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Quasi-static problem of a crack in an elastic strip subject to antiplane state of strain(*)

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THE PAPER considers the quasi-static problem of an infinite elastic medium weakened by an infinite number of semi-infinite, rectilinear, parallel and uniformly spaced cracks. Edges of these cracks are acted on by identic forces harmonically variable in time and satisfying the conditions of an antiplane state of strain. The stress intensity factor at the crack tip is determined. The solution is applied to the analysis of the problem of an infinite elastic strip containing a semi-infinite crack.

W pracy rozważono quasistatyczne zagadnienie nieograniczonego ośrodka sprężystego osłabionego nieskończoną liczbą półnieskończonych, prostoliniowych, równoległych i jednakowo od siebie odległych szczelin. Brzegi tych szczelin poddane są działaniu jednakowych sił zmieniających się harmonicznie w czasie i spełniających warunki antypłaskiego stanu naprężenia. Wyznaczono współczynnik intensywności naprężenia w końcu szczeliny. Rozwiązania zastosowano do analizy problemu pasma nieskończonego osłabionego półnieskończoną szczeliną.

В работе рассмотрена квазистатическая задача о бесконечном упругом теле, ослабленном бесконечным числом полубесконечных прямолинейных параллельных трещин, расположенных на равных расстояниях друг от друга. Края трещин подвержены воздействиям одинаковых усилий, изменяющихся гармонически во времени и удовлетворяющих условиям антиплоского деформированного состояния. Для случая нагрузки с произвольной амплитудой колебаний определен коэффициент интенсивности напряжений.

1. General formulation

IT IS KNOWN that in the antiplane state of strain the only non-vanishing component of the elastic displacement vector is, in a rectangular coordinate system (x, y, z), the displacements w parallel to the z-axis, $w = w(x, y, t)$; the non-vanishing stress components are $\sigma_{xz} = \sigma_{xz}(x, y, t)$ and $\sigma_{yz} = \sigma_{yz}(x, y, t)$. These stresses are expressed in terms of w as

$$\sigma_{xz} = \mu \frac{\partial w}{\partial x}, \quad \sigma_{yz} = \mu \frac{\partial w}{\partial y}, \tag{1.1}$$

and the equations of motion reduce, under the assumption of zero body forces, to the single equation

$$\nabla^2 w = \frac{1}{c_2^2} \frac{\partial^2 w}{\partial t^2}. \tag{1.2}$$

Here $c_2 = \sqrt{\mu/\varrho}$ denotes the velocity of propagation of transversal elastic waves.

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In this paper we shall use the complex integral Fourier transform defined by the relations

$$
(1.3)\qquad
\begin{aligned}
F(\alpha, y) &= \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} f(x, y) e^{i\alpha x} dx,\\
f(x, y) &= \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty+ic}^{\infty+ic} F(\alpha, y) e^{-i\alpha x} d\alpha.
\end{aligned}
$$

Here α is the complex transform parameter and the path of integration in Eq. $(1.3)_2$ lies within the strip $\alpha_1 < \mathrm{Im}\,\alpha < \alpha_2$ which represents the region of regularity of $F(\alpha, y)$. α_1 and α_2 denote certain real constants.

From the theory of Fourier integral transforms [1] it is known that $F(\alpha, y)$ may also be represented in the form

$$
(1.4)\qquad F(\alpha, y) = F^-(\alpha, y) + F^+(\alpha, y)
$$

the functions

$$
(1.5)\qquad
\begin{aligned}
F^-(\alpha, y) &= \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{0} f(x, y) e^{i\alpha x} dx,\\
F^+(\alpha, y) &= \frac{1}{\sqrt{2\pi}} \int\limits_{0}^{\infty} f(x, y) e^{i\alpha x} dx,
\end{aligned}
$$

being analytic in the respective lower ($\mathrm{Im}\,\alpha < \alpha_2$) and upper ($\mathrm{Im}\,\alpha > \alpha_1$) halfplanes of the complex variable α. In the quasi-static case when the displacement and stresses vary harmonically in time, all the magnitudes under consideration may be expressed in the form

$$
(1.6)\qquad g(x, y, t) = g^*(x, y) e^{i\omega t},
$$

where ω is the harmonic vibration frequency.

