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Joanna Sokół-Supel

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Joanna Sokół-Supel
Institute of Fundamental Technological Research
Świętokrzyska 21, 00-049 Warsaw, Poland.

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1. Introduction.

The dynamic response of structures subjected to moving loads was considered by several authors. In most cases, however, only elastic behaviour was considered. An account on the available results can be found in [1]. For quasi-static loadings of structures it appears that, the design criterion requiring the stresses in the structure which do not exceed the yield value anywhere, is not economical. A more rational approach is needed, utilizing the fact that the material of the structure is elasto-plastic. The general dynamic elasto-plastic analysis is particularly difficult, but by ignoring the elastic deformation, the rigid plastic problem can be solved easily, although it is necessary to place restrictions on applications of the solutions to engineering materials.

The present report applies the yield line method [2,3] to the problem of a rigid-plastic plate subjected to a moving transverse load which is too large for the plate to support under static conditions. The question is whether there is a critical value for the moving load, above which the crossing can not be made at a finite speed. Such a critical value has been obtained for a massless beam [4] . It has also been shown that a critical load does not exist if the mass of the beam is taken into account [5]. This paper shows that for a massless plate there is a critical value for the moving load.

As an illustration of the method of solution a rectangular plate simply supported along the boundary is considered. The limit load with respect to its speed and the displacements distribution are computed. It will be shown that, for practical values of the ratios of plate dimensions, displacements are small.

2. Notation.

We consider an isotropic plate with a reference yield moment M_o. The transverse point load Q moves along a straight line /Fig. 1/.

We introduce the following dimensionless quantities

$$x=\frac{X}{A},\quad y=\frac{Y}{A},\quad s=\frac{S}{A},\quad w=\frac{W}{A},\quad q=\frac{Q}{M},\quad \beta=\frac{B}{A},\quad \gamma=\frac{H}{A}. \qquad (1)$$

where A is the reference length, H is the thickness of the plate, X, Y denote the cartesian coordinates of an arbitrary point of the plate, S is the distance from the edge of the plate to the travelling load, W is the vertical deflection. Moreover the following quantities are used:

$t\ [sec]$ - time

$v=\frac{V}{A},\ \frac{1}{sec}$ - speed of the load

$\dot{w}=\frac{dw}{dt}=\frac{dw}{ds}\cdot v = w'\cdot v,\ \left[\frac{1}{sec}\right]$ - transverse displacement velocity of the plate point (2)

θ - angular deformation

$\dot{\theta}=\frac{d\theta}{dt}=\frac{d\theta}{ds}\cdot v=\theta'\cdot v$ - angular deformation velocity

$\dot{w}_m$ - transverse displacement velocity of the moving load

$$\ddot{\Theta} = \frac{d^2\Theta}{dt^2} = \frac{d^2\Theta}{ds^2}\, v^2 = \Theta'' \cdot v^2$$ - angular deformation acceleration

(3)

$$a = A\,\frac{d^2 w_m}{dt^2} = A\,\frac{d^2 w_m}{ds^2} \cdot v^2 , \left[\frac{m}{sec^2}\right]$$ - transverse acceleration

$$m = \frac{Q}{g}, \left[\frac{kg \cdot sec^2}{m}\right]$$ - travelling mass

3. Formulation of the problem.

We shall adopt the following assumptions

a. The mass of the plate is negligible as compared to the mass of the moving load.

b. The transient yield pattern, i.e. the network of hinge lines, is adopted from the yield line theory and moves with the load.

c. The displacements are so small that distances along the plate can be replaced by their horizontal projections.

d. The load moves at a constant speed.

Under the above assumptions the problem is described by the following virtual power equation.

$$(Q - ma)\,\delta \dot{w} = M_o \sum_i \delta \phi_i \, l_i \tag{4}$$

where the summation is performed over all the yield lines of respective length l_i ; $\delta \dot{w}$ is the virtual deflection velocity, $\delta \Phi_i$ is the virtual vector of rotation rate which coincides with the yield line.

4. Solution

As an example, without loss of generality, we consider a rectangular simply supported plate. The transverse point load, Q, moves along $Y = Y_o$ /Fig. 2/.

