Equations of flow of a one-component, three-phase mixture

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THE PAPER considers two models of a one-component mixture containing three phases: gas, liquid and solid, density of the gaseous phase being small in comparison with densities of the remaining phases. The corresponding flow equations are derived for these two models on the basis of the flow equations presented in [1] and concerning multi-phase and multicomponent mixtures. The first model assumes no relative motion of phases being in thermodynamic equilibrium. The second model admits the linear perturbation of equilibrium, assuming at the same time that the mixture consists solely of two types of spherical particles (liquid and solid) uniformly dispersed in the gaseous phase. Surface tension is disregarded in both models.

Rozpatrywane są dwa modele mieszaniny jednoskładnikowej, zawierającej trzy fazy: gazową, ciekłą i stałą, przy czym gęstość fazy gazowej jest mała w stosunku do gęstości faz pozostałych. Dla tych dwóch modeli — na podstawie uprzednio wyprowadzonych równań przepływu mieszanin wieloskładnikowych i wielofazowych [1], wyprowadzone są odpowiednie równania przepływu. W pierwszym modelu zakłada się, że fazy nie przemieszczają się względem siebie i pozostają w równowadze termodynamicznej. W drugim modelu dopuszcza się liniowe zaburzenie równowagi, zakładając jednocześnie o mieszaninie, że składa się ona jedynie z dwóch rodzajów kulek (ciekłych i stałych) rozproszonych równomiernie w spójnej fazie gazowej. W obu modelach pominięto wpływ napięcia powierzchniowego.

Рассмотрены две модели однокомпонентной смеси, содержащей три фазы: газовую, жидкую и твердую. Предполагается, что плотность газовой фазы мала по отношению к плотностям остальных фаз. Для предложенных моделей выводятся уравнения течения, основанные на выведенных ранее уравнениях течения многокомпонентных и многофазных смесей [1]. В рамках первой модели предполагается, что отсутствует взаимное перемещение фаз, которые остаются в термодинамическом равновесии. Во второй модели допустимы линейные возмущения равновесия, причем принимается, что смесь состоит лишь из шариков двух видов (жидких и твердых), равномерно распределенных по связной газовой фазе. В обоих моделях не учитывается влияние поверхностного натяжения.

Notations

p, T pressure and temperature,

ξ^n mass ratios of the n-th phase,

ϱ density ($1/\varrho$ — specific volume),

e, h, s specific energy, enthalpy and entropy,

$\mathbf{f}$ mass force,

$\mathbf{u}$ velocity,

$\mathbf{w}^n$ relative velocities (3.3), (3.6),

μ^n chemical potential of the n-th phase,

$r^n (n = 1, 2)$ radius of drop or spherical solid particle,

R gas constant,

c_p^0 specific heat of the gaseous phase at constant pressure,

c^1, c^2 specific heats of the liquid and solid phases,

C^n non-dimensional specific heats (3.17),

a^n, b^n, Q^n $(n = 1, 2)$ thermodynamical constants (2.6), (3.5),

$\eta^0, \varkappa^0$ coefficients of viscosity and heat conduction,

$\tau_v^0, \mathbf{q}^0$ viscous stress tensor and heat flux in the gaseous phase,

$X^n, \mathbf{Y}^n, Z^n$ mass, momentum, and energy fluxes from the gaseous phase to the unity of mass of the n-th dispersed phase ($n = 1, 2$),

$\mathcal{Z}^n$ reduced energy flux (3.10),

$\mathcal{A}^n, \mathcal{K}_i^n, \mathcal{L}^n, \mathcal{M}^n, \mathcal{N}^n$ Onsager constants in (3.10),

α^n coefficient of condensation,

β^n effective accommodation coefficient of molecular energy transport,

γ^n effective coefficient of molecular transport of momentum,

$\Delta p, \Delta T^n$ parameters of perturbed state (3.3),

t time variable,

∇ Hamiltonian operator;

superscripts 0, 1, 2, denote gaseous, liquid and solid phase, respectively; a bar denotes equilibrium values at the triple point state.