Performing the transformation (1.6) and the integral transform (1.3) in Eqs. (1.1) and (1.2) we obtain

$$
(1.7)\qquad
\begin{aligned}
&\Sigma^*_{xz}(\alpha, y) = -i\alpha\mu W^*(\alpha, y), \qquad \Sigma^*_{yz}(\alpha, y) = \mu \frac{dW^*(\alpha, y)}{dy},\\
&\frac{d^2 W^*(\alpha, y)}{dy^2} - (\alpha^2 - \sigma^2) W^*(\alpha, y) = 0,
\end{aligned}
$$

where $\sigma = \omega/c_2$. Solution of Eq. $(1.7)_3$ yields then the transforms of functions w, σ_{xz}, σ_{yz}:

$$
(1.8)\qquad
\begin{aligned}
W^*(\alpha, y) &= A(\alpha) \mathrm{sh}\, y\sqrt{\alpha^2-\sigma^2} + B(\alpha) \mathrm{ch}\, y\sqrt{\alpha^2-\sigma^2},\\
\Sigma^*_{xz}(\alpha, y) &= -i\mu\alpha \left[A(\alpha) \mathrm{sh}\, y\sqrt{\alpha^2-\sigma^2} + B(\alpha) \mathrm{ch}\, y\sqrt{\alpha^2-\sigma^2}\right],\\
\Sigma^*_{yz}(\alpha, y) &= \mu\sqrt{\alpha^2-\sigma^2}\left[A(\alpha) \mathrm{ch}\, y\sqrt{\alpha^2-\sigma^2} + B(\alpha) \mathrm{sh}\, y\sqrt{\alpha^2-\sigma^2}\right].
\end{aligned}
$$

The unknown functions $A(\alpha)$ and $B(\alpha)$ are to be determined from the boundary conditions of the problem considered.

2. Infinite medium with cracks

Let us consider an infinite elastic medium weakened by an infinite number of semi-infinite, rectilinear, parallel and equally spaced cracks (Fig. 1). The crack edges are assumed to be loaded by identic forces harmonically varying in time and satisfying the conditions of antiplane state of strain.

Owing to the symmetry properties the problem may be reduced to that of an infinite elastic strip of thickness $2h$ weakened in its middle plane $y = 0$ by a semi-infinite crack

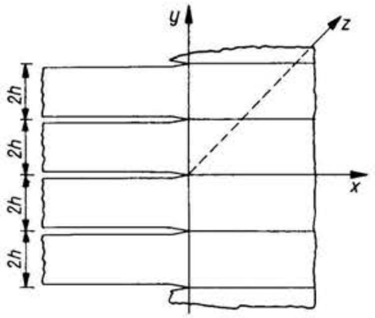

FIG. 1.

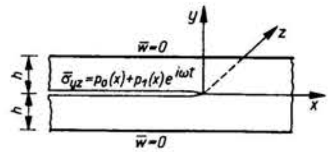

FIG. 2.

$x < 0$ (Fig. 2). The strip is rigidly clamped at the sutfaces $y = \pm h$, and the crack edges are acted on by forces $\bar{\sigma}_{yz} = p_0(x)+p_1(x)\exp i\omega t$, an additional assumption being made that the harmonic vibration frequency $\omega < \pi c_2/2h$.

Making use of the superposition principle which enables the static and quasi-static problems to be considered separately and applying the symmetry properties, the problem is reduced to that of an infinite elastic strip of thickness h with the following boundary conditions

$$\begin{aligned} w(x, y) &= 0 \quad &&\text{for} \quad |x| < \infty, \ y = h, \\ w(x, y) &= 0 \quad &&\text{for} \quad x > 0, \quad y = 0, \\ \sigma_{yz}(x, y) &= p(x)\exp(i\omega t) \quad &&\text{for} \quad x < 0, \quad y = 0. \end{aligned}$$