The transient hinge pattern is shown in heavy lines in Fig. 2 ; $\dot{w}(x, y_o) = \dot{\theta}_2 \cdot (1-x)$. The load is moving from the left to the right. It follows that the part of the plate which the load has not yet crossed, has remained flat. The position of the load can be defined by the length of this part, $1-s$, and the angle θ_2, thus the transient transverse displacement of the moving load is

$$w_m = \theta_2 \cdot (1-s), \tag{5}$$

The transverse displacement velocity is

$$\dot{w}_m = \dot{\theta}_2\ (1-s) - \theta_2 \cdot v \tag{6}$$

where

$$\dot{s} = v$$

and transverse acceleration is

$$a = A\,\ddot{w}_m = A[\ddot{\theta}_2 (1-s) - 2\dot{\theta}_2 \cdot v] \tag{7}$$

Introducing the notation given by Eqs (1) and (2), into both sides of the virtual power equation (4), substituting (7) and using a more convenient nondimensional time scale $s = v \cdot t$, we obtain

$$(Q - ma)\,\delta w' = \left\{ q \cdot M_0 - \frac{qM_0}{g} A v^2 [\theta_2''(1-s) - 2\theta_2'] \right\} \delta w' \qquad (8)$$

$$M_0 \sum_i \delta \phi_i \, l_i = M_0 \beta \left(\frac{1}{s(1-s)} + Z \right) \delta w' \qquad (9)$$

where $Z = \frac{1}{y_0(\beta - y_0)}$, $0 < \beta \leqslant 1$.

From (8) and (9) we obtain the following ordinary differential equation

$$\frac{A v^2}{g} \left(\theta_2'' - \frac{2}{1-s} \theta_2' \right) = \frac{1}{q} \left\{ q - \beta \left[\frac{1}{s(1-s)} + Z \right] \right\} \frac{1}{1-s} \qquad (10)$$

where the upper commas denote differentiation with respect to s. This equation may be easily integrated in a closed form.

The initial conditions for (10) are

$$\theta_2(s_0) = \theta'(s_0) = 0 \qquad (11)$$

Motion will start when the moving load becomes the collapse load. This happens at a value of $s = s_0$ given by the root of the equation

$$q - \beta \left(\frac{1}{s_0(1-s_0)} + Z \right) = 0 \qquad (12)$$

It may be noted that this is equivalent to making the left hand side of Eq. (10) zero. Thus

$$s_0 = \frac{1}{2} \left(1 - \sqrt{1 - \frac{4\beta}{q - \beta Z}} \right) \qquad (13)$$

under the assumption that $0 \leqslant s_0 \leqslant \frac{1}{2}$. It follows from Eq. (13) that the motion of the plate is only possible for

$$q > \beta (Z + 4) \qquad (14)$$

By integrating (10) as a linear equation in θ_2', it is found that

$$\theta_2' = \exp\left(-\int_{s_0}^{s_f} P(s)\,ds\right) \cdot \int_{s_0}^{s_f} \left(R(s) \exp\left(\int_{s_0}^{s_f} P(s)\,ds \right) \right) ds \qquad (15)$$

where

$$P(s) = -\frac{2}{1-s}\ ,\quad R(s) = \frac{g}{Av^2}\cdot\frac{1}{q}\ \left[q-\beta\left(Z+\frac{1}{s(1-s)}\right)\right]\frac{1}{1-s}$$

The expressions for the angular velocity θ_2' and the rotation θ_2 are obtained from (15) in the form

$$\frac{Av^2}{g}\theta_2' = \frac{1}{q}\left\{[q-\beta Z](s-s_0-\frac{s^2-s_0^2}{2})-\beta\ln\frac{s}{s_0}\right\}\frac{1}{(1-s)^2} \qquad (16)$$

$$\frac{Av^2}{g}\theta_2 = \frac{1}{q}\left\{[q-\beta Z]\left[\frac{(s-s_0)^2}{2(1-s)}\right]-\beta\left[\frac{s}{1-s}\ln\frac{s}{s_0}+\ln\frac{1-s}{1-s_0}\right]\right\} \qquad (17)$$

The motion ceases when the relative angular velocity at the vertex of the yield pattern becomes zero, i.e. when $\theta_2'=0$. The corresponding final value of s may be found by setting the right-hand side of (16) equal to zero. When load q and its speed v are limited, substituting $q-\beta Z = \frac{\beta}{(1-s_0)s_0}$ into (16) we are to solve following equation

$$(s-s_0-\frac{s^2-s_0^2}{2})\frac{1}{s_0(1-s_0)}-\ln\frac{s}{s_0}=0 \qquad (18)$$

to find the value of . It can be seen that s_f is evidently only function of s_0, /Fig. 3/, and because of that function of q and β too. The values of s_0 and s_f corresponding to a given value of $\frac{Q}{Q_c}$ can be read off Fig. 4 a, b; Q_c is the magnitude of the static limit load when applied at the midpoint of the plate and equal to $Q_c = 4\beta+\frac{4}{\beta}$. The final rotation θ_f is found by putting $s=s_f$ in (17). It will be noted in Fig. 5 that for certain value of Q, dependent on β, θ_f increases indefinitely irrespective of the speed v except $v\to\infty$ when θ_f is indeterminate.