1. Introduction

ACCORDING to Gibbs phase rule, at the triple point states of a one-component mixture, the pressure p and the temperature T are constant. Only one triple point, where gaseous, liquid and solid phases coexist, will be the object of our interest. Four diagrams in Fig. 1 present characteristics of the equilibrium state with phases separated by plane surfaces
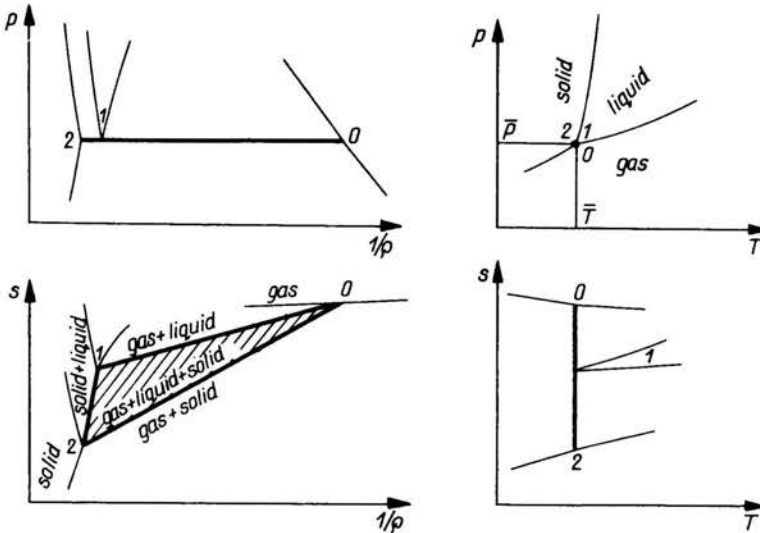

Fig. 1.

of contact. The triple point in the (p, T) space corresponds to the straight line sections in the (T, s) and $(p, 1/\varrho)$ spaces, and to the inside of the triangle 012 in the $(s, 1/\varrho)$ space.

The experimental data from [2, 3] for water, carbon dioxide and ammonia at the triple point state are given in the Table 1. It may be observed that in cases presented the

Table 1

		R	$\overline{T}$	$\overline{p}$	$\overline{\varrho}_0/\varrho^1$	$\overline{\varrho}^0/\varrho^2$
		$\dfrac{m^2}{s^{2.0}K}$	°K	$\dfrac{N}{m^2}$	—	—
H_2O	[2]	462	273.16	611.2	$4.85 \cdot 10^{-6}$	$5.28 \cdot 10^{-6}$
CO_2	[3]	188.6	216.55	518 000		$9.15 \cdot 10^{-3}$
NH_3	[3]	480	195.42	6 070	$8.8 \cdot 10^{-6}$	

density ratios are very small:

$$(1.1) \qquad \frac{\overline{\varrho}^0}{\overline{\varrho}^1} \ll 1, \qquad \frac{\overline{\varrho}^0}{\overline{\varrho}^2} \ll 1,$$

and disregarding specific volumes of condensed phases, we may simplify the $(s, 1/\varrho)$ diagram for the triple point triangle (Fig. 2).

In the three phase mixture, the phases are dispersed in the form of small particles, drops or bubbles and the influence of surface tension phenomena may change the results presented on Fig. 1 and Fig. 2 for plane surfaces of contact between phases. In the internal

FIG. 2.

energy of a mixture, an additional term proportional to the surface of separation appears, and also on both sides of a drop or bubble surface a difference of pressure arises which is proportional to the mean radius of curvature of the contact surface. Quantitatively, however, only in highly dispersed mixtures are the increment of energy and the difference of pressures important, but such mixtures are in general not permanent, due to the intense mass exchange. In the following considerations, the influence of surface tension will be disregarded.

While flowing, the three-phase mixture may not fulfil the equilibrium conditions. The pressure may change and the temperatures and the velocities of each phase, and even of each fraction of drops or bubbles may be different. The irreversible processes and the dissipation of energy occurring therein depend notably on the internal structure of the

mixture and are very difficult for theoretical description. Thus only one idealised model of a non-equilibrium mixture will be considered here. The liquid and solid spheres are taken to be dispersed in a coherent gaseous phase.