Applying now the transform (1.6) and the integral Fourier transform (1.3) and using, in accordance with Eqs. (1.4), (1.5), the notations

$$\begin{aligned} W^*(\alpha, 0) &= \Psi^-(\alpha)+\Psi^+(\alpha), \\ \Sigma^*_{yz}(\alpha, 0) &= \Phi^-(\alpha)+\Phi^+(\alpha), \\ P(\alpha) &= \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{0} p(x)e^{i\alpha x}dx, \end{aligned} \tag{2.1}$$

the solution of the problem considered [Eqs. (1.8) being used] reduces to the solution of the integral Wiener-Hopf equation

$$\Psi^-(\alpha) = -\frac{1}{\mu} \frac{\operatorname{th} h\sqrt{\alpha^2-\sigma^2}}{\sqrt{\alpha^2-\sigma^2}} [\Phi^+(\alpha)+P(\alpha)]. \tag{2.2}$$

The region of existence of this equation is the region of regularity of functions appearing in Eq. (2.2). Owing to our former assumption concerning the vibration frequency ω, i.e. for $\sigma < \pi/2h$, this region is represented by a strip

$$-\sqrt{\pi^2/4h^2-\sigma^2} < -\varepsilon < \operatorname{Im}\alpha < 0.$$

Equation (2.2) will be solved by means of the factorization method [2]. First of all, the function

$$H(\alpha) = \frac{\operatorname{th} h\sqrt{\alpha^2-\sigma^2}}{\sqrt{\alpha^2-\sigma^2}} \tag{2.3}$$

has to be factorized. Using the procedure described in [3] let us represent the function (2.3) in the form

$$H(\alpha) = \overline{H}(\alpha) H_1(\alpha), \tag{2.4}$$

The function $\overline{H}(\alpha)$ is required to behave at infinity ($|\alpha| \to \infty$) and at zero ($|\alpha| \to 0$) in the same manner as $H(\alpha)$; the auxiliary function $H_1(\alpha)$ should be non-zero and possess no singularities within the strip $|\operatorname{Im}\alpha| < \varepsilon_1$, where $0 < \varepsilon \leqslant \varepsilon_1 < \sqrt{\pi^2/4h^2-\sigma^2}$.

The function $H(\alpha)$ defined by Eq. (2.3) has no zeros and singularities in the strip $|\operatorname{Im}\alpha| < \varepsilon$, therefore — according to the assumptions concerning $H(\alpha)$ — we may assume

$$H(\alpha) = R^-(\alpha) R^+(\alpha), \tag{2.5}$$

Here

$$R^{\pm}(\alpha) = \frac{1}{\sqrt{\alpha \pm iA}}, \quad A = \frac{\sigma}{\operatorname{tg} h\sigma}. \tag{2.6}$$

It consequently follows that the assumptions concerning $H_1(\alpha)$ are satisfied and in view of the fact that $H_1(\alpha) \to 1$ in the strip $|\operatorname{Im}\alpha| < \varepsilon_1$ for $|\alpha| \to \infty$, the function may be represented in the form [2]

$$H_1(\alpha) = \frac{H_1^+(\alpha)}{H_1^-(\alpha)}, \tag{2.7}$$

Here

$$\begin{aligned} \ln H_1^+(\alpha) &= \frac{1}{2\pi i} \int_{-\infty+i\gamma_2}^{\infty+i\gamma_2} \frac{\ln H_1(\xi)}{\zeta-\alpha} d\zeta, \\ \ln H_1^-(\alpha) &= \frac{1}{2\pi i} \int_{-\infty+i\gamma_1}^{\infty+i\gamma_1} \frac{\ln H_1(\zeta)}{\zeta-\alpha} d\zeta. \end{aligned} \tag{2.8}$$

The parameters appearing here fulfil the inequality $-\varepsilon_1 < \gamma_2 < \gamma_1 < \varepsilon_1$.

The functions $H_1^{\pm}(\alpha)$ defined in this manner possess no zeros and singular points within the respective halfplanes $\operatorname{Im}\alpha > \gamma_2$ and $\operatorname{Im}\alpha < \gamma_1$. It also follows from Eqs. (2.8) and from the fact that $H_1(0) = H_1(\infty) = 1$ that these functions satisfy the additional condition $H_1^{\pm}(0) = H_1^{\pm}(\pm\infty) = 1$.

