{P}_X = \{\Lambda(X, t), \Pi(X, t), \Gamma(X, t)\}, \tag{3.2}$$

given for every $t \in (t_p, t_k)$, which satisfies two laws of thermodynamics in local form,

$$\operatorname{Div}(FT)+\varrho b = \varrho\ddot{x}, \quad T = T^T,$$

(3.3)
$$\frac{1}{2}\operatorname{tr}(T\dot{C})-\operatorname{Div}q-\varrho(\dot{\psi}+\vartheta\dot{\eta}+\dot{\vartheta}\eta)+\varrho r = 0,$$

$$-\dot{\psi}-\dot{\vartheta}\eta+\frac{1}{2\varrho}\operatorname{tr}(T\dot{C})-\frac{1}{\varrho\vartheta}q\cdot\nabla\vartheta \geqslant 0,$$

where $\Lambda(X,t)$ is the deformation-temperature-radiation configuration of a particle X, $\Pi(X,t)$ represents the variables

(3.4)
$$\Pi(X,t) = \{\psi(X,t), \eta(X,t), T(X,t), q(X,t)\}$$

— namely the specific free energy per unit mass ψ, the specific entropy per unit mass η, the Piola-Kirchhoff stress tensor T, and the heat flux vector per unit surface in the reference configuration q; $\Gamma(X,t)$ stands for the functions

(3.5)
$$\Gamma(X,t) = \{b(X,t), r(X,t)\}$$

— i.e., for the body force per unit mass $b(X,t)$ and for the heat supply per unit mass and unit time $r(X,t)$. The operator Div is computed with respect to the material coordinates, the dot denotes the material differentiation with respect to the time t, ϱ is the mass density in the reference configuration, and F denotes the deformation gradient.

A thermodynamic process at X of a body $\mathscr{B}$ is described by a set of functions which are of three different types. The first are functions represented by Λ which describe the local configuration of a particle X, the second are those dependent variables given in Π which will be related to Λ by the constitutive assumption. The third type are functions represented by Γ, which should be determined by the fields involved into considerations independently of constitutive relations.

A collection of values which take the functions $\mathscr{P}_X$ for particular time $t \in (t_p, t_k)$ we shall call a thermo-radiation-mechanical state at the instant t of a material point X of a body $\mathscr{B}$.

A material for a body $\mathscr{B}$ will be defined by the set of constitutive equations in which Π and Λ will be involved. Thus, to describe a thermo-radiation-mechanical state at time t of a material point X of a body $\mathscr{B}$, we do not need all the values of functions $\mathscr{P}_X$.

We assume that the material structure of a body is compatible with specification for a rheological material with internal changes presented by PERZYNA [26, 27] (see review by PERZYNA and SAWCZUK [30]).

In the case of rheological material with internal changes, there are two sources of internal dissipation. The first is associated with viscous properties and the second is related to structural changes generated by plastic deformations. To describe both effects simultaneously, and to take into account changes of thermo-mechanical properties due to irradiation, we postulate that a thermo-radiation-mechanical state of particle X at time t is described by the value of the function:

(3.6)
$$g(X,t) = \{\Lambda(X,t), \alpha(X,t), \omega(X,t)\}$$

at $t \in (t_p, t_k)$, and by the initial-value problem

$$\dot{\alpha}(X,t) = A(g(X,t)), \quad \alpha(X,t_0) = \alpha_0(X),$$
$$\dot{\omega}(X,t) = \Omega(g(X,t)), \quad \omega(X,t_0) = \omega_0(X), \tag{3.7}$$

where t_0 is also from (t_p, t_k) and $t_0 < t$.

The material structure in a body $\mathscr{B}$ is defined by the functional relation

$$\Pi(X,t) = \mathscr{R}(g(X,t)), \tag{3.8}$$

where

$$\mathscr{R} = \{\Psi, N, T, Q\} \tag{3.9}$$

represents the constitutive functions for the free energy Ψ, entropy N, stress T and heat flux Q.

To be sure that we can have the collection of functions $\mathscr{P}_X$ for every $t \in (t_p, t_k)$ which describes a local thermodynamic process at a material point X, it is sufficient to assume that the initial-value problem (3.7) for the internal state variables $\alpha(X,t)$ and $\omega(X,t)$ has unique solutions for $t \in [t_0, t_k)$. This condition imposes certain restrictions on the functions A and Ω. Namely, the functions A and Ω should be Lipschitz continuous functions with respect to α and ω and continuous functions with respect to Λ.