In the limiting case when $s_f\to 1$, $s_0=0.2847$ is the solution of (18). It can be seen that in this case the value of an initial point of motion is constant and independent of dimensions of a plate.

The critical magnitude of the moving load obtained from (12) is

$$q_{cr} = \beta\left(4.9108+\frac{1}{y_0(\beta-y_0)}\right) \qquad (19)$$

q_{cr} is shown as a function of β in Fig. 6 compared with the value of the static limit load at midpoint of the plate.

Such a critical value equal 4.9108 has been obtained for a massless beam by Parkes [4] . Analysis of the load path explains in a pictorial manner why such a result has been obtained

Substituting (17) into (5) and ordering, the following equation which describes the displacement of the moving load is obtained

$$\frac{q}{\beta}\,\frac{Av^2}{g}\,w_m = \frac{1}{2}\,\frac{(s-s_0)^2}{(1-s_0)s_0} + s\ln\frac{(1-s)\,s_0}{(1-s_0)\,s} - \ln\frac{1-s}{1-s_0} \qquad (20)$$

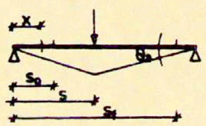
From mathematical point of view the above equation is valid for $0 < s_o < 0.2847$ too. Then the load path can be plotted even if the load is greater than the critical one. Shapes of the paths shown in Fig. 7 a, b, c indicate that for $Q > Q_{cr}$ the continuity between plate and support would be broken off, but such a solution cannot be admitted.

5. Final displacements

According to the assumed hinge pattern shown in Fig. 2 the transient velocity of displacement along the line $y=y_o$ during the motion can be described in a following way

for $0<x<s_0$

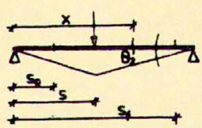
$$W'(x, y_0, s) = \frac{1-s}{s}\cdot\theta_2'\cdot x$$



when $s_0<s<s_f$

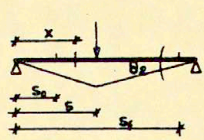
for $s_0<x<s_f$

$$W'(x, y_0, s) = (1-x)\cdot\theta_2'$$



when $s_0<s<x$

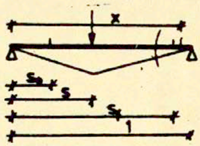
$$W'(x, y_0, s) = \frac{1-s}{s}\;\theta_2'\cdot x$$



when $x<s_0<s_f$

for $s_1 < x < 1$

$W'(x, y_0, s) = (1-x)\,\theta_2'$



when $s_0 < s < s_1$

Similarly we can write suitable expressions for $w'(s)$ in each point (x, y) of the plate.

The final displacement w of any point (x, y) of the plate after the load has moved out is given by

$$w(x,y) = \int_{s_0}^{s_1} w'(x,y,s)\,ds \tag{22}$$

For instance: an integration of (21) yields the final displacements along the line $y=y_0$

$$w(x,y_0) = \begin{cases} x \cdot \displaystyle\int_{s_0}^{s_1} \frac{1-s}{s}\,\theta_2'(s)\,ds & \text{for } 0 < x \leqslant s_0 \\ (1-x)\theta_2(x) + x \displaystyle\int_{x}^{s_1} \frac{1-s}{s}\,\theta_2'(s)\,ds & \text{for } s_0 \leqslant x \leqslant s_1 \\ (1-x)\,\theta_2(s_1) & \text{for } s_1 \leqslant x \leqslant 1 \end{cases} \tag{23}$$

Substituting (16) and (17) into (23) and integrating we obtain the following expression for the displacements along the line $y=y_0$ and $x \in (s_0, s_1)$

$$\frac{Av^2}{g}\,w(x,y_0) = \frac{\beta}{q}\left\{(1-x)\left[K\frac{(x-s_0)^2}{2(1-x)} - \frac{x}{1-x}\ln\frac{x}{s_0} - \ln\frac{1-x}{1-s_0}\right] + \right.$$

$$+x\left[\frac{K}{2}\left(s-\ln(1-s)+s_0(2-s_0)\ln\frac{1-s}{s}\right)-\ln\frac{s}{1-s}\ln\frac{s}{s_0}+\frac{1}{2}\ln^2 s+\sum^{\infty}\frac{s^n}{n^2}\right]_x^{s_f}\Bigg\} \qquad (24)$$

where $K=\frac{1}{(1-s_0)s_0}$

For $0 < x < s_0$ and $s_f < x < 1$ the displacement (24) remains a linear function of x.

In Fig. 8 a, b, c the variation of $\frac{Av^2}{g}w(x,y_0)$ with x is shown for some representative values of Q and β .

It should be noted that the shape of the permanently deformed plate is asymmetric.

The point of maximum deflection is displaced to the right of the midpoint of the plate i.e. in the direction of the speed of the load, as might be expected, and is nearer to the right support for higher loads.

The permanent maximum deflection $\frac{A\,v^2}{g}w_{max}$ of the plate is plotted in Fig. 9 for ratios β = 0.5, 0.75, 1. It increases together with Q and remains finite for $Q = Q_{cr}$.

The solution presented above is derived under the assumption of very small displacements i.e. the final slope .

In Fig. 10 and 11 relations between load, its speed and β, for a final slope of 1 in 20 and span of plate A = 8m are shown. They were calculated from (17) .

To test directly whether the obtained displacements are small in fact, i.e., in the practical case if $W < 0.5H$, we compute them for some practical values of ratio $\frac{H}{A}$, q and β with respect to the speed of the moving load. In Fig. 12 a, b we can see that the maximum permanent displacements after the load moved out the plate are still very small.

6. Conclusions

This paper presents an approximation to the solution of a rigid-plastic plate under a moving load being larger than the load under which the plate would collapse if the load were stationary at the plate midpoint. It has been shown for a massless plate that the moving load cannot exceed the critical magnitude given by (19) .

The reasons of this surprising result are the assumptions that a plate is massless and the transient yield pattern moves together with load at the same speed, thus the process of deformation ceases when the load goes off the plate.
Therefore one can draw the conclusion that mass of the plate must be taken into account to obtain the critical magnitude of the moving load in dynamic problem, even for the case in which the load is large compared with the plate weight.

On the other hand the present results can be utilized as an estimation for the ponderable plate's displacements. They indicate that for the massless plate the assumption of very small deflection is reasonable, thus it can be adopted to a ponderable plate.

7. References

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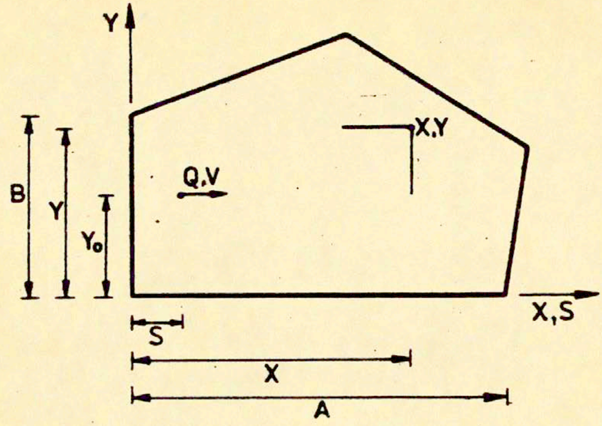