Application of the Eqs. (2.4), (2.5), (2.7) enables us to rewrite the Eq. (2.2) in the form

$$\Psi^-(\alpha) = -\frac{1}{\mu} R^-(\alpha) R^+(\alpha) \frac{H_1^+(\alpha)}{H_1^-(\alpha)} [\Phi^+(\alpha) + P(\alpha)].$$

The procedure used in [4] yields a new form of that equation

$$-\frac{\mu H_1^-(\alpha)\Psi^-(\alpha)}{R^-(\alpha)} = R^+(\alpha)H_1^+(\alpha)\Phi^+(\alpha)+E(\alpha), \tag{2.9}$$

where

$$E(\alpha) = R^+(\alpha)H_1^+(\alpha)P(\alpha). \tag{2.10}$$

Assuming the function $E(\alpha)$ to be regular at least in the region of existence of Eq. (2.2), it may be represented in the form [2]

$$E(\alpha) = E^+(\alpha)-E^-(\alpha), \tag{2.11}$$

where

$$\begin{aligned} E^+(\alpha) &= \frac{1}{2\pi i}\int\limits_{-\infty-i\delta_2}^{\infty-i\delta_2}\frac{E(\zeta)}{\zeta-\alpha}\,d\zeta, \\ E^-(\alpha) &= \frac{1}{2\pi i}\int\limits_{-\infty-i\delta_1}^{\infty-i\delta_1}\frac{E(\zeta)}{\zeta-\alpha}\,d\zeta, \end{aligned} \tag{2.12}$$

Here $0 < \delta_1 < \delta_2 < \varepsilon$ and the functions $E^\pm(\alpha)$ are regular in the respective halfplanes $\operatorname{Im}\alpha > -\varepsilon$ and $\operatorname{Im}\alpha < 0$. In view of the relation (2.11), Eq. (2.9) may now be rewritten as

$$-\frac{\mu H_1^-(\alpha)\Psi^-(\alpha)}{R^-(\alpha)} + E^-(\alpha) = R^+(\alpha)H_1^+(\alpha)\Phi^+(\alpha)+E^+(\alpha).$$

Both sides of this equation represent functions which are regular in the respective halfplanes $\operatorname{Im}\alpha < 0$ and $\operatorname{Im}\alpha > -\varepsilon$, and hence by applying the generalized Liouville theorem its solution is written in the form

$$\begin{aligned} \Psi^-(\alpha) &= \frac{1}{\mu}\frac{R^-(\alpha)E^-(\alpha)}{H_1^-(\alpha)} \quad \text{reg. for} \quad \operatorname{Im}\alpha < 0, \\ \Phi^+(\alpha) &= -\frac{E^+(\alpha)}{R^+(\alpha)H_1^+(\alpha)} \quad \text{reg. for} \quad \operatorname{Im}\alpha > -\varepsilon. \end{aligned} \tag{2.13}$$

A very important (from the point of view of crack stability) result of the analysis will be the determination of the stress intensity factor [5]. This factor as also the crack boundary displacement in the vicinity of the crack tip is determined by using the Abel theorem concerning Fourier transforms [6]; according to that theorem, the asymptotic behaviour of expressions (2.13) for $|\alpha| \to 0$ and $|\alpha| \to \infty$ determines the behaviour of inverse transforms of these functions at $|x| \to \infty$ and $|x| \to 0$, respectively.