Assumption that the constitutive function Ψ is piecewise continuously differentiable (jointly) with respect to $g(X,t)$ and the inequality $(3.3)_4$ lead to the following results:

$$\partial_{\nabla\vartheta}\Psi = 0, \quad \partial_{\phi}\Psi = 0,$$
$$T = 2\varrho\partial_C\Psi(g^*), \quad \eta = -\partial_\vartheta\Psi(g^*), \tag{3.10}$$
$$\partial_\alpha\Psi(g^*)\cdot A(g) + \partial_\omega\Psi(g^*)\cdot\Omega(g) + \frac{1}{\varrho\vartheta}q\cdot\nabla\vartheta \leqslant 0,$$
$$g^* = \{C, \vartheta; \alpha, \omega\}.$$

The internal dissipation of a material is determined by the function

$$\sigma(g(X,t)) = -\frac{1}{\vartheta}\{\partial_\alpha\Psi(g^*)\cdot A(g) + \partial_\omega\Psi(g^*)\cdot\Omega(g)\}. \tag{3.11}$$

According to two sources of internal dissipation in a rheological material with internal structural changes, the function $\sigma(g(X,t))$ consists of two general terms.

4. Elastic-viscoplastic material

The main object of this section is to show how to interpret internal state variables introduced to describe an elastic-viscoplastic irradiated material in an entire range of strain rate and temperature changes. As was shown in the Sec. 2, physical microscopic theories have suggested that two mechanisms — namely, the thermal activated process (Region II) and the damping of dislocation motion by phonon viscosity (Region IV) — are most important for proper explanation of the strain rate and temperature sensitivity of an irradiated metal.

Let us postulate

$$\alpha = \{\alpha^{(i)}\}, \quad A = \{A^{(i)}\}, \qquad i = 1, 2, \ldots, n, \tag{4.1}$$

where $\alpha^{(i)}$, $i = 1, 2, \ldots, n$, are scalars and $A^{(i)}$, $i = 1, 2, \ldots, n$, are acalar functions, and

$$\omega = \{\omega_1, \varkappa_1, \xi_1; \omega_2, \varkappa_2, \xi_2\}, \quad \Omega = \{\Omega_1, K_1, \Xi_1; \Omega_2, K_2, \Xi_2\}, \tag{4.2}$$

where ω_1, ω_2 are the second-order tensors and Ω_1, Ω_2 are the second order tensor functions, $\varkappa_1, \xi_1, \varkappa_2, \xi_2$ are scalars and K_1, Ξ_1, K_2, Ξ_2 are scalar functions.

Additionally, we assume the relation for the inelastic deformation tensor $P(X, t)$ as follows:

$$\dot{P} = \begin{cases} \dot{\omega}_1 & \text{in Region II,} \\ \lambda + \dot{\omega}_2 & \text{in Region IV,} \end{cases} \tag{4.3}$$

where λ is a constant which can be determined from the continuity for $\dot{P}$.

We assume that the internal parameters $\{\omega_1, \varkappa_1, \xi_1\}$ describe the thermal activation mechanism and $\{\omega_2, \varkappa_2, \xi_2\}$ the mechanism of the damping of dislocation motion by phonon viscosity. The tensors $\dot{\omega}_1$ and $\dot{\omega}_2$ are rates of inelastic deformation, the scalars $\varkappa_1$ and $\varkappa_2$ represent the work hardening parameters, and the scalars ξ_1 and ξ_2 can be interpreted as concentration of defects, respectively, for these two mechanisms(7).

A material considered before yielding has rheological properties which, as we assume, can be described by internal state variables $\alpha^{(i)}$. Thus, the quasi-static yield condition for an elastic-viscoplastic irradiated material can be defined as follows(8):

$$\mathscr{F}_1(g) = \frac{f_1(T, \vartheta, \phi, P, \alpha)}{\varkappa_1} - 1. \tag{4.4}$$

It is postulated that the following differential equation determines the internal state tensor ω_1 for an elastic-viscoplastic irradiated material

$$\dot{\omega}_1 = \gamma_1(\vartheta, \xi)\left\langle\left\langle \Phi\left(\frac{f_1}{\varkappa_1} - 1\right)\right\rangle\right\rangle M_1(g), \tag{4.5}$$

where $\gamma_1(\vartheta, \xi)$ is a viscosity coefficient of a material; the dimensionless function $\Phi(\mathscr{F}_1)$ may be chosen to represent results of tests on the dynamic behaviour of irradiated materials. The function $M_1(g)$ is a second-order tensor function. The symbol is defined as follows:

$$\langle\langle [\ ] \rangle\rangle = \begin{cases} 0 & \text{if} \quad f_1 \leqslant \varkappa_1 \quad \text{or} \quad f_2 > \varkappa_2, \\ [\ ] & \text{if} \quad f_1 > \varkappa_1 \quad \text{and} \quad f_2 < \varkappa_2. \end{cases} \tag{4.6}$$

We assume (for active irradiation process)

$$\dot{\varkappa}_1 = \mathrm{tr}[K_{11}(g^*)\dot{P}] + K_{12}(g^*)\phi, \quad \dot{\xi}_1 = \mathrm{tr}[\Xi_{11}(g^*)\dot{P}] + \Xi_{12}(g^*)\phi, \tag{4.7}$$

(7) This interpretation of internal state variables has been earlier presented by the author in Ref. [28].