Fig.1. Arbitrary plate under moving load

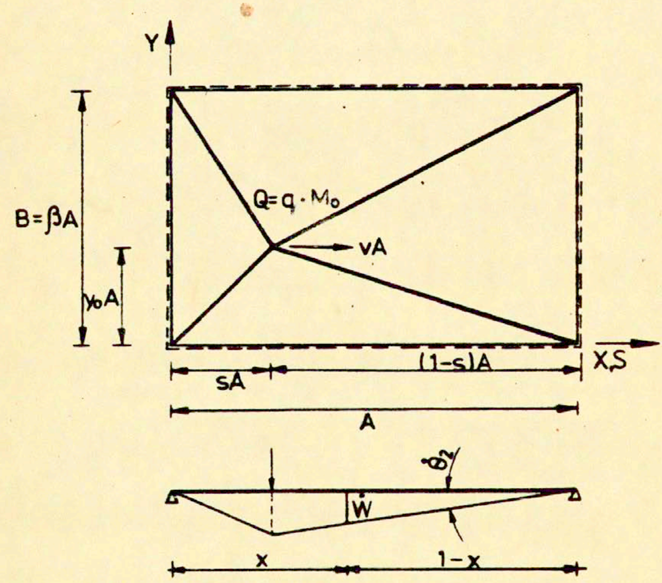

Fig.2. Rectangular simply supported plate under moving load

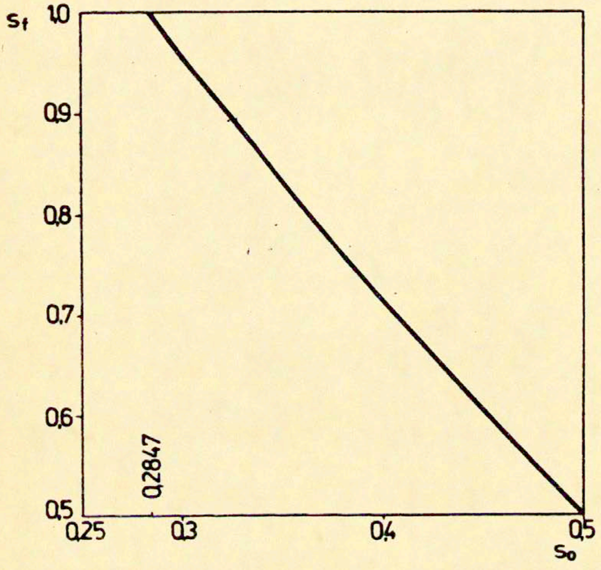

Fig.3. Relation between final and initial points of motion

a)

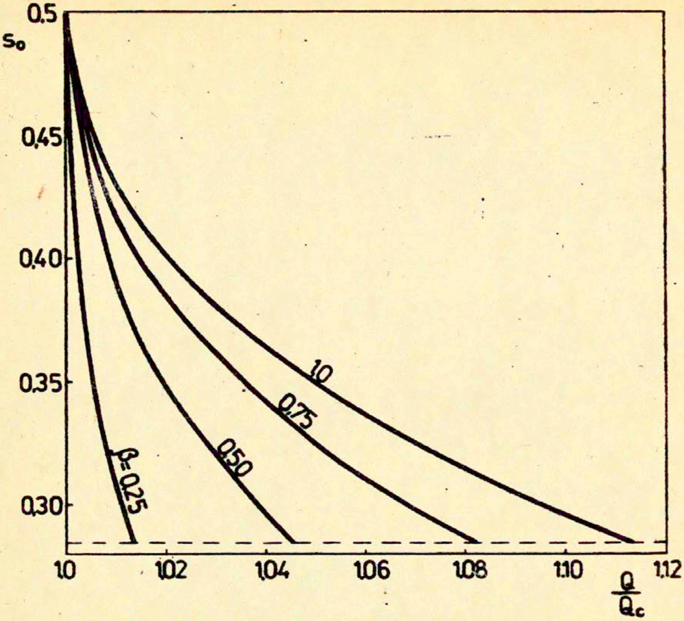

Fig.4a. Initial point of motion

b)

