EVOLUTIONARY IDENTIFICATION OF LAMINATES' STOCHASTIC PARAMETERS

***W. Beluch*[1], *T. Burczyński*[1,2] *and P. Orantek*[1]**
[1]*Department for Strength of Materials and Computational Mechanics, Silesian University of Technology, Gliwice, Poland*
[2]*Institute of Computer Modelling, Cracow University of Technology, Cracow, Poland*

1. Introduction

Laminates are a group of fibre-reinforced composites made of many stacked and permanently joined layers (plies). Laminates have great strength/weight ratio and it is possible to obtain desired properties of laminate by manipulating the components materials, stacking sequence, fibers orientation and layer thicknesses.

The aim of the paper is to identify material constants in multi-layered, fibre-reinforced laminates. Simple and hybrid (with laminas made of different materials) laminates are considered. The hybrid laminates are in a form of interply hybrids, with plies composed of two different materials [1]. The internal layers are made of a low-strength and less expensive material while the outer layers are made of a more expensive but better material.

Usually, laminates can be treated as orthotropic thin plates with four independent elastic constants: axial Young's modulus E_1, transverse Young's modulus E_2, axial-transverse shear modulus G_{12} and axial-transverse Poisson ratio ν_{12}.

2. Formulation of the stochastic identification problem

A non-linear stochastic optimization problem is a searching for a random vector [4]:

$$\mathbf{X}(\gamma) = [X_1(\gamma),\ X_2(\gamma), ..., X_i(\gamma), ...,\ X_n(\gamma)] \tag{1}$$

which minimizes the objective function $F(\gamma) = F[\mathbf{X}(\gamma)]$ and satisfies the constraints:

$$P\left[g_j(\mathbf{X}) \geqslant 0\right] \geqslant p_j, j = 1, 2, ..., m \tag{2}$$

where: $(\Gamma, \mathfrak{F}, P)$ - the probability space; Γ - the space of elementary events; $\mathfrak{F}$ - σ-algebra of subset of the set Γ; P - the probability defined on $\mathfrak{F}$.

In the present paper evolutionary algorithm (EA) is used as the optimization method [2]. The vector $\mathbf{X}(\gamma)$ (chromosome) consists of random genes. Each gene is represented by a random variable. It is also assumed that each gene has a n-dimensional Gaussian distribution function and that random genes are independent random variables. Eventually, the original stochastic problem can be reduced to the deterministic one. Random chromosome $\mathbf{X}(\gamma)$ is replaced by a deterministic chromosome $ch(\mathbf{x})$. Each gene x_i is a stochastic variable represented by a mean value m_i and a standard deviation σ_i [3].

Identification can be treated as the minimization of the objective function F with respect to the vector of the design variables $\mathbf{x}$:

$$\min : \left[F(\mathbf{x}) = \sum_{k=1}^{N} \left|\frac{\hat{\mathbf{q}}_k - \mathbf{q}_k}{\hat{\mathbf{q}}_k}\right|\right] \tag{3}$$

where: $\mathbf{x} = (x_k)$ - the parameters representing the identified constants; $\hat{\mathbf{q}}_\mathbf{k}$ - the measured values of state fields; $\mathbf{q}_\mathbf{k}$ - the values of the same state fields calculated from the solution of the direct problem; $k = 1..N$, N - the number of sensor points.

The **x** vector has the form: i) for simple laminates: $\mathbf{x} = (E_1, E_2, G_{12}, \nu_{12})$; ii) for hybrid laminates: $\mathbf{x} = (E_1^1, E_2^1, G_{12}^1, \nu_{12}^1, \rho^1, E_1^2, E_2^2, G_{12}^2, \nu_{12}^2, \rho^2)$ (superscripts specify the material number).

Direct problems for the identification tasks are solved by means of a finite element method software (MSC.PATRAN/NASTRAN). To reduce the number of sensor points, the modal analysis methods are employed. In present paper the eigenfrequencies are used as the measurement data. The numbers of plies, their thicknesses, fibres orientation and the number of layers made of each material are assumed to be known. External layers hybrid laminates are made of material M_e and the core layers are made of material M_i. The number of layers made of each material is also known.

3. Numerical example

A rectangular simple laminate 0.5x0.2m with one of shorter sides fixed is made of the glass-epoxy. Each ply of the symmetrical laminate has the same thickness h_i=0.002m. The stacking sequence of the symmetrical laminate is: (0/45/90/-45/0/90/0/90)s. The plate is divided into 200 4-node plane finite elements. The first 10 eigenfrequencies of the plate are the measurement data. It is assumed that measurements are random variables with the Gaussian distribution. The measurements were repeated 200 times to collect data. The population in EA consists of 200 chromosomes of 4 genes each. The identification results after 1000 generations are collected in Table 1.

	E_1 [Pa]		E_2 [Pa]		ν_{12}		G_{12} [Pa]	
	m	σ	m	σ	m	σ	m	σ
Min	2.00E10	0.00E9	4.00E9	0.00E9	0.00	0.00	2.00E9	0.10E8
Max	6.00E10	0.30E9	9.00E9	0.30E9	0.50	0.10	6.00E9	0.70E8
Actual	3.86E10	0.12E9	8.28E9	0.20E9	0.26	0.02	4.14E9	0.50E8
Found	3.92E10	0.11E9	8.14E9	0.17E9	0.27	0.04	4.07E9	0.22E8

Table 1. A simple laminate - identification results.

4. Final conclusions

An identification method based on the stochastic representation of the identified parameters has been presented. The Evolutionary Algorithm has been employed to solve the identification task for simple and hybrid laminates. Positive identification results have been obtained for both kinds of laminates.

5. Acknowledgments

The research is partially financed from the Polish science budget resources as the research project and the Foundation for Polish Science (2005-2008).

6. References

[1] S. Adali et.al. Optimal design of symmetric hybrid laminates with discrete ply angles for maximum buckling load and minimum cost, *Composite Structures*, **32**, 409–415, 1995.

[2] J. Arabas. *Lectures on Evolutionary Algorithms*, WNT, 2001 (in Polish).

[3] P. Orantek and T. Burczyński. The identification of stochastic parameters in mechanical structures, *CMM-2007 Conference Proceedings*, CD-Edition, Lodz-Spala, 2007.

[4] K. Sobczyk. *Stochastic Wave Propagation*. PWN, Warsaw, 1984.