Meanwhile let us moreover observe that by using the relation (2.10) and in view of the fact that $E^\pm(\alpha)$ defined by Eqs. (2.12) are assumed to be regular within the strip $-\varepsilon < \operatorname{Im}\alpha < 0$, these functions may be represented in the form

$$E^\pm(\alpha) = -\frac{1}{\alpha}\left[B-\frac{1}{2\pi i}\int\limits_{-\infty-i\delta}^{\infty-i\delta}\frac{\zeta E(\zeta)}{\zeta-\alpha}\,d\zeta\right], \tag{2.14}$$

where

$$B = \frac{1}{2\pi i} \int\limits_{-\infty - i\delta}^{\infty - i\delta} E(\zeta) d\zeta, \quad \delta_1 < \delta < \delta_2. \tag{2.15}$$

Using then the relations (2.6) and (2.14), and taking into account the properties of $E^{\pm}(\alpha)$ and $H_{\zeta}^{\pm}(\alpha)$ it may easily be demonstrated that for $|\alpha| \to \infty$ the functions $\Psi^{-}(\alpha)$ and $\Phi^{+}(\alpha)$ given by (2.13) are expressed by the formulae

$$\Psi^{-}(\alpha) = -\frac{B}{\mu} \frac{1}{\alpha\sqrt{\alpha}}, \quad \Phi^{+}(\alpha) = \frac{B}{\sqrt{\alpha}}.$$

The Abel theorem quoted before yields, by means of the transform (1.6), the displacement w of the upper edge of the crack and the stresses σ_{yz} along the positive x-axis for small values of $|x| \to 0$,

$$\begin{aligned} w(x) &= \frac{2N^*(p, \omega)e^{i\omega t}}{\mu} \sqrt{-x} \quad \text{for} \quad x \to (-0), \\ \sigma_{yz}(x) &= \frac{N^*(p, \omega)e^{i\omega t}}{\sqrt{x}} \quad \text{for} \quad x \to (+0), \end{aligned} \tag{2.16}$$

where

$$N^*(p, \omega) = -\sqrt{-2i}\, B. \tag{2.17}$$

This equation may be used to determine the exact value of the stress intensity factor for arbitrary load varying harmonically in time and applied to the edges of the crack; it is assumed that $\omega < \pi c_2/2h$. From the considerations thus presented it follows that the displacement of the upper edge of the crack $\overline{w}$ and the stress $\overline{\sigma}_{yz}$ along the positive x-axis for $|x| \to 0$ in the problem illustrated by Fig. 2 are given by Eqs. (2.16), the stress intensity factor being equal to

$$\overline{N}(\omega) = N^*(p_0, 0) + N^*(p_1, \omega)\cos\omega t. \tag{2.18}$$

The function $N^*(p, \omega)$ is given by Eq. (2.17).

3. Particular cases

3.1. Constant amplitude of vibrations.

To illustrate the results of preceding sections let us consider the case when in the problem shown in Fig. 2 the load acting on the edges of the crack is equal to $\overline{\sigma}_{yz} = p_0 + p_1 \cos\omega t$, with $p_0, p_1 = \text{const}$. In view of $(2.1)_3$ we have

$$P(\alpha) = \frac{p_1}{i\sqrt{2\pi}} \frac{1}{\alpha}. \tag{3.1}$$

All the assumptions concerning the parameters introduced previously are satisfied, and therefore in the quasi-static problem we obtain, according to Eq. (2.17),

$$N^*(p_1, \omega) = -\frac{p_1\sqrt{i}}{\sqrt{\pi}} \frac{1}{2\pi i} \int\limits_{-\infty - i\delta}^{\infty - i\delta} \frac{H_1^+(\zeta)}{\zeta\sqrt{\zeta + iA}} d\zeta. \tag{3.2}$$

After integration and using the relations (2.6), owing to the fact that $H^+(0) = 1$, Eq. (3.2) takes now the form

$$N^*(p_1, \omega) = -p_1 \sqrt{\frac{\operatorname{tg} h\sigma}{\pi\sigma}}. \tag{3.3}$$

With $\omega = 0$ this formula reduces to the stress intensity factor in a static case [7], and with $p_1 = p_0$ the factor is equal to

$$N^*(p, 0) = -p_0 \sqrt{\frac{h}{\pi}}. \tag{3.4}$$

Utilizing then the relations (2.18), (3.3), (3.4) we obtain the final form of the stress intensity factor $\overline{N}(\omega)$ in the case of a constant vibration amplitude,

$$\overline{N}(\omega) = -p_0 \sqrt{\frac{h}{\pi}} \left[1 + \frac{p_1}{p_0} \sqrt{\frac{\operatorname{tg} \sigma'}{\sigma'}} \cos \omega t \right]. \tag{3.5}$$

Here $\sigma' = h\sigma = h\omega/c_2$.