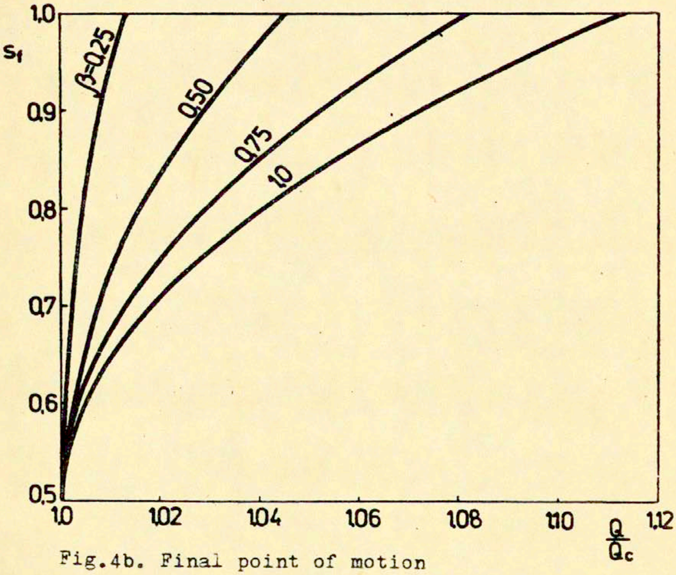

Fig.4b. Final point of motion

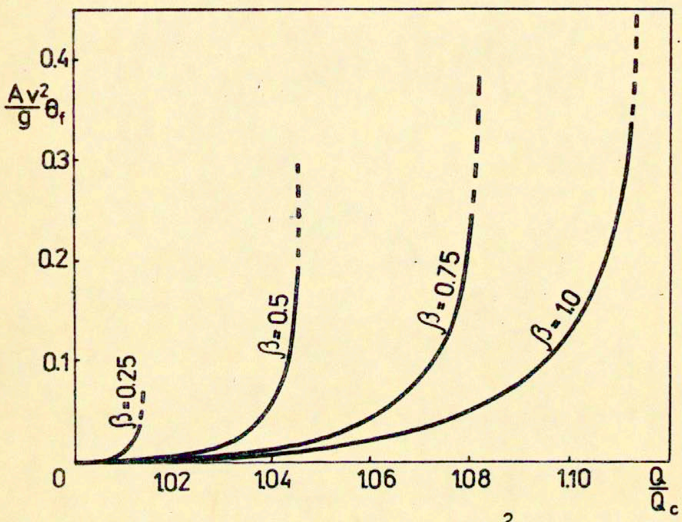

Fig.5. Final angular rotation $\frac{Av^2}{g}\Theta_f$

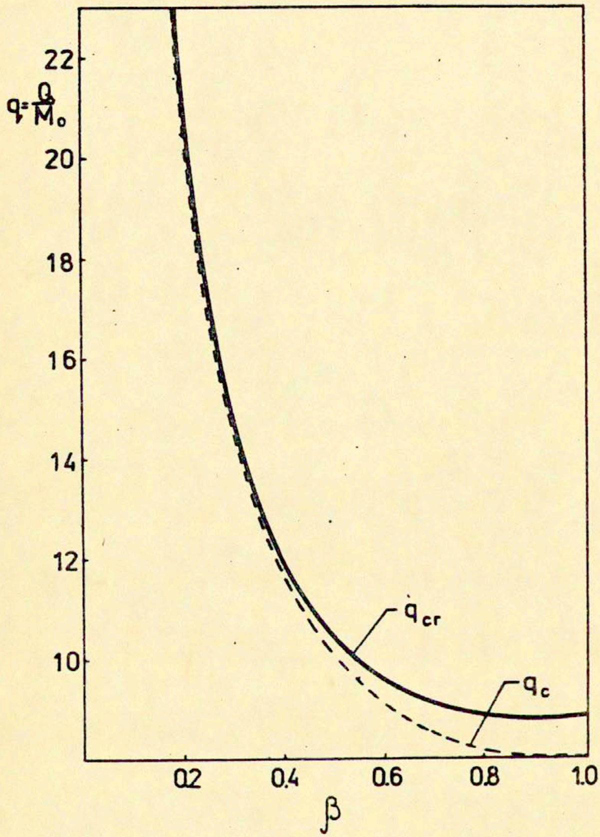

Fig.6. Critical moving load q_{cr} and limit load q_c