We shall deduce here the flow equations for two models of one-component three-phase mixture composed of incompressible solid and liquid phases and of perfect gas:

$$(1.2) \qquad \varrho^1 = \text{const}, \quad \varrho^2 = \text{const}, \quad p/\varrho^0 = RT^0.$$

The first model shall fulfil equilibrium conditions $p = \bar{p}$, $T^0 = T^1 = T^2 = \bar{T}$, $\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{u}^2 = \mathbf{u}$.

In the second model, the irreversible processes in linear approximation will be taken into account, but the mixture will be assumed to be composed of two sorts of spherical particles only. The equations will be deduced from the previously obtained equations of multiphase mixture [1] considered as a macroscopically homogeneous medium.

2. Flow in equilibrium

Let us consider the three-phase mixture in equilibrium conditions as a macroscopically homogeneous fluid. We shall assume also that the velocities of all phases are equal.

At the triple point state, the temperature and the pressure are constant:

$$(2.1) \qquad \bar{T} = \text{const}, \quad \bar{p} = \text{const},$$

and the density $\bar{\varrho}$, the specific entropy $\bar{s}$ and the mass ratios ξ^0, ξ^1, ξ^2, fulfil the following relations:

$$(2.2) \qquad \begin{aligned} \frac{1}{\bar{\varrho}} &= \frac{1}{\bar{\varrho}^0}\xi^0 + \frac{1}{\varrho^1}\xi^1 + \frac{1}{\varrho^2}\xi^2, \\ \bar{s} &= \bar{s}^0\xi^0 + \bar{s}^1\xi^1 + \bar{s}^2\xi^2, \\ 1 &= \xi^0 + \xi^1 + \xi^2. \end{aligned}$$

Diseregarding all dissipative phenomena, we may write for the mixture under consideration the isobaric momentum equation:

$$(2.3) \qquad \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \mathbf{f},$$

and "classical" continuity and isentropy equations:

$$(2.4) \qquad \begin{aligned} \frac{\partial}{\partial t}\left(\frac{1}{\bar{\varrho}}\right) + \mathbf{u} \cdot \nabla\left(\frac{1}{\bar{\varrho}}\right) - \frac{1}{\bar{\varrho}}\nabla \cdot \mathbf{u} &= 0, \\ \frac{\partial \bar{s}}{\partial t} + \mathbf{u} \cdot \nabla\bar{s} &= 0. \end{aligned}$$

Taking into account (2.2), we may replace $\bar{\varrho}$, $\bar{s}$, by ξ^1, ξ^2, reducing (2.4) to equivalent equations of a three-phase mixture:

$$(2.5) \qquad \frac{\partial \xi^n}{\partial t} + \mathbf{u} \cdot \nabla\xi^n + a^n(1 - b^1\xi^1 - b^2\xi^2)\nabla \cdot \mathbf{u} = 0 \qquad n = 1, 2,$$

where

(2.6)
$$b^n = 1 - \frac{\bar{\varrho}^0}{\bar{\varrho}^n} \approx 1, \qquad\qquad n, m = 1, 2,$$

$$a^n = \frac{\bar{s}^0 - \bar{s}^m}{b^n(\bar{s}^0 - \bar{s}^m) - b^m(\bar{s}^0 - \bar{s}^n)} \approx \frac{\bar{s}^0 - \bar{s}^m}{\bar{s}^n - \bar{s}^m}, \quad n \neq m.$$

The constants a^n and b^n fulfil the evident relation:

(2.7)
$$a^1 b^1 + a^2 b^2 = 1 \approx a^1 + a^2.$$

It should be emphasised that the validity of the idealised model of a three-phase mixture in equilibrium seems to be restricted to a very narrow class of problems. The isobaric flow with $\bar{p} = \text{const}$ seems to be difficult to realise and, more realistically, the dissipative phenomena should be taken into account. A simple model of three-phase mixture in non-equilibrium conditions will be presented below. Further comparison of solutions of equilibrium and non-equilibrium equations may give some idea of the region of validity of the idealised Eqs. (2.2) and (2.5).

