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Large deflections of viscoelastic anisotropic plates

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IN THE PAPER is considered the problem of relatively large deflections of viscoelastic anisotropic plates. Deflections are assumed to be not small in comparison with the thickness of the plate but small as compared with the other dimensions. The system of nonlinear integro-differential equations has been obtained. This system can be solved by the method of successive approximations.

Rozpatruje się zagadnienie stosunkowo dużych ugięć lepkosprężystych płyt anizotropowych. Zakłada się, że ugięcia nie są małe w porównaniu z grubością płyty, ale pozostają małe w stosunku do pozostałych wymiarów. Otrzymano układ nieliniowych równań całkowo-różniczkowych, który można rozwiązać metodą kolejnych przybliżeń.

Рассмотрена задача об относительно больших прогибах вязкоупругих анизотропных пластинок. Предполагается, что прогибы не малы по сравнению с толщиной пластинки, но значительно меньше остальных ее размеров. Получена система нелинейных интегрально-дифференциальных уравнений, которую можно решить по методу последовательных приближений.

1. Introduction

THE MODERN nonlinear theory of viscoelasticity is marked by the utmost generality. It was formulated by GREEN and RIVLIN [1] and NOLL [2] in their papers on general stress-deformation relations for materials with memory. The general constitutive equations express the stress by an operator applied to the deformation history, the form of dependence being restricted only by certain general invariance requirements. It is evident that these general constitutive equations are too complicated for the solution of boundary value problems. Therefore, it is necessary to deal with approximations of general operators. GREEN and RIVLIN [1] have shown that these operators may be expressed to any desired approximation by the n-tuple integral operators.

The first-order approximation given by a single integral operator generalizes the infinitesimal theory of viscoelasticity to the finite deformation theory of viscoelasticity, in which rotations and displacements can be large [3-5].

In this paper, we shall deal with further simplification of the theory for two-dimensional bodies corresponding to the large deflection theory of elastic plates and shells. Thus we shall assume that the deflections are not small in comparison with the thickness of the plate but are still small as compared with the other dimensions.

2. Constitutive equations

Consider quasi-static problems in which inertia forces due to deformation are negligible. The constitutive equation of an arbitrary linear non-polar viscoelastic material can be written in the form:

(2.1) $$\sigma^{iJ} = H^{iJkl}\varepsilon_{kl},$$

where H^{iJkl} represents a tensor operator. In the case of non-polar materials, this operator is symmetric and positive definite. It can have a differential or integral form. In the case of the differential form, the constitutive equations (2.1) assume the following form [6–7]:

$$H^{(r)}\sigma^{iJ} = H^{iJkl}_{(s)}\varepsilon_{kl}, \tag{2.2}$$

or

$$Q^{(r)}\varepsilon_{iJ} = Q^{(s)}_{iJkl}\sigma^{kl}, \tag{2.3}$$

where

$$H^{(r)} = \prod_{n=1}^{r}\left(\frac{\partial}{\partial t}+\varkappa_n\right) \quad Q^{(r)} = \prod_{n=1}^{r}\left(\frac{\partial}{\partial t}+\lambda_n\right) \tag{2.4}$$

are scalar operators and

$$H^{iJkl}_{(s)} = \sum_{n=0}^{s} H^{iJkl}_{(n)}\frac{\partial^n}{\partial t^n}, \qquad Q^{(s)}_{iJkl} = \sum_{n=0}^{s} Q^{(n)}_{iJkl}\frac{\partial^n}{\partial t^n} \tag{2.5}$$

are tensor operators, $\varkappa_n \geqslant 0$ are inverse relaxation times, $\lambda_n \geqslant 0$ are inverse retardation times and $H^{(0)} = 1$, $Q^{(0)} = 1$. As has been proved by the present author [6], tensor operators cannot be on both sides of (2.2)–(2.3).

In the case of a homogeneous relaxation spectrum, Eqs. (2.5) assume the form:

$$H^{iJkl}_{(s)} = H^{iJkl}\sum_{n=1}^{s} K_n\frac{\partial^n}{\partial t^n}, \qquad Q^{(s)}_{iJkl} = Q_{iJkl}\sum_{n=1}^{s} Q_n\frac{\partial^n}{\partial t^n}. \tag{2.6}$$

In the case of an integral form, the constitutive equations can be written in the form:

$$\sigma^{iJ} = \int_0^t G^{iJkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}\,d\tau, \tag{2.7}$$

or

$$\varepsilon_{iJ} = \int_0^t J_{iJkl}(t-\tau)\frac{\partial \sigma^{kl}}{\partial \tau}\,d\tau, \tag{2.8}$$

where $G^{iJkl}(t-\tau)$ is a tensor of relaxation functions and $J_{iJkl}(t-\tau)$ a tensor of creep functions.

3. Plane stress equation

Consider a viscoelastic anisotropic thin plate of constant thickness h. We choose the rectangular Cartesian coordinate system x_i with $x_3 = 0$ in the middle plane of the undeformed plate. We restrict our attention to those plates which are elastically symmetric about the middle plane. According to the theory of large deflections, we assume that the transverse displacement u_3 is relatively large as compared with the thickness h, and the displacements u_1, u_2 in the midplane are so small that the equilibrium equations can be expressed in the coordinates of the undeformed plate and squares and products of derivatives of u_1, u_2 can be disregarded.

The tensor of finite strain can be written in the form

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j}+u_{j,i}+u_{k,i}u_{k,j}), \quad i,j,k = 1,2,3. \tag{3.1}$$

We shall consider the midplane components of this tensor $\varepsilon_{\alpha\beta}$, where α, β assume the values 1 and 2. According to the assumption indicated, we have:

$$\varepsilon_{\alpha\beta} = \frac{1}{2}(u_{\alpha,\beta}+u_{\beta,\alpha}+u_{3,\alpha}+u_{3,\beta}), \quad \alpha,\beta = 1,2. \tag{3.2}$$

Using the infinitesimal strain tensor

$$e_{\alpha\beta} = \frac{1}{2}(u_{\alpha,\beta}+u_{\beta,\alpha}), \tag{3.3}$$

we can write

$$\varepsilon_{\alpha\beta} = e_{\alpha\beta}+\frac{1}{2}u_{3,\alpha}u_{3,\beta} \tag{3.4}$$

or

$$e_{\alpha\beta} = \varepsilon_{\alpha\beta}-\frac{1}{2}u_{3,\alpha}u_{3,\beta}. \tag{3.5}$$

According to the definition (3.3); the infinitesimal strain tensor fulfils the compatibility equation:

$$\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}e_{\alpha\beta,\gamma\delta} = 0, \tag{3.6}$$

where $\epsilon_{\alpha\beta}$ is the alternating tensor.

Inserting (3.5) into (3.6), we obtain:

$$\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}\varepsilon_{\alpha\beta,\gamma\delta} = \frac{1}{2}\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}u_{3,\alpha\delta}u_{3,\beta\gamma}. \tag{3.7}$$

From the constitutive equation (2.1) we find that

$$\varepsilon_{\alpha\beta} = G_{\alpha\beta\gamma\delta}\sigma^{\gamma\delta}, \tag{3.8}$$

where

$$G_{\alpha\beta\gamma\delta} = (H^{\alpha\beta\gamma\delta})^{-1}. \tag{3.9}$$

Introducing the stress function

$$\sigma_{\alpha\beta} = \epsilon_{\alpha\gamma}\epsilon_{\beta\delta}F_{,\gamma\delta}, \tag{3.10}$$

we can write:

$$\varepsilon_{\alpha\beta} = G_{\alpha\beta\gamma\delta}\epsilon_{\gamma\mu}\epsilon_{\delta\gamma}F_{,\mu\nu}. \tag{3.11}$$

Using (3.11), the compatibility equation takes the form:

$$\epsilon_{\alpha\mu}\epsilon_{\beta\nu}\epsilon_{\gamma\varkappa}\epsilon_{\delta\rho}G_{\alpha\beta\gamma\delta}F_{,\mu\nu\varkappa\rho} = \frac{1}{2}\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}u_{3,\alpha\delta}u_{3,\beta\gamma}, \tag{3.12}$$

which is the compatibility equation written in terms of the stress function.

