ON SOME PROPERTIES OF BONE FUNCTIONAL ADAPTATION PHENOMENON USEFUL IN MECHANICAL DESIGN

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1. Introduction

Contemporary design methods include optimization procedures on each of design stage. In case of structural design the optimization assists the engineers from the earliest design idea up to the end of the design process. In case of the living entities all kind of the optimization must be simultaneous. The example of such simultaneous adaptation is the phenomenon of the trabecular bone remodeling process.

2. Trabecular bone remodeling process

There are many models of bone remodeling. In the Huiskes' 'regulatory model' [1] conception based on clinical observation of trabecular bone tissue behavior, the main assumption of this model is existence of homeostasis (perfect balance between bone gain and loss). This equilibrium can occur only in presence of mechanical stimulation. The network of osteocytes plays the role of sensors of the mechanical energy distribution along the trabecular bone tissue. The model used here postulates strain energy density (SED) on the surface of trabecular bone, as a scalar measure of mechanical stimulation and distinguished value of SED, corresponding to bone remodeling homeostasis.

3. The Principle of the Constant Strain Energy Density

The notable assumption of the presented model is existence of the homeostasis of the remodeling process described by the distinguished value of SED. It is interesting, that SED, as a energy measure, is also emphasized in optimization research, distant from biomechanical applications. In Pedersen's [2] considerations the optimal shape of the structure, minimizing the strain energy is thought.
The derivative of the total potential Π with respect to an arbitrary parameter h is:

(1) $$\frac{d\Pi}{dh} = \frac{\partial U_\varepsilon}{\partial h}$$

For a local design parameter h_e that only changes the design in the domain e of the structure:

(2) $$\frac{\partial U_\varepsilon}{\partial h_e} = \frac{\partial \left(\bar{u}_e V_e\right)}{\partial h_e}$$

where $\bar{u}_e$ is the mean strain energy density in the domain of, and V_e is the corresponding volume.
Assuming two parameters h_i, h_j, the total volume V of the structure is:

(3) $$\Delta V = \frac{dV}{dh_{\mathrm{i}}}\Delta h_{\mathrm{i}} + \frac{dV}{dh_{\mathrm{j}}}\Delta h_{\mathrm{j}} = 0$$

then the increment of the elastic energy:

(4) $$\Delta U_{\varepsilon} = \frac{dU_{\varepsilon}}{dh_{\mathrm{i}}}\Delta h_{\mathrm{i}} + \frac{dU_{\varepsilon}}{dh_{\mathrm{j}}}\Delta h_{\mathrm{j}}$$

for design independent loads, and when only the local energies are involved:

(5) $$\Delta U_{\varepsilon} = \bar{u}_i \frac{dV_i}{dh_i}\Delta h_i + \bar{u}_j \frac{dV_j}{dh_j}\Delta h_j$$
$$\Delta U_{\varepsilon} = -(\bar{u}_i - \bar{u}_j)\frac{dV_i}{dh_i}\Delta h_i$$

and with the constant volume assumption the a necessary condition for optimality $\Delta U = 0$ with constraint $\Delta V = 0$ leads to the conclusion, that the strain energy densities must be equal. Similar, with all design parameters, the total energy change equation leads to the conclusion, that a necessary condition for optimality is constant value of the strain energy density. Thus for the stiffest design the energy density along the shape to be designed must be constant:

(6) $$u_s = \text{const.}$$

4. Conclusions

From our resent research in the area of numerical simulation of bone remodeling phenomenon and studies on structural optimization, the astonishing conclusions can be formulated. The bone remodeling phenomenon is a biological realization of the optimal structure principle, requiring equal value of surface energy distribution. On the other hand, the optimization scenario based on the osteoclasts, osteoblasts activity and osteocytes mechanosensitivity assumption leads to the optimization results identical to these obtained by traditional optimization methods based on the minimal potential assumption [3].

6. References

[1] Huiskes R. et al. (2000). Effects of mechanical forces on maintenance and adaptation of form in trabecular bone, *Nature* **404**, pp. 704-706.

[2] Pedersen P., (2003). Optimal Design - Structures and Materials - Problems and Tools Book ISBN 87-90416-06-6.

[3] Nowak. M, (2006). Structural optimization system based on trabecular bone surface adaptation, *Journal of Structural and Multidisciplinary Optimization*, Springer Berlin/ Heidelberg, Volume **32**, Number 3, pp. 241-249.