STRAIN SOFTENING COSSERAT PLASTICITY FOR FGM SHELLS IN NONLINEAR 6-PARAMETER SHELL THEORY

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1. Introduction

Decreasing of strength during growing deformation in solids in known as strain softening phenomena, which can occur in materials like concrete, rock, ceramics etc. [1]. Strain softening plasticity, along with brittle damage, are two possible ways to predict ultimate behaviour of enlisted materials. Plates and shells made of Functionally Graded Materials (FGM) are usually composed from ceramics and metal constituents. During extreme loading, some cracks may occur in ceramics layer, then ductile metal constituent is responsible for stopping their development into structure depth. While a lot of research was done in the topic of elastic behaviour of FGM plates and shells investigations to cover materially nonlinear FGM shell structures deformations are limited. As a rule the TTO mixture model was applied (e.g. [2,3,4]), which assumes elasto-plastic material law with linear hardening and ceramics is considered as perfectly elastic constituent, without any limit stress defined to enrich its material law. In this study, the elasto-plastic model with strain softening attributed to ceramic constituent is proposed, as a possible way to simulate brittle behaviour. Since strain softening exhibits strong mesh dependence in numerical calculations with classic FEM codes, it is reasonable and appropriate to implement present idea in code based on Cosserat type kinematics, where drilling rotation and its' stiffness expressed by the characteristic length l have influence on regularization (reduction of mesh dependence) of result [5,6]. All present calculations are carried out in authors own FEM code named CAM, written in Fortran language.

2. Material law

It is assumed that shell is built from two constituents: the first is ceramic and the second metallic type. Material mixture proportions vary through the thickness of the shell with well-known power law. At each point of the shell reference surface Cosserat plane stress is assumed

$$(1)\qquad \left\{\begin{array}{c}\sigma_{aa}\\ \sigma_{bb}\\ \sigma_{ab}\\ \sigma_{ba}\\ \hline m_a\\ m_b\end{array}\right\}=\left[\begin{array}{cccc:cc} Ea_1 & Ea_2 & 0 & 0 & 0 & 0\\ Ea_2 & Ea_1 & 0 & 0 & 0 & 0\\ 0 & 0 & G+k & G-k & 0 & 0\\ 0 & 0 & G-k & G+k & 0 & 0\\ \hdashline 0 & 0 & 0 & 0 & 2Gl^2 & 0\\ 0 & 0 & 0 & 0 & 0 & 2Gl^2\end{array}\right]\left\{\begin{array}{c}\varepsilon^{aa}\\ \varepsilon^{bb}\\ \varepsilon^{ab}\\ \varepsilon^{ba}\\ \hline \kappa_a\\ \kappa_b\end{array}\right\},\quad a_1=\frac{1}{1-\nu^2},\quad a_2=\nu a_1,\quad a,b=1,2$$

with $k = N^2/(1-N^2)G$ where $0 < N < 1$ is the so-called coupling number. All material parameters in through-the-thickness direction follow simple rule of mixture $X = V_cX_c + V_mX_m$, where V_i denotes constituents relative volume in shell layer, subscripts c and m refer to ceramic and metal parameters. Each of materials is assumed to be elasto-plastic with piece-wise linear hardening (m) or softening rule (c). Plastic flow theory, based on J_2 plasticity extended to Cosserat continua is applied [5]. Closest Point Projection Method is used to numerical integration of plasticity equations at single integration point.

3. Numerical example

Consider FGM plate with initial imperfect geometry. Parametric study of similar plate was conducted in [3] with TTO mixture model. Dimensions of the plate are defined as $a = 111.125$, $b = 127$, $h = 3.175$. In Fig. 1. geometry, load, mesh (division into 16-node CAM finite elements) is shown. In present study double

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symmetry is taken into account. Here, metal constituent is assumed to be perfectly elasto-plastic, with material data $E_m = 206200$, $\nu_m = 0.3$, $l_m = 0.0001$, $H_m = 0$, $\sigma_{Ym} = 250$. Ceramics is assumed as strain softening elasto-plastic with $E_c = 340000$, $\nu_c = 0.25$, $l_c = 0.0001$ or $l_c = 0.01$. Softening curve, defined as pairs of parameters (σ_Y, ε_{eff}) with ε_{eff} as the effective plastic strain, is displayed in Fig 1. When $\varepsilon_{eff} \geq 0.02$, yield stress remains constant $\sigma_Y = 50$. Thickness integration is performed with 7 point Gauss-Lobatto rule. Graph in Fig. 1 shows equilibrium paths. Values on vertical axis are an average compression stress normalized with $S_Y = \sigma_Y$ while on horizontal axis displacement of loaded edge is shown. Equilibrium paths reveal that mesh refinement do not affect limit load, differences however are visible in global softening curves after limit point is passed. There is a small effect of ceramics characteristic length visible in the finest mesh. Contour plot of effective plastic strain shows localization phenomena, typical for strain softening, in which plastified zones are narrow and divide structure into elastic subregions.

Example proves capability of the author's CAM code to analyze deformations in FGM structures with novel strain softening ceramics behaviour description. Limit load and further course of deformation could be investigated, with characteristic for strain softening localization phenomena.

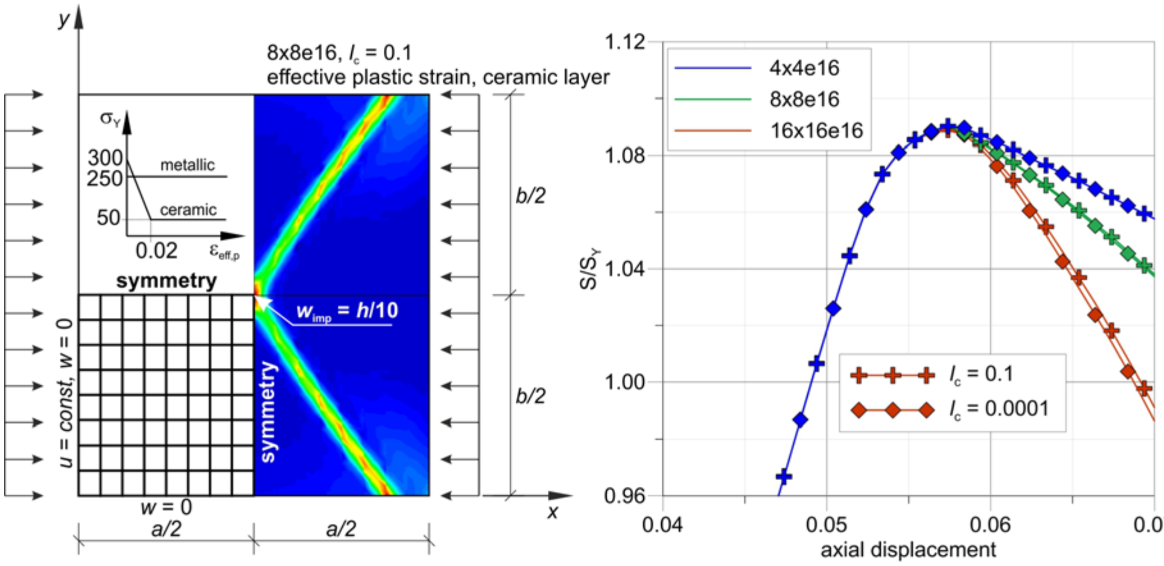

Fig. 1. FGM plate under in-plane load: geometry, discretization, equilibrium paths.

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