(8) The main conclusion from the experimental investigations of DUDDERAR and DUFFY [7] for neutron irradiated copper is that neutron irradiation produces an effect which, for engineering purposes may be described as analogous to an isotropic strainhardening, though of very different origin as regards microstructure. This conclusion justifies the static yield condition in the form $f_1(T, \vartheta, P) = \varkappa_1$, cf. PERZYNA [29].

— i.e., that changes of the work-hardening parameter and the concentration of defects are due to inelastic deformation and to irradiation.

On the basis of the phonon viscosity damping mechanism in Region IV, we assume the differential equation for ω_2 as follows:

$$\dot{\omega}_2 = \gamma_2(\vartheta, \xi_2)\left\langle\left[\frac{f_2}{\varkappa_2}-1\right]\right\rangle M_2(g), \tag{4.8}$$

where $\gamma_2(\vartheta, \xi_2)$ is a viscosity coefficient, M_2 is the second-order, symmetric tensor function and the function

$$\mathscr{F}_2(g) = \frac{f_2}{\varkappa_2}-1 \tag{4.9}$$

defines the transition criterion from Region II to Region IV. The symbol $\langle[\ ]\rangle$ is defined as follows:

$$\langle[\ ]\rangle = \begin{cases} 0 & \text{if} \quad f_2 \leqslant \varkappa_2, \\ [\ ] & \text{if} \quad f_2 > \varkappa_2. \end{cases} \tag{4.10}$$

Similarly, we assume (for active irradiation process):

$$\begin{aligned} \dot{\varkappa}_2 &= \mathrm{tr}[K_{21}(g^*)(\dot{P}-\lambda)]+K_{22}(g^*)\phi, \\ \dot{\xi}_2 &= \mathrm{tr}[\Xi_{21}(g^*)(\dot{P}-\lambda)]+\Xi_{22}(g^*)\phi. \end{aligned} \tag{4.11}$$

Equations (4.5) and (4.8) give the following yield conditions, respectively, for Region II:

$$f_1 = \varkappa_1\left\{1+\Phi^{-1}\left[\frac{(\mathrm{tr}\,\dot{P}^2)^{1/2}}{\gamma_1(\vartheta, \xi_1)}(\mathrm{tr}\,M_1^2)^{-\frac{1}{2}}\right]\right\}, \tag{4.12}$$

and for Region IV:

$$f_2 = \varkappa_2\left\{1+\frac{[\mathrm{tr}(\dot{P}-\lambda)^2]^{1/2}}{\gamma_2(\vartheta, \xi_2)}(\mathrm{tr}\,M_2^2)^{-\frac{1}{2}}\right\} \tag{4.13}$$

The comparison of the phenomenological dynamical relations (4.12) and (4.13) with the physically justified relations for stress (2.5) and (2.9) gives simple interpretations for internal state variables $\varkappa_1$ and $\varkappa_2$.

The internal dissipation is determined by the relations

$$\begin{aligned} \sigma_{\mathrm{II}} = -\frac{1}{\vartheta}\Big\{\sum_{i=1}^{n}\partial_{\alpha^{(i)}}\Psi(g^*)A^{(i)}(g)+\mathrm{tr}\big[\big(\partial_{\omega_1}\Psi(g^*)+\partial_{\varkappa_1}\Psi(g^*)K_{11}(g) \\ +\partial_{\xi_1}\Psi(g^*)\Xi_{11}(g)\big)\dot{P}\big]+[\partial_{\varkappa_1}\Psi(g^*)K_{12}(g)+\partial_{\xi_1}\Psi(g^*)\Xi_{12}(g)]\phi\Big\}, \end{aligned} \tag{4.14}$$

$$\begin{aligned} \sigma_{\mathrm{IV}} = -\frac{1}{\vartheta}\Big\{\sum_{i=1}^{n}\partial_{\alpha^{(i)}}\Psi(g^*)A^{(i)}(g)+\mathrm{tr}\big[\big(\partial_{\omega_2}\Psi(g^*)+\partial_{\varkappa_2}\Psi(g^*)K_{21}(g) \\ +\partial_{\xi_2}\Psi(g^*)\Xi_{21}(g)\big)(\dot{P}-\lambda)\big]+[\partial_{\varkappa_2}\Psi(g^*)K_{21}(g)+\partial_{\xi_2}\Psi(g^*)\Xi_{22}(g)]\phi\Big\}, \end{aligned} \tag{4.15}$$

for Region II and Region IV, respectively.

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