PROBABILISTIC SOLUTIONS OF THE STRETCHED BEAM SYSTEMS FORMULATED BY FINITE DIFFERENCE SCHEME AND EXCITED BY FILTERED GAUSSIAN WHITE NOISE

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1. Abstract

The multi-degree-of-freedom (MDOF) and nonlinear stochastic dynamical (NSD) system about the nonlinear random vibration of the stretched beam is formulated by finite difference scheme. The effectiveness and efficiency of state-space-split (SSS) method [1,2] and exponential-polynomial-closure (EPC) method [3] are studied in analyzing the probability density functions of responses of the formulated systems which are excited by filtered Gaussian white noises. The Kanai-Tajimi seismic ground acceleration is adopted as the filtered Gaussian white noise in numerical analysis. Numerical results are obtained about the probabilistic solutions of the beam with pin supports at its two ends and excited by the filtered Gaussian white noise which is uniformly distributed over the beam or concentrated in the middle of the beam. The numerical analyses show that the SSS-EPC method works well for accurately and efficiently analyzing the probabilistic solutions of the stretched Euler-Bernoulli beam excited by distributed filtered Gaussian white noise when the MDOF-NSD system is formulated by finite difference scheme.

2. Nonlinear stochastic dynamical system of stretched beam

Consider the stretched beam and its finite difference discretization shown by Figure 1.

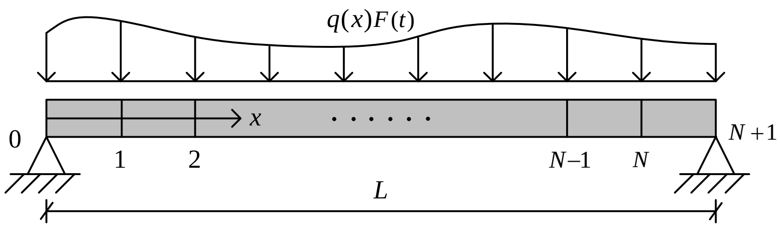

Figure 1: The stretched beam discretized by finite difference scheme

The equation of motion of the beam is

(1) $$\rho\ddot{Y}(x,t)+c\dot{Y}(x,t)+EIY^{(4)}(x,t)-\frac{EA}{2L}Y''(x,t)\int_0^L Y'^2(x,t)dx=qF(t)$$

where $Y(t)$ is the deflection of beam at time t and at the location with distance x to the left-hand end of the beam; ρ is the mass density of material; c is the damping constant; E is the Young's modulus of beam material; I is the moment inertia of cross section of the beam; A is the area of cross section of the beam; L is the beam length; $q(x)F(t)$ is the distributed loading laterally applied on the beam. By finite difference scheme as shown in Figure. 1, Equation (1) can be discretized into the following system.

(2) $$\ddot{Y}_k+\frac{c}{\rho}\dot{Y}_k+\alpha(Y_{k+2}-4Y_{k+1}+6Y_k-4Y_{k-1}+Y_{k-2})-\beta(Y_{k+1}-2Y_k+Y_{k-1})\sum_{i=1}^{N+1}(Y_{i+1}^2+Y_i^2$$
$$+Y_{i-1}^2+Y_{i-2}^2+Y_{i+1}Y_i-2Y_{i+1}Y_{i-2}-Y_{i+1}Y_{i-2}-Y_iY_{i-1}-2Y_iY_{i-2}+Y_{i-1}Y_{i-2})=\frac{q_k}{\rho}F(t)$$

(3) $$\ddot{Z}(t)+2\varsigma\omega_0\dot{Z}(t)+\omega_0^2Z(t)=W(t)$$

where $k=1, 2,\ldots,N$; $F(t)=2\varsigma\omega_0\dot{Z}(t)+\omega_0^2 Z(t)$; $\alpha = EI/(h^4\rho)$; $\beta = EA/(24Lh^3\rho)$, $h = L/(N+1)$; N is the number of unknowns in finite difference scheme; Y_k is the deflection of beam at node k; q is a constant reflecting the distributed load density; ς and ω_0 are the parameters in the filter (3); $W(t)$ is the Gaussian white noise with power spectral density S.

3. Numerical results

Based on Equations (2) and (3), the probability density functions (PDFs) of deflections and velocities at the nodes are analyzed. The polynomial degree n equals 4 in EPC solution procedure. The given values of system parameters are $E = 2.1\times10^{11} Pa$, $L = 7m$, $A = 8.61\times10^{-3} m^2$, $I = 2.17\times10^{-4} m^4$, $c = 10^3 Ns/m$. $\rho = 7.850 kg/m^3$, $\varsigma = 0.3$, $\omega_0 = 50 rad/s$, $S = 0.05 m^2/s$ and $q_k = 50{,}000 kg/m$. For $N = 11$, Equations (2) and (3) formulate a 12-DOF system. In this case, the node 6 is in the middle of the beam. The PDFs and logarithm of PDFs of Y_6 obtained by SSS-EPC, Monte Carlo simulation (MCS), and equivalent linearization (EQL), respectively, are shown and compared in Figure. 2. The sample size in MCS is 10^8. In Figure 2, σ_{y_6} is the standard deviation of Y_6 obtained by EQL method. It is seen that the results obtained by SSS-EPC method are close to MCS while those obtained by EQL deviate a lot from MCS. The behaviors of the probabilistic solutions at the other nodes are similar to those at node 6. The computational time needed by MCS is about 500 times more than that needed by the SSS-EPC method for this 12-DOF system. The value of this ratio can further increase as the number of system degrees of freedom or samples in MCS increases. As the system nonlinearity increases, the required sample size and therefore the computational effort also increase with MCS.

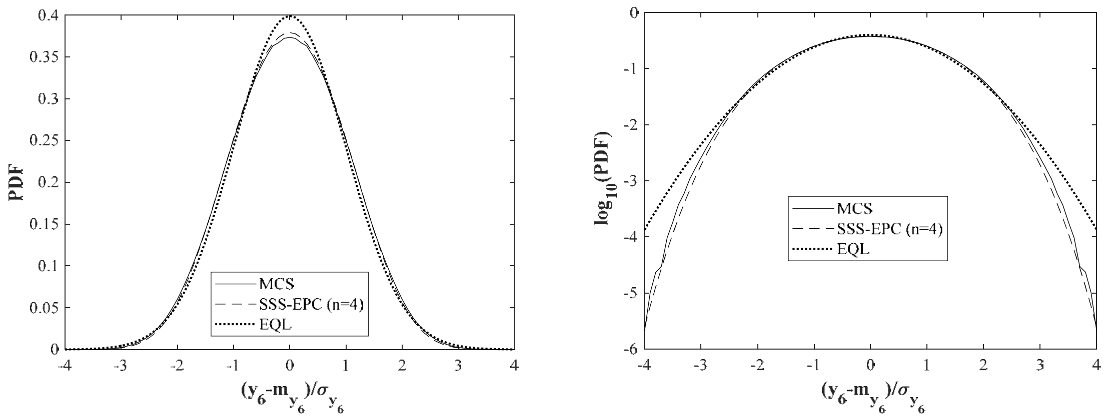

Figure 2: PDFs and logarithm of PDFs of the deflection in the middle of beam

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