EXISTENCE AND UNIQUENESS THEOREMS FOR CUSPED POROUS ELASTIC PRISMATIC SHELLS IN THE ZERO APPROXIMATION OF THE HIERARCHICAL MODELS

N. Chinchaladze[1]

[1] *I.Vekua Institute of Applied Mathematics & Faculty of Exact and Natural Sciences of I. Javakhishvili Tbilisi State University, Tbilisi, Georgia*

e-mail: natalia.chinchaladze@tsu.ge

In [1] hierarchical models for porous elastic and viscoelastic Kelvin-Voigt prismatic shells on the basis of linear theories is constructed. Using I. Vekuas dimension reduction method ([2,3]), governing systems are derived and in the Nth approximation boundary value problems are set. The ways of investigation of boundary value problems (BVPs), including the case of cusped prismatic shells, are indicated and some preliminary results are presented.

Let us consider prismatic shells occupying 3D domain Ω with the projection ω (on the plane $x_3 = 0$) and the face surfaces

$$x_3 = \overset{(+)}{h}(x_1, x_2) \in C^2(\omega) \cap C(\overline{\omega}) \text{ and } x_3 = \overset{(-)}{h}(x_1, x_2) \in C^2(\omega) \cap C(\overline{\omega}), \quad (x_1, x_2) \in \omega.$$

In what follows we assume that

$$2h(x_1, x_2) := \overset{(+)}{h}(x_1, x_2) - \overset{(-)}{h}(x_1, x_2) = \begin{cases} > 0, & \text{for } (x_1, x_2) \in \omega, \\ \geq 0, & \text{for } (x_1, x_2) \in \partial\omega \end{cases}$$

is the thickness of the prismatic shell. Prismatic shells are called cusped shells if a set γ_0, consisting of $(x_1, x_2) \in \partial\omega$ for which $2h(x_1, x_2) = 0$, is not empty (see, e.g., Figures 1, 2).

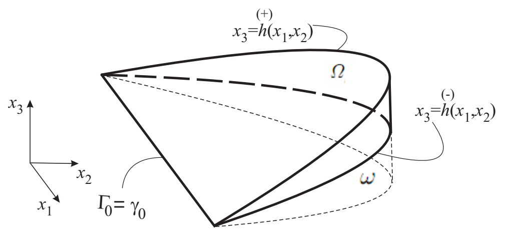

Figure 1: A sharp cusped prismatic shell with a semicircle projection. $\partial\Omega$ is a Lipschitz boundary

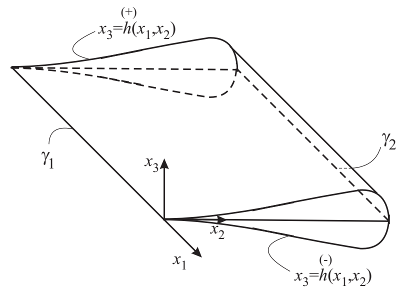

Figure 2: A cusped plate with sharp γ_1 and blunt γ_2 edges, $\gamma_0 = \gamma_1 \cup \gamma_2$. $\partial\Omega$ is a non-Lipschitz boundary

The governing system of porous elastic prismatic shells in case of zero approximation has the following form (see [1])

(1) $$(\mu h v_{\alpha 0,\beta})_{,\alpha} + (\mu h v_{\beta 0,\alpha})_{,\alpha} + (\lambda h v_{\gamma 0,\gamma})_{,\beta} + (bh\psi_0)_{,\beta} + \overset{0}{X}_\beta = \rho h \ddot{v}_{\beta 0}, \quad \beta = 1, 2;$$

(2) $$(\mu h v_{30,\alpha})_{,\alpha} + \overset{0}{X}_3 = \rho h \ddot{v}_{30};$$

(3) $$(\tilde{\alpha} h \psi_{0,\alpha})_{,\alpha} - bhv_{\gamma 0,\gamma} - \xi h \psi_0 + \overset{0}{H} = \rho \ddot{\varphi}_0 - \mathcal{F}_0,$$

where λ, μ, $\tilde{\alpha}$, b, ξ are the constitutive coefficients, ρ is the reference mass density, $v_{i0} := \dfrac{u_{i0}}{h}$ $(i = 1, 2, 3)$, $\psi_0 := \dfrac{\varphi_0}{h}$, u_{i0} and φ_0 are the zero moments of the displacements vector components and of the changes of the volume fraction from the matrix reference volume fraction, correspondingly, $\overset{0}{X}_i$ $(i = 1, 2, 3)$, $\overset{0}{H}$, F_0 are given functions (see [1]). The points as superscripts mean differentiation with respect to the time, and Einsteins summation convention is used; indices after comma mean differentiation with respect to the corresponding variables of the Cartesian frame $Ox_1x_2x_3$.