In order to simplity the writing and to obtain the common form of this equation, we introduce the operator:

(3.13) $$K_{\mu\nu\kappa\rho} = \epsilon_{\alpha\mu}\epsilon_{\beta\nu}\epsilon_{\gamma\kappa}\epsilon_{\delta\rho}G_{\alpha\beta\gamma\delta}.$$

Hence, the compatibility equation becomes:

(3.14) $$K_{\alpha\beta\gamma\delta}F_{,\alpha\beta\gamma\delta} = \frac{1}{2}\epsilon_{\alpha\nu}\epsilon_{\beta\delta}u_{3,\alpha\delta}u_{3,\beta\gamma},$$

which yields:

(3.15) $$K_{1111}F_{,1111}+4K_{1112}F_{,1112}+2(K_{1122}+4K_{1212})F_{,1122}+4K_{1222}F_{,1222} +K_{2222}F_{,2222} = u_{3,12}^2-u_{3,11}u_{3,22}$$

or

(3.16) $$G_{2222}F_{,1111}-4G_{1222}F_{,1112}+(2G_{1122}+4G_{1212})F_{,1122}-4G_{1112}F_{,1222} +G_{1111}F_{,2222} = u_{3,12}^2-u_{3,11}u_{3,22}.$$

In contracted notation [8] we obtain:

(3.17) $$G_{22}F_{,1111}-2G_{26}F_{,1112}+(2G_{12}+G_{66})F_{,1122}-2G_{16}F_{,1222} +G_{11}F_{,2222} = u_{3,12}^2-u_{3,11}u_{3,22}.$$

Using the constitutive equation (2.3), the equation becomes

(3.18) $$Q_{2222}F_{,1111}-4Q_{1222}F_{,1112}+(2Q_{1122}+4Q_{1212})F_{,1122}-4Q_{1112}F_{,1222} +Q_{1111}F_{,2222} = Q(u_{3,12}^2-u_{3,11}u_{3,22}),$$

where Q is a scalar operator.

4. Plate equation

Now we are concerned with the transverse displacements of the plate of constant thickness h loaded by the transverse load $q(x_1, x_2)$. In addition to the fixed Cartesian axes x_i in the undeformed plate, we take the coordinates y_i in the deflected plate. We put the coordinate surface $y_3 = 0$ in the deflected midplane of the plate. Then, according to assumptions on the displacement of the large deflection theory, we put

(4.1) $$x_1 = y_1, \quad x_2 = y_2, \quad x_3 = y_3+w,$$

where

(4.2) $$w = u_3.$$

The covariant base vectors can be obtained from the formula:

(4.3) $$\mathbf{g}_r = \frac{\partial x^s}{\partial y^r}\mathbf{i}_s,$$

which yields:

(4.4) $$\mathbf{g}_1 = \mathbf{i}_1+w_{,1}\mathbf{i}_3, \quad \mathbf{g}_2 = \mathbf{i}_2+w_{,2}\mathbf{i}_3, \quad \mathbf{g}_3 = \mathbf{i}_3.$$

Then the components of the metric tensor become:

(4.5)
$$\mathbf{g}_{ij} = \mathbf{g}_i \mathbf{g}_j = \begin{bmatrix} 1+w_{,1}^2 & w_{,1}w_{,2} & w_{,1} \\ w_{,1}w_{,2} & 1+w_{,2}^2 & w_{,2} \\ w_{,1} & w_{,2} & 1 \end{bmatrix},$$

(4.6)
$$g^{ij} = \begin{bmatrix} 1 & 0 & -w_{,1} \\ 0 & 1 & -w_{,2} \\ -w_{,1} & -w_{,2} & w_{,1}^2+w_{,2}^2 \end{bmatrix}.$$

In the absence of body forces, the equilibrium equations are

(4.7)
$$\sigma^{ij}/i = 0,$$

where the symbol / denotes the covariant differentiation. This equation can be written in the form:

(4.8)
$$\sigma^{ij}_{,i}+\Gamma^i_{ki}\sigma^{kj}+\Gamma^j_{ki}\sigma^{ik} = 0,$$

where

(4.9)
$$\Gamma^i_{jk} = g^{im}\Gamma_{jkm}$$

and

(4.10)
$$\Gamma_{ijk} = \frac{1}{2}(g_{ik,j}+g_{jk,i}-g_{ij,k}),$$

Γ_{ijk}, Γ^i_{jk} are Christoffel symbols of the first and second kind, respectively.