3. Non-equilibrium flow

Let us consider now the three-phase mixture composed of a coherent volumetrically dominant gaseous phase

(3.1)
$$\frac{\varrho^0}{\xi^0}\left(\frac{\xi^1}{\varrho^1} + \frac{\xi^2}{\varrho^2}\right) \ll 1,$$

in which is dispersed a constant number of numerous small particles: spherical drops of radius r^1 and solid spheres of radius r^2. The spheres always belong to the same phase.

In non-equilibrium conditions, the temperatures T^0, T^1, T^2, and the velocities $\mathbf{u}^0$, $\mathbf{u}^1$, $\mathbf{u}^2$ of each phase and the pressure p may be different from their equilibrium values $\bar{T}$, $\mathbf{u}$, $\bar{p}$. Also, the phenomena of the viscosity and the heat conduction in the gaseous phase may be considered.

According to [1], for the non-equilibrium conditions considered we shall introduce two sorts of parameters: the reference parameters

(3.2)
$$\xi^1, \xi^2, \mathbf{u} = \mathbf{u}^0 + \xi^1(\mathbf{u}^1 - \mathbf{u}^0) + \xi^2(\mathbf{u}^2 - \mathbf{u}^0),$$

and the parameters characterising the perturbation of equilibrium:

(3.3)
$$\Delta T^0 = T^0 - \bar{T}, \quad \Delta T^1 = T^1 - \bar{T}, \quad \Delta T^2 = T^2 - \bar{T}, \quad \Delta p = p - \bar{p},$$
$$\mathbf{w}^1 = \mathbf{u}^1 - \mathbf{u}, \quad \mathbf{w}^2 = \mathbf{u}^2 - \mathbf{u},$$

enabling us to introduce the linearised thermodynamical forces:

(3.4)
$$\frac{T^0 - T^n}{\bar{T}} = \frac{\Delta T^0 - \Delta T^n}{\bar{T}}, \quad -T\Delta\frac{\mu^n}{T^n} = -\bar{h}^0\frac{\Delta T^0 - \Delta T^n}{\bar{T}} + R\bar{T}\left(\frac{\Delta p}{\bar{p}} - Q^n\frac{\Delta T^n}{\bar{T}}\right),$$
$$\mathbf{w}^0 - \mathbf{w}^n = -\frac{(1-\xi^m)\mathbf{w}^n + \xi^m\mathbf{w}^m}{1 - \xi^1 - \xi^2}, \quad n, m = 1, 2, \quad n \neq m,$$

where

$$(3.5) \qquad Q^n = \frac{\overline{T}}{\overline{p}}\frac{dp_{cc}^{n-0}(\overline{T})}{dT} \approx \frac{\overline{s}^0 - \overline{s}^n}{R} = \frac{\overline{h}^0 - \overline{h}^n}{R\overline{T}}, \qquad n = 1, 2,$$

is the logarithmic derivative of the condensation or sublimation curves on the (p, T) diagram (Fig. 1), according to the Clapeyron-Clausius equation. We shall use also the auxiliary quantities:

$$(3.6) \qquad \xi^0 = 1 - \xi^1 - \xi^2, \qquad \mathbf{w}^0 = \mathbf{u}^0 - \overline{\mathbf{u}} = -\frac{\xi^1 \mathbf{w}^1 + \xi^2 \mathbf{w}^2}{1 - \xi^1 - \xi^2}.$$