Passing in the formula (3.5) to the limit with $\omega \to 0$ we obtain the value of the stress intensity factor in the static case in which the edges of the crack are loaded by $p_0 + p_1$.

In the case when the vibration frequency $\omega \to \pi c_2/2h$, a resonance-type phenomenon occurs in the problem: arbitrarily small values of the load component p_1 lead to infinite stress intensity factors $\overline{N}(\omega) \to \infty$.

From the relation (3.5) it also follows that decreasing thickness of the strip decreases —

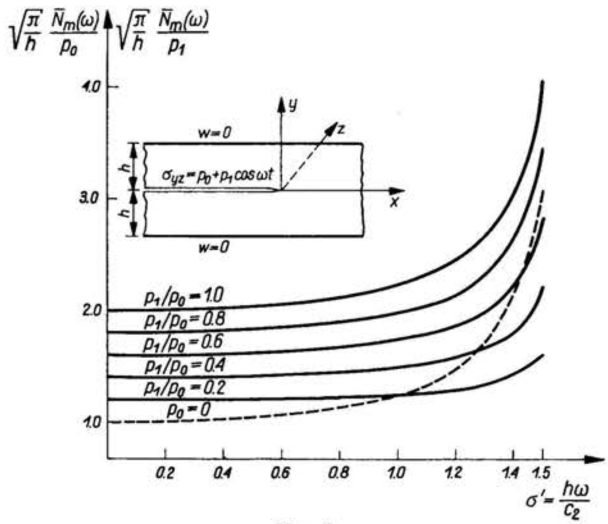

FIG. 3.

at a constant vibration frequency ω — the stress intensity factor. The maximum value of $\overline{N}_m(\omega)$ calculated at a constant vibration frequency ω is given by the formula

$$\overline{N}_m(\omega) = -p_0 \sqrt{\frac{h}{\pi}} \left[1 + \frac{p_1}{p_0} \sqrt{\frac{\operatorname{tg} \sigma'}{\sigma'}} \right].$$

Variation of this function is shown in Fig. 3.

3.2. Crack with stress-free edges

The solution derived above (Eq. (3.5)) may be used to determine the stress intensity factor in the problem of crack with stress-free edges, the elastic strip having prescribed values of displacements at its boundary surfaces: $w(x, \pm h) = \mp(w_0 + w_1 \cos\omega t)$, with $\omega < \pi c_2/2h$ and $w_0, w_1 = \text{const}$ (Fig. 4). Proceeding as in the first part of this paper we shall consider the static and quasi-static cases separately. By means of the superposition method the solution of the quasi-static case (Fig. 5a) may again be represented as a sum

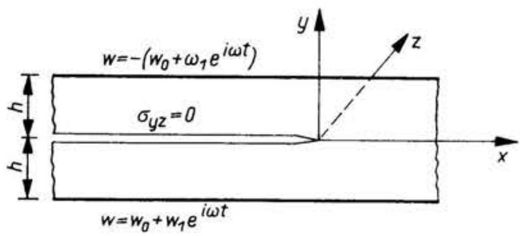

FIG. 4.

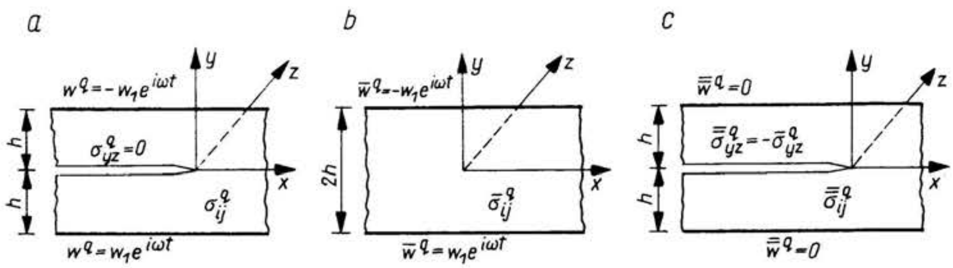

FIG. 5.