Equations (4.8) represents three equilibrium equations in y_1, y_2 and y_3 directions. We shall deal separately with the equations of equilibrium in the midplane

(4.11)
$$\sigma^{i\alpha}_{,i}+\Gamma^i_{ki}\sigma^{k\alpha}+\Gamma^\alpha_{ki}\sigma^{ik} = 0$$

and in y_3 direction

(4.12)
$$\sigma^{i3}_{,i}+\Gamma^i_{ki}\sigma^{k3}+\Gamma^3_{ki}\sigma^{ik} = 0.$$

According to the assumptions of the large deflection theory of plates and according to (4.1), we replace y_α by x_α, which are orthogonal Cartesian coordinates. Multiplying (4.11) by y_3 and then integrating it with respect to y_3 through the thickness of the plate, we have:

(4.13)
$$M_{\alpha\beta,\alpha}-Q_\beta = 0,$$

where we have denoted

(4.13)
$$M_{\alpha\beta} = \int_{-h/2}^{h/2} \sigma^{\alpha\beta} y_3 \, dy_3$$

and

(4.15)
$$Q_\beta = \int_{-h/2}^{h/2} \sigma^{3\beta} \, dy_3.$$

Similarly, after integration of (4.12) with respect to y_3, we obtain:

(4.16)
$$Q_{\alpha,\alpha}+w_{,\alpha\beta}N_{\alpha\beta}+q = 0,$$

where

$$N_{\alpha\beta} = \int_{-h/2}^{h/2} \sigma^{\alpha\beta} dy_3 . \tag{4.17}$$

Eliminating the shearing forces Q_α from (4.13) and (4.16) yields:

$$M_{\alpha\beta,\alpha\beta} + w_{,\alpha\beta} N_{\alpha\beta} + q = 0. \tag{4.18}$$

Assuming that normals to the midplane remain normal during deformation and do not change their length, we can write:

$$\varepsilon_{\alpha\beta} = -y_3 w_{,\alpha\beta} . \tag{4.19}$$

Inserting (4.19) into the constitutive equation (2.1), and integrating it with respect to y_3, we obtain:

$$M_{\alpha\beta} = -D_{\alpha\beta\gamma\delta} w_{,\gamma\delta}, \tag{4.20}$$

where

$$D_{\alpha\beta\gamma\delta} = \frac{1}{12} h^3 H_{\alpha\beta\gamma\delta} \tag{4.21}$$

are operators corresponding to stiffnesses of an elastic plate.

Hence (4.18) can be written in the form:

$$D_{\alpha\beta\gamma\delta} w_{,\alpha\beta\gamma\delta} = q + h \in_{\alpha\gamma} \in_{\beta\delta} w_{,\alpha\beta} F_{,\gamma\delta}, \tag{4.22}$$

where we have put

$$N_{\alpha\beta} = h \in_{\alpha\gamma} \in_{\beta\delta} F_{,\gamma\delta} . \tag{4.23}$$

Equations (3.14) and (4.22) correspond to Kármán equations for large deflections of elastic plates.

Equation (4.22) can be written in the form:

$$\begin{aligned} D_{1111} w_{,1111} + 4D_{1112} w_{,1112} + 2(D_{1122} + 2D_{1212}) w_{,1122} + 4D_{1222} w_{,1222} \\ + D_{2222} w_{,2222} = q + h(w_{,11} F_{,22} - 2w_{,12} F_{,12} + w_{,22} F_{,11}), \end{aligned} \tag{4.24}$$

or in the contracted notation

$$\begin{aligned} D_{11} w_{,1111} + 4D_{16} w_{,1112} + 2(D_{12} + 2D_{66}) w_{,1222} + 4D_{26} w_{,1222} + D_{22} w_{,2222} = q \\ + h(w_{11} F_{,22} - 2w_{,22} F_{,12} + w_{,22} F_{,11}). \end{aligned} \tag{4.25}$$

In the case of a Voigt plate,

$$\sigma^{\alpha\beta} = \left(E^{\alpha\beta\gamma\delta} + \eta^{\alpha\beta\gamma\delta} \frac{\partial}{\partial t} \right) \varepsilon_{\gamma\delta}, \tag{4.26}$$

where $E^{\alpha\beta\gamma\delta}$, $\eta^{\alpha\beta\gamma\delta}$ are moduli of elasticity and moduli of viscocity, respectively.

In the form of Laplace transform, (4.26) becomes

$$\tilde{\sigma}^{\alpha\beta} = (E^{\alpha\beta\gamma\delta} + p\eta^{\alpha\beta\gamma\delta}) \tilde{\varepsilon}_{\gamma\delta} , \tag{4.27}$$

where symbols with tildas denote Laplace transforms.

Solving this equation, we find:

$$A(p) \tilde{\varepsilon}_{\alpha\beta} = A_{\alpha\beta\gamma\delta}(p) \tilde{\sigma}^{\gamma\delta}, \tag{4.28}$$

where in the general case $A(p) = |E^{\alpha\beta\gamma\delta} + p\eta^{\alpha\beta\gamma\delta}|$ is a polynomial of degree in p and the adjoint matrix $A_{\alpha\gamma\delta\delta}(p)$ is a p-matrix of degree 2.

Inserting (4.26) and the inverse of (4.28) in (3.14) and (4.23), we obtain:

$$K_{\alpha\beta\gamma\delta}\left(\frac{\partial}{\partial t}\right)F_{,\alpha\beta\gamma\delta} = \frac{1}{2}\in_{\alpha\gamma}\in_{\beta\delta}A\left(\frac{\partial}{\partial t}\right)w_{,\alpha\delta}w_{,\beta\gamma} \tag{4.29}$$

and

$$\left(D_{\alpha\beta\gamma\delta}+\Omega_{\alpha\beta\gamma\delta}\frac{\partial}{\partial t}\right)w_{,\alpha\beta\gamma\delta} = g+h\in_{\alpha\gamma}\in_{\beta\delta}w_{,\alpha\beta}F_{,\gamma\delta}, \tag{4.30}$$

where

$$(4.31)\quad \begin{aligned} K_{\alpha\beta\gamma\delta}\left(\frac{\partial}{\partial t}\right) &= \in_{\alpha\mu}\in_{\beta\nu}\in_{\gamma\varkappa}\in_{\delta\rho}A_{\mu\nu\varkappa\rho}\left(\frac{\partial}{\partial t}\right),\\ D_{\alpha\beta\gamma\delta} = \frac{1}{12}h^3E_{\alpha\beta\gamma\delta}, &\quad \Omega_{\alpha\beta\gamma\delta} = \frac{1}{12}h^3\eta_{\alpha\beta\gamma\delta}.\end{aligned}$$

In the case of the homogeneous spectrum

$$\eta_{\alpha\beta\gamma\delta} = KE_{\alpha\beta\gamma\delta}, \tag{4.32}$$

we obtain:

$$(4.33)\quad \begin{aligned} K_{,\alpha\beta\gamma\delta}F_{,\alpha\beta\gamma\delta} &= \frac{1}{2}\in_{\alpha\gamma}\in_{\beta\delta}\left(1+K\frac{\partial}{\partial t}\right)w_{,\alpha\delta}w_{,\beta\gamma},\\ D_{\alpha\beta\gamma\delta}\left(1+K\frac{\partial}{\partial t}\right)w_{,\alpha\beta\gamma\delta} &= q+h\in_{\alpha\gamma}\in_{\beta\delta}w_{,\alpha\beta}F_{,\gamma\delta},\end{aligned}$$

where

$$K_{\alpha\beta\gamma\delta} = \in_{\alpha\mu}\in_{\beta\nu}\in_{\gamma\varkappa}\in_{\delta\rho}A_{\mu\nu\varkappa\rho} \tag{4.34}$$

and

$$A_{\alpha\beta\gamma\delta} = (E_{\alpha\beta\gamma\delta})^{-1}. \tag{4.35}$$

5. Integro-differential equations for large deflections of viscoelastic plates

When dealing with boundary value problems of large deflection theory of viscoelastic plates, it is advantageous to replace boundary value problems by solutions of nonlinear integro-differential equations.