From the derived previously equations of transport of mass, momentum and energy of a multiphase mixture [1], we obtain in our case the following particular form:

for the mixture as a whole:

$$\frac{\partial}{\partial t}\left(\frac{1}{\varrho}\right) + \mathbf{u}\cdot\nabla\left(\frac{1}{\varrho}\right) - \frac{1}{\varrho}\nabla\cdot\mathbf{u} = 0,$$

$$(3.7) \qquad \frac{\partial\mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = \mathbf{f} - \frac{1}{\varrho}\nabla p + \frac{1}{\varrho}\nabla\cdot\boldsymbol{\tau}_v^0,$$

$$\left(\frac{\partial h}{\partial t} + \mathbf{u}\cdot\nabla h\right) - \frac{1}{\varrho}\left(\frac{\partial p}{\partial t} + \mathbf{u}\cdot\nabla p\right) = \frac{1}{\varrho}\nabla\cdot[-\varrho(\xi^0 e^0 \mathbf{w}^0 + \xi^1 e^1 \mathbf{w}^1 + \xi^2 e^2 \mathbf{w}^2)$$

$$- p\mathbf{w}^0 - \mathbf{q}^0 + \boldsymbol{\tau}_v^0\cdot\mathbf{w}^0] + \frac{1}{\varrho}\boldsymbol{\tau}_v^0 : \nabla\mathbf{u},$$

and for the dispersed phase for $n = 1, 2$:

$$\frac{\partial\xi^n}{\partial t} + \mathbf{u}^n\cdot\nabla\xi^n = -\xi^n\frac{\nabla\cdot(\varrho\mathbf{w}^n)}{\varrho} + \xi^n X^n,$$

$$(3.8) \qquad \frac{\partial\mathbf{u}^n}{\partial t} + (\mathbf{u}^n\cdot\nabla)\mathbf{u}^n = \mathbf{f} + \mathbf{Y}_-^n,$$

$$c^n\left(\frac{\partial T^n}{\partial t} + \mathbf{u}^n\cdot\nabla T^n\right) = Z^n - e^n X_{\mathbf{a}}^n.$$

The flux of mass X^n causes growth of spherical particles, described by the equation

$$(3.9) \qquad 3\frac{1}{r_{\mathbf{a}}^n}\left(\frac{\partial r^n}{\partial t} + \mathbf{u}^n\cdot\nabla r^n\right) = X^n, \qquad n = 1, 2.$$

The fluxes X^n, $\mathbf{Y}^n$, Z^n of mass, momentum and energy from the coherent gaseous phase to the unit of mass of each dispersed phase are, according to [1], the linear functions of thermodynamical forces (3.4) and may be presented in the form:

$$X^n = \mathcal{K}^n\frac{\Delta T^n - \Delta T^0}{\overline{T}} + \mathcal{L}_{\mathbf{i}}^n\left(\frac{\Delta p}{\overline{p}} - Q^n\frac{\Delta T^n}{\overline{T}}\right),$$

$$(3.10) \qquad \mathcal{Z}^n = \frac{Z^n - e^n X^n}{R\overline{T}} = -\mathcal{M}^n\frac{\Delta T^n - \Delta T^0}{\overline{T}} + \mathcal{N}^n\left(\frac{\Delta p}{\overline{p}} - Q^n\frac{\Delta T^n}{\overline{T}}\right),$$

$$\mathbf{Y}^n = -\mathcal{A}^n\frac{(1 - \xi^m)\mathbf{w}^n + \xi^m\mathbf{w}^m}{1 - \xi^1 - \xi^2},$$

where the constants $\mathscr{A}^n$, $\mathscr{K}^n$, $\mathscr{L}^n$, $\mathscr{M}^n$, $\mathscr{N}^n$ ($n = 1, 2$) are determined by the irreversible interaction phenomena between the coherent phase and singular particles[1]. These constants should fulfil the relation

(3.11)
$$Q^n\mathscr{L}^n - \mathscr{N}^n = \mathscr{K}^n,$$

resulting from the Onsager symmetry principle applied to the model of multiphase mixture under consideration [1].

The infuence of the viscosity and heat conduction phenomena will be here considered in the first linear approximation only:

(3.12)
$$\nabla \cdot \boldsymbol{\tau}_v^0 = \eta^0(\nabla \cdot \nabla)\mathbf{u}^0 + \frac{\eta^0}{3}\nabla(\nabla \cdot \mathbf{u}^0)$$
$$-\nabla \mathbf{q}^0 = \varkappa^0(\nabla \cdot \nabla)\varDelta T^0,$$

disregarding $\tau_v^0 : \nabla\mathbf{u}$ and $\nabla \cdot (\boldsymbol{\tau}_v^0 \cdot \mathbf{w}^0)$ as higher order terms.