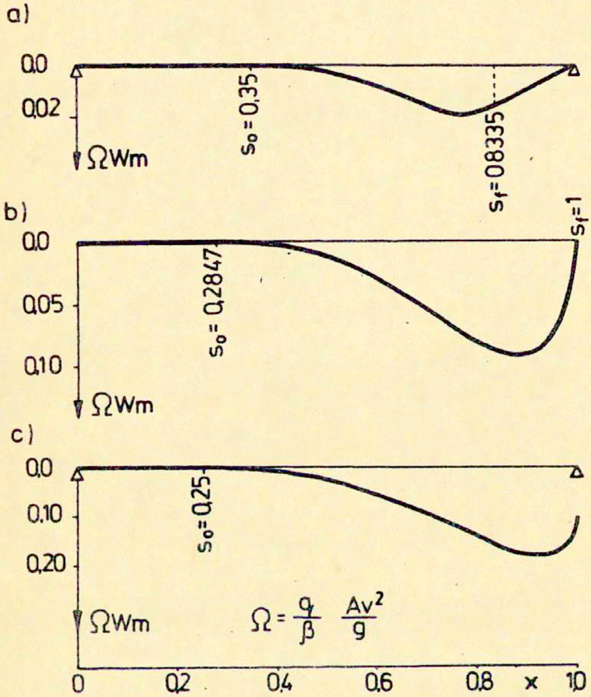

Fig.7. Paths of the moving load- w_m

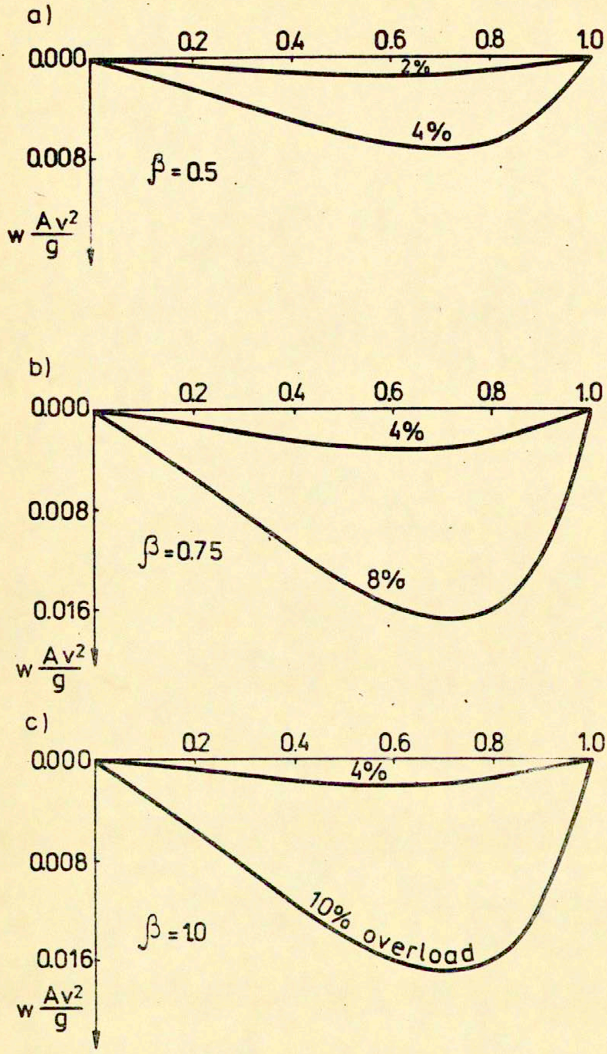

Fig.8. Final displacement $w\frac{Av^2}{g}$ of the line $y=y_o$

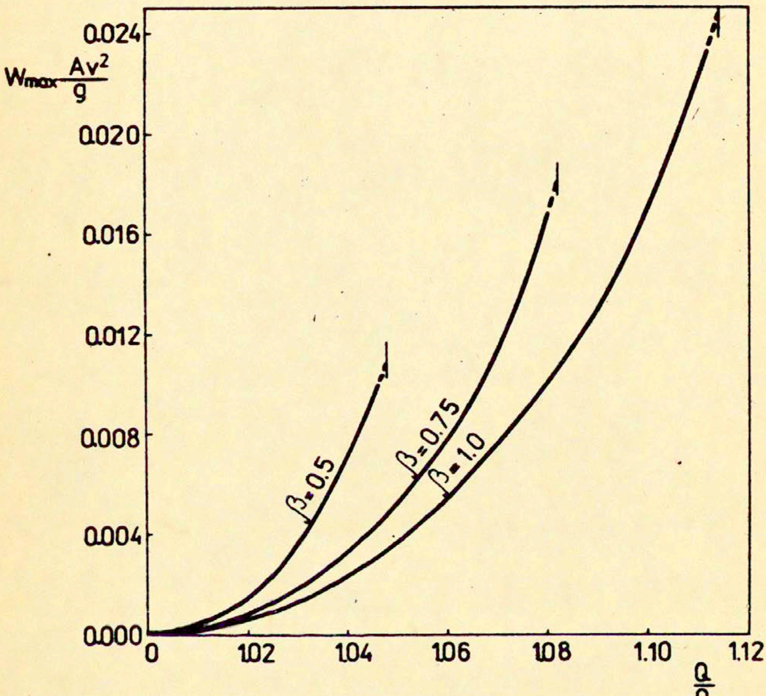

Fig.9. Maximum final displacement $w_{max}\frac{Av^2}{g}$