of solutions of the continuous strip with prescribed boundary surface displacement $\bar{w}^q = \mp w_1 \exp(i\omega t)$, Fig. 5b, and of the rigidly clamped strip with a crack loaded at its edges by $\bar{\bar{\sigma}}^q_{yz}(x, 0) = -\bar{\sigma}^q_{yz}(x, 0)$ (Fig. 5c). Displacements $\bar{w}^q$ and stresses $\bar{\sigma}^q_{xz}$, $\bar{\sigma}^q_{yz}$ in the problem illustrated by Fig. 5b are given by the formulae

$$\bar{w}^q(x, y) = -w_1 \frac{\sin y\sigma}{\sin h\sigma},$$

$$\bar{\sigma}^q_{xz}(x, y) = 0, \quad \bar{\sigma}^q_{yz}(x, y) = -\mu w_1 \frac{\sigma \cos y\sigma}{\sin h\sigma},$$

whence

$$\bar{\bar{\sigma}}^q_{yz}(x, 0) = -\bar{\sigma}^q_{yz}(x, 0) = p_1 = \frac{\mu w_1 \sigma}{\sin h\sigma}.$$

Using the formula (3.3) we obtain the stress intensity factor in the problem shown in Fig. 5a,

$$N^q = -\frac{\mu w_1}{\sqrt{\pi h}} \sqrt{\frac{2\sigma'}{\sin 2\sigma}}\, e^{i\omega t}. \tag{3.6}$$

With $\omega = 0$ the above formula yields the solution of the corresponding static case [7]; substituting $w_0 = w_1$, we obtain

$$N^s = -\frac{\mu w_0}{\sqrt{\pi h}}. \tag{3.7}$$

The final form of the stress intensity factor for the problem shown in Fig. 4 is now obtained by using the Eqs. (3.6), (3.7):

$$N = -\frac{\mu w_0}{\sqrt{\pi h}}\left[1+\frac{w_1}{w_0}\sqrt{\frac{2\sigma'}{\sin 2\sigma'}}\cos\omega t\right]. \tag{3.8}$$

Passing here to the limit with $\omega \to 0$, we obtain the stress intensity factor in the static case, and in the case when $\omega \to \pi c_2/2h$ again a resonance-type phenomenon occurs.

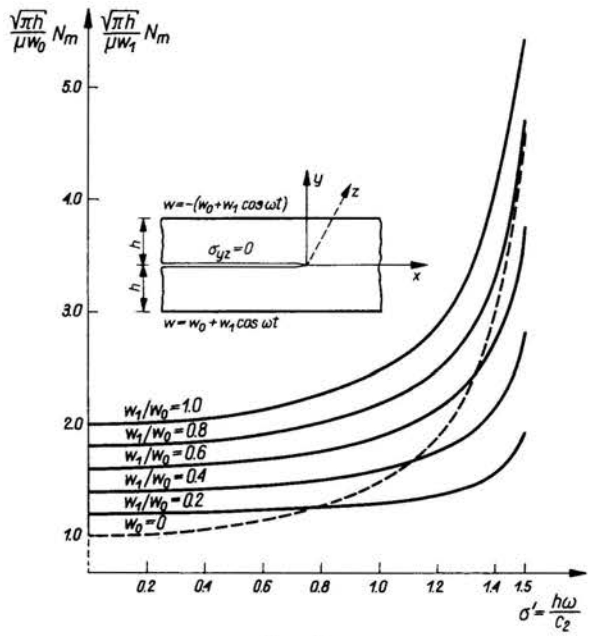

FIG. 6.

From the relation (3.8) it also follows that, at a constant vibration frequency ω, smaller thickness of the strip leads to larger values of the stress intensity factor. The maximum value of N is given, due to Eq. (3.8), by the formula

$$N_m(\omega) = -\frac{\mu w_0}{\sqrt{\pi h}}\left[1+\frac{w_1}{w_0}\sqrt{\frac{2\sigma'}{\sin 2\sigma'}}\right],$$

its variation being illustrated, for various values of w_1/w_0 and ω, by Fig. 6.

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