Taking into account the assumptions introduced above we may transform the equations previously derived and linearise them with respect to the non-equilibrium parameters $\varDelta T^0$, $\varDelta T^1$, $\varDelta T^2$, $\varDelta p$, $\mathbf{w}^1$, $\mathbf{w}^2$.

The linearised specific volume $1/\varrho$ and enthalpy h are

(3.13)
$$\frac{1}{\varrho} = \frac{1}{\bar{\varrho}}\left(1 + \frac{\varDelta T^0}{\bar{T}} - \frac{\varDelta p}{\bar{p}}\right), \quad \frac{\bar{\varrho}^0}{\bar{\varrho}} = 1 - \xi^1 - \xi^2 = \xi^0,$$
$$h = \bar{h} + c_p^0\xi^0\varDelta T^0 + c^1\xi^1\varDelta T^1 + c^2\xi^2\varDelta T^2,$$
$$\bar{h} = \bar{h}^0 - R\bar{T}(Q^1\xi^1 + Q^2\xi^2).$$

The transformed and linearised equations may now be presented in the following form:

the momentum equation (3.7):

(3.14)
$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \mathbf{f} + \xi^0\left\{-R\bar{T}\nabla\frac{\varDelta p}{\bar{p}} + \frac{\eta^0}{\bar{\varrho}^0}\nabla^2(\mathbf{u} + \mathbf{w}^0) + \frac{1}{3}\frac{\eta^0}{\bar{\varrho}^0}\nabla[\nabla \cdot (\mathbf{u} + \mathbf{w}^0)]\right\},$$

the continuity and energy equations (3.7):

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right)\left[\xi^0\left(1 + \frac{\varDelta T^0}{\bar{T}} - \frac{\varDelta p}{\bar{p}}\right)\right] - \xi^0\left(1 + \frac{\varDelta T^0}{\bar{T}} - \frac{\varDelta p}{\bar{p}}\right)\nabla \cdot \mathbf{u} = 0,$$

(3.15)
$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right)\left[-Q^1\xi^1 - Q^2\xi^2 + C^0\xi^0\frac{\varDelta T^0}{\bar{T}} + C^1\xi^1\frac{\varDelta T^1}{\bar{T}} + C^2\xi^2\frac{\varDelta T^2}{\bar{T}}\right]$$
$$-\xi^0\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right)\frac{\varDelta p}{\bar{p}} = \xi^0\left\{\nabla\left[\frac{1}{\xi^0}(Q^1\xi^1\mathbf{w}^1 + Q^2\xi^2\mathbf{w}^2)\right] + \frac{\varkappa^0}{R\bar{\varrho}^0}\nabla^2\left(\frac{\varDelta T^0}{\bar{T}}\right)\right\},$$

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[1] For example, in the case in which the diameter of particles is very small in comparison with the mean free path of gas molecules, we may apply the Knudsen model of interaction (cf. [4, 5]), obtaining

$$2\mathscr{K}^n = \mathscr{L}^n = \mathscr{N}^n\bigg/\left(Q^n - \frac{1}{2}\right) = 3\frac{\bar{\varrho}^0}{\varrho^n}\sqrt{\frac{R\bar{T}}{2\pi}}\frac{\alpha^n}{r^n},$$

$$\mathscr{M}_*^n = \left[\left(\frac{c_p^0}{R} - \frac{1}{2}\right)\beta^n - \left(Q^n - \frac{1}{2}\right)\frac{\alpha^n}{2}\right]3\frac{\bar{\varrho}^0}{\varrho^n}\sqrt{\frac{R\bar{T}}{2\pi}}\frac{1}{r^n}, \quad \mathscr{A}^n = 3\frac{\bar{\varrho}^0}{\varrho^n}\sqrt{\frac{R\bar{T}}{2\pi}}\frac{\gamma^n}{r^n},$$

where $0 \leqslant \alpha^n \leqslant 1$, $0 \leqslant \beta^n \leqslant 1$, $1,19 \leqslant \gamma^n \leqslant 2$ are constants characterising the molecular transport of mass, energy and momentum between phases.