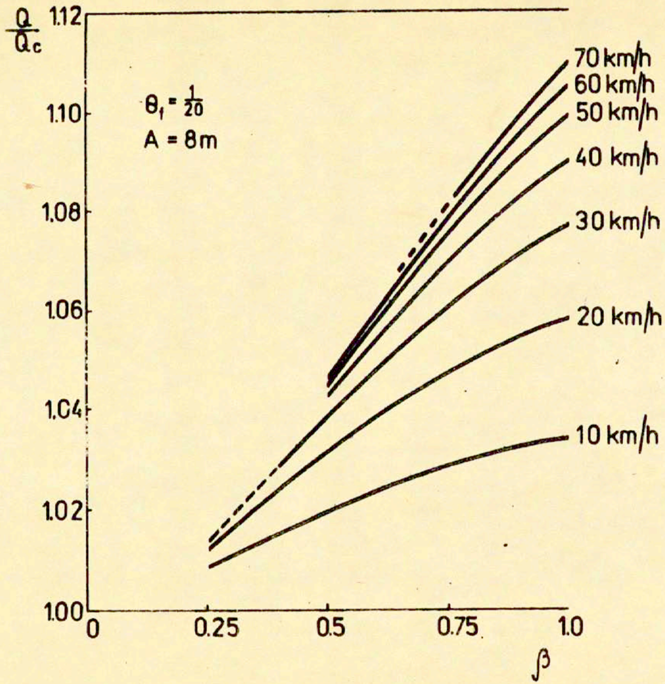

Fig.10. Moving load as a function of β for vary speed of load

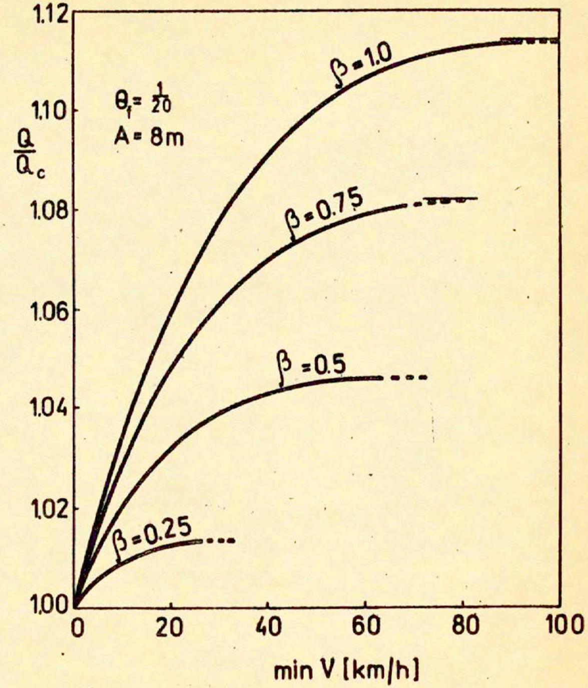

Fig.11. Moving load as a function of speed for vary value of β

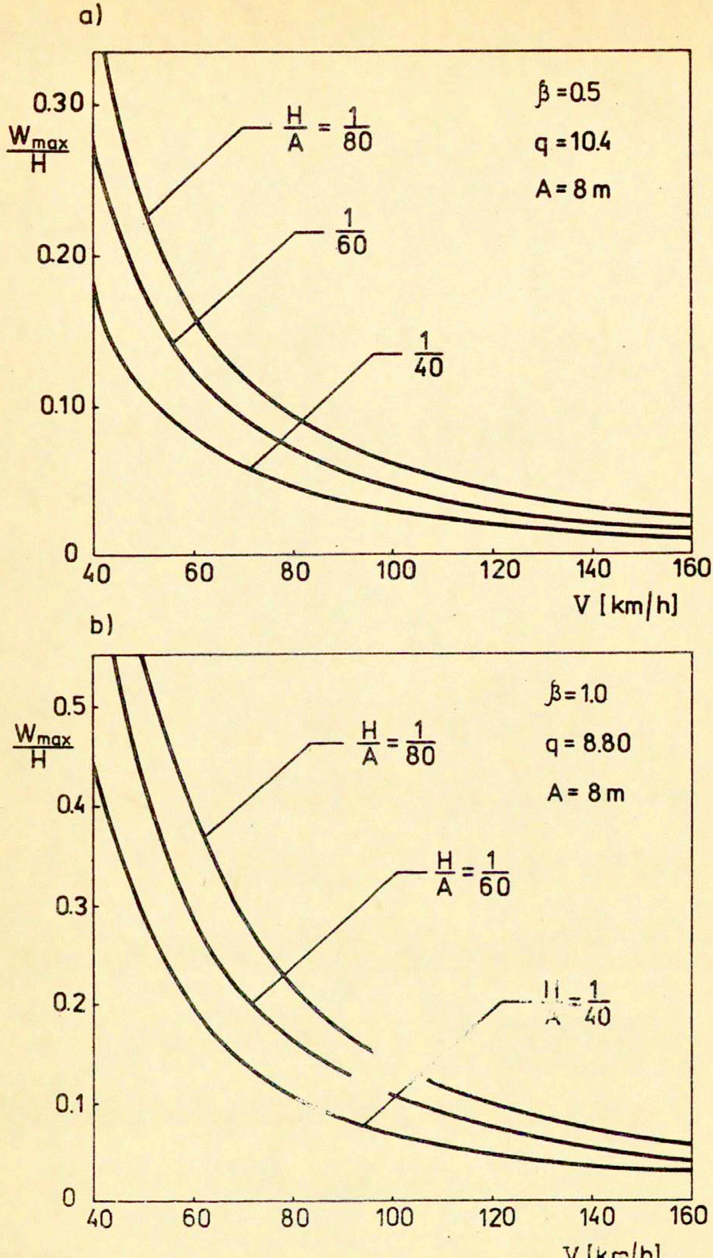

Fig.12. Maximum final displacement $\frac{W}{H}$