We shall consider the combination of the following boundary conditions:

$$w = 0, \quad w_{,n} = 0, \quad \text{on} \quad \partial S, \tag{5.1}$$

or

$$w = 0, \quad M_{nn} = 0, \quad \text{on} \quad \partial S \tag{5.2}$$

and

$$N_{nn} = 0, \quad N_{ns} = 0, \quad \text{on} \quad \partial S, \tag{5.3}$$

or

$$u_1 = 0, \qquad u_2 = 0, \quad \text{on} \quad \partial S, \tag{5.4}$$

where ∂S denotes the boundary of the plate.

We consider the basic equations of the large deflection theory of viscoelastic anisotropic plates:

$$\begin{aligned} D_{\alpha\beta\gamma\delta} w_{,\alpha\beta\gamma\delta} &= q + h \in_{\alpha\gamma} \in_{\beta\delta} w_{,\alpha\beta} F_{,\gamma\delta}, \\ K_{\alpha\beta\gamma\delta} F_{,\alpha\beta\gamma\delta} &= \frac{1}{2} \in_{\alpha\gamma} \in_{\beta\delta} w_{,\alpha\delta} w_{,\beta\gamma}. \end{aligned} \tag{5.5}$$

Applying, formally, Laplace transformation, we arrive at:

$$\tilde{D}_{\alpha\beta\gamma\delta} \tilde{w}_{,\alpha\beta\gamma\delta} = \tilde{q} + h \in_{\alpha\gamma} \in_{\beta\delta} \widetilde{w_{,\alpha\beta} F_{,\gamma\delta}}, \tag{5.6}$$

$$\tilde{K}_{\alpha\beta\gamma\delta} \tilde{F}_{,\alpha\beta\gamma\delta} = \frac{1}{2} \in_{\alpha\gamma} \in_{\beta\delta} \widetilde{w_{,\alpha\delta} w_{,\beta\gamma}}, \tag{5.6}$$

where the Laplace transformation, as denoted by tildas, is applied to the whole of the products of nonlinear terms and not to single terms separately.

Denoting Green functions of the left-hand sides of (5.6) with appropriate boundary conditions by $\tilde{G}_1$, $\tilde{G}_2$, we obtain

$$\begin{aligned} \tilde{w} &= \iint\limits_s \tilde{G}_1(x_1 - \xi_1, x_2 - \xi_2, p)(\tilde{q} + h \in_{\alpha\gamma} \in_{\beta\delta} \widetilde{w_{,\alpha\beta} F_{,\gamma\delta}}) d\xi_1 \, d\xi_2, \\ \tilde{F} &= \frac{1}{2} \iint\limits_s \tilde{G}_2(x_1 - \xi_1, x_2 - \xi_2, p) \in_{\alpha\gamma} \in_{\beta\delta} \widetilde{w_{,\alpha\delta} w_{,\beta\gamma}} d\xi_1 \, d\xi_2. \end{aligned} \tag{5.7}$$

Using the convolution theorem, we find that

$$\begin{aligned} w &= \int\limits_0^t \iint\limits_s G_1(x_1 - \xi_1, x_2 - \xi_2, t - \tau)(q + h \in_{\alpha\gamma} \in_{\beta\delta} w_{,\alpha\beta} F_{,\gamma\delta}) d\xi_1 \, d\xi_2 \, d\tau, \\ F &= \frac{1}{2} \int\limits_0^t \iint\limits_s G_2(x_1 - \xi_1, x_2 - \xi_2, t - \tau) \in_{\alpha\gamma} \in_{\beta\delta} w_{,\alpha\delta} w_{,\beta\gamma} d\xi_1 \, d\xi_2 \, d\tau. \end{aligned} \tag{5.8}$$

Thus we have obtained a system of nonlinear integro-differential equations for large deflections of viscoelastic anisotropic plates.

This system can be solved by the method of successive approximation. As the first approximation we take the linear solution for

$$w_1 = \int\limits_0^t \iint\limits_s G_1(x_1 - \xi_1, x_2 - \xi_2, t - \tau) q(\xi_1, \xi_2, \tau) d\xi_1 \, d\xi_2 \, d\tau. \tag{5.9}$$

Then the first approximation of F is given by the formula:

$$F_1 = \frac{1}{2} \int\limits_0^t \iint\limits_s G_2(x_1 - \xi_1, x_2 - \xi_2, t - \tau) \in_{\alpha\gamma} \in_{\beta\delta} w_{1,\alpha\delta} w_{1,\beta\gamma} d\xi_1 \, d\xi_2 \, d\tau. \tag{5.10}$$

Hence the second approximation can be written in the form:

(5.11)
$$w_2 = \int_0^t \iint_s G_1(x_1-\xi_1, x_2-\xi_2, t-\tau)(q+h\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}w_{1,\alpha\beta}F_{1,\gamma\delta})d\xi_1 d\xi_2 d\tau,$$
$$F_2 = \frac{1}{2}\int_0^t \iint_s G_2(x_1-\xi_1, x_2-\xi_2, t-\tau)\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}w_{2,\alpha\delta}w_{2,\beta\gamma}d\xi_1 d\xi_2 d\tau.$$

Continuing this procedure, we find that the n-th approximation is given by

(5.12)
$$w_n = \int_0^t \iint_s G_1(x_1-\xi_1, x_2-\xi_2, t-\tau)(q+h\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}w_{n-1,\alpha\beta}F_{n-1,\gamma\delta})d\xi_1 d\xi_2 d\tau,$$
$$F_n = \frac{1}{2}\int_0^t \iint_s G_2(x_1-\xi_1, x_2-\xi_2, t-\tau)\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}w_{n,\alpha\delta}w_{n,\beta\gamma}d\xi_1 d\xi_2 d\tau.$$

In the case of the homogeneous relaxation spectrum, the constitutive equations (2.2) can be written in them form:

(5.13)
$$H\left(\frac{\partial}{\partial t}\right)\sigma^{ij} = E^{iJkl}E\left(\frac{\partial}{\partial t}\right)\varepsilon_{kl},$$

where H, E are scalar polynomials in $\frac{\partial}{\partial t}$, and E^{iJkl} is a tensor of elastic moduli.

Thus the equations for large deflections of viscoelastic plates become

(5.14)
$$D_{\alpha\beta\gamma\delta}E\left(\frac{\partial}{\partial t}\right)w_{,\alpha\beta\gamma\delta} = H\left(\frac{\partial}{\partial t}\right)(q+h\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}w_{,\alpha\beta}F_{,\gamma\delta}),$$
$$K_{\alpha\beta\gamma\delta}H\left(\frac{\partial}{\partial t}\right)F_{,\alpha\beta\gamma\delta} = \frac{1}{2}\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}E\left(\frac{\partial}{\partial t}\right)w_{,\alpha\delta}w_{,\beta\gamma}.$$

Then the corresponding integro-differential equations assume the form:

(5.15)
$$E\left(\frac{\partial}{\partial t}\right)w = \iint_s G_1(x_1-\xi_1, x_2-\xi_2)H\left(\frac{\partial}{\partial t}\right)(q+h\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}w_{,\alpha\beta}F_{,\gamma\delta})\,d\xi_1 d\xi_2,$$
$$H\left(\frac{\partial}{\partial t}\right)F = \frac{1}{2}\iint G_2(x_1-\xi_1, x_2-\xi_2)\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}E\left(\frac{\partial}{\partial t}\right)w_{,\alpha\delta}w_{,\beta\gamma}d\xi_1 d\xi_2,$$

where G_1, G_2 are Green functions of the corresponding elastic plate. This system of integro-differential equations can also be solved by the method of successive approximation. The n-th approximation is then given by the formulae:

(5.16)
$$E\left(\frac{\partial}{\partial t}\right)w_n = \iint_s G_1(x_1-\xi_1, x_2-\xi_2)H\left(\frac{\partial}{\partial t}\right)(q+\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}w_{n-1,\alpha\beta}F_{n-1,\gamma\delta})d\xi_1 d\xi_2,$$
$$H\left(\frac{\partial}{\partial t}\right)F_n = \frac{1}{2}\iint_s G_2(x_1-\xi_1, x_2-\xi_2)\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}E\left(\frac{\partial}{\partial t}\right)w_{n,\alpha\delta}w_{n,\beta\gamma}d\xi_1 d\xi_2.$$

The method of successive approximation appears a convenient method also for numerical solution of the problem. The convergence of this method will be analysed in an another paper.

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