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and the linearised Eqs. (3.8) for $n = 1, 2$:

(3.16)
$$\frac{\partial \xi^n}{\partial t} + \mathbf{u} \cdot \nabla \xi^n = -\xi^0 \nabla \cdot \frac{\xi^n \mathbf{w}^n}{\xi^0} + \xi^n X^n,$$

$$C^n \left(\frac{\partial}{\partial t} \frac{\Delta T^n}{\bar{T}} + \mathbf{u} \cdot \nabla \frac{\Delta T^n}{\bar{T}} \right) = \mathscr{Q}^n,$$

$$\frac{\partial \mathbf{w}^n}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{w}^n + (\mathbf{w}^n \cdot \nabla) \mathbf{u} = \mathbf{Y}^n + \xi^0 \left\{ R\bar{T} \nabla \frac{\Delta p}{\bar{p}} \right.$$
$$\left. - \frac{\eta^0}{\varrho^0} \nabla^2 (\mathbf{u} + \mathbf{w}^0) - \frac{1}{3} \frac{\eta^0}{\varrho^0} \nabla [\nabla \cdot (\mathbf{u} + \mathbf{w}^0)] \right\}.$$

We have introduced here the non-dimensional constants

(3.17)
$$C^0 = \frac{c_p^{0j}}{R}, \quad C^1 = \frac{c^1}{R}, \quad C^2 = \frac{c^2}{R}.$$

By means of (3.16), we shall now transform the Eqs. (3.15) to the form:

(3.18)
$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \left(\frac{\Delta T^0}{\bar{T}} - \frac{\Delta p}{\bar{p}} \right) = (\nabla \cdot \mathbf{u}) \left(\frac{\Delta T^0}{\bar{T}} - \frac{\Delta p}{\bar{p}} \right) + \nabla (\mathbf{u} + \mathbf{w}^0) + \frac{\xi^1 X^1 + \xi^2 X^2}{\xi^0},$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \left(C^0 \frac{\Delta T^0}{\bar{T}} - \frac{\Delta p}{\bar{p}} \right) = \frac{1}{\xi^0} [\xi^1 (Q^1 X^1 - \mathscr{Q}^1) + \xi^2 (Q^2 X^2 - \mathscr{Q}^2)]$$
$$+ \frac{\varkappa^0}{R\varrho^0} \nabla^2 \frac{\Delta T^0}{\bar{T}}.$$

The values of Q^n, C^n, calculated from [2], [3] are given in Table 2.

Table 2

		Q^1	$Q^2 - Q^1$	C^0	C^1	C^2
H_2O	[2]	19.8	2.93	4.18	9.15	4.6
CO_2	[3]	8.56	4.88	5.15	11.1	
NH_3	[3]	15.8	3.53	10	6.5	

The set of 12 scalar and 5 vector Eqs. (3.14), (3.18), (3.16), (3.10), (3.9) for the unknown functions $\mathbf{u}$, ξ^1, ξ^2, $\Delta T^0/\bar{T}$, $T^1/\bar{T}$, $\Delta T^2/\bar{T}$, $\Delta p/\bar{p}$, $\mathbf{w}^1$, $\mathbf{w}^2$, X^1, X^2, $\mathscr{Q}^1$, $\mathscr{Q}^2$, $\mathbf{Y}^1$, $\mathbf{Y}^2$, r^1, r^2 are the flow equations sought for of the model under consideration of one-component, three-phase mixture in non-equilibrium conditions. Comparing them with the equations resulting from the equilibrium model (2.3), (2.5), we see that their number is larger and that they are more complicated. It should be emphasised, however, that the considered model of the case of non-equilibrium is highly simplified also — for instance, by the assumption that only two fractions of spherical particles exist in the mixture. These particles should always belong to the same phase and their number should not change during the flow.

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