Let the thickness is given by the following expression

$$2h(x_1, x_2) = h_0 x_2^{\kappa}, \quad x_2 \in [0, l] \quad h_0,\ \kappa, l = const > 0.$$

In case of harmonic vibration with the oscillation frequency ϑ system (1)-(3) can be rewritten as follows

$$\begin{aligned}
&-\mu\Big[(h\overset{\circ}{u}_{\alpha,\beta})_{,\alpha} + (h\overset{\circ}{u}_{\beta,\alpha})_{,\alpha}\Big] - \lambda(h\overset{\circ}{u}_{\gamma,\gamma})_{,\beta} - b(h\overset{\circ}{u}_4)_{,\beta} - \rho h\vartheta^2 \overset{\circ}{u}_\beta = F_\beta, \quad \beta = 1, 2,\\
(4)\qquad &-\mu(h\overset{\circ}{u}_{3,\alpha})_{,\alpha} - \rho h\vartheta^2 \overset{\circ}{u}_{30} = F_3,\\
&-\tilde{\alpha}(h\overset{\circ}{u}_{4,\alpha})_{,\alpha} + bh\overset{\circ}{u}_{\gamma,\gamma} + \xi h\overset{\circ}{u}_4 - \rho h\vartheta^2 \overset{\circ}{u}_4 = F_4,
\end{aligned}$$

where

$$\overset{\circ}{u}_i := \overset{\circ}{v}_{i0}, \quad \overset{\circ}{u}_4 := \overset{\circ}{\psi}_0, \quad F_i := \overset{\circ}{X}_i, \quad F_4 := \overset{\circ}{H} + \mathcal{F}_0,$$

$$\overset{\circ}{H} + \mathcal{F}_0 = e^{\iota\vartheta t} F(x_1, x_2), \quad \overset{\circ}{X}_3 = e^{\iota\vartheta t} \overset{\circ}{X}{}^0{}_3(x_1, x_2), \quad v_{30} = e^{\iota\vartheta t} \overset{\circ}{v}_{30}(x_1, x_2), \quad \psi_0 = e^{\iota\vartheta t} \overset{\circ}{\psi}_0(x_1, x_2).$$

BCs for the weighted displacements and the weighted volume fraction are nonclassical in the case of cusped prismatic shells. Namely, we are not always able prescribe them at cusped edges.

For the particular case of deformation when

$$\overset{\circ}{u}_\alpha \equiv 0, \quad \alpha = 1, 2; \quad \overset{\circ}{u}_3,\ \overset{\circ}{u}_4 \not\equiv 0$$

the following theorem can be proved

Theorem. *If*

$$\xi - \rho\vartheta^2 \geq 0,$$

(i) $\kappa < 1$, the Dirichlet problem (find $\overset{\circ}{u}_3,\ \overset{\circ}{u}_4 \in C^2(\omega) \cap C(\bar{\omega})$ by their values prescribed on $\partial\omega$) is well-posed;

(ii) if $\kappa \geq 1$, the Keldysh problem (find bounded $\overset{\circ}{u}_3,\ \overset{\circ}{u}_4 \in C^2(\omega) \cap C(\omega \cup (\partial\omega \setminus \overline{\gamma}_0))$ by their values prescribed only on the arc $\partial\omega \setminus \overline{\gamma}_0$) is well-posed.

The talk is devoted to the homogeneous Dirichlet problem for the general system (4). The classical and weak setting of the problem are formulated. For arbitrary $\kappa \geq 0$, we introduce appropriate function spaces X^κ, which are crucial in our analysis. We show coerciveness of the corresponding bilinear form and prove uniqueness and existence results for the variational problem. We describe in detail the structure of the spaces X^κ and establish their connection with weighted Sobolev spaces. Moreover, we give some sufficient conditions for a linear functional arising in the right-hand side of the variational equation to be bounded.

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References

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