FE-SIMULATIONS OF DYNAMIC SHEAR LOCALIZATION IN GRANULAR BODIES USING AN ARBITRARY LAGRANGIAN-EULERIAN FORMULATION

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1. Introduction

Algorithms of continuum mechanics usually use two descriptions of motion: the Lagrangian and Eulerian one [1]. In a pure Lagrangian formulation each individual node of the computational mesh is permanently connected to the same material points during motion. There are no convective effects and the material derivative reduces to a simple time derivative. Such formulation allows an easy tracking of free surfaces and interfaces between different materials, and treatment of materials with history-dependent constitutive relations. The formulation, however, is restricted to a certain deformation range because the element mesh may be severely distorted or entangled due to the fact that elements deform with the material. Thus, the FE-analysis usually looses accuracy, size of the time increment has to be significantly reduced or simply terminates due to convergence problems. A remeshing may not be even efficient. In a pure Eulerian formulation, the computational mesh is fixed spatially and the continuum moves with respect to the grid (elements retain their original shape). The convective effects appear because of the relative motion between the deforming material and computational grid, which makes the analysis computationally expensive. The formulation leads to difficulties when free surface conditions, prescribed boundary conditions or deformation history dependent material properties are considered as the element mesh is not connected to the material. In order to combine the advantages of both formulations and to minimize their limitations, an Arbitrary Lagrangian-Eulerian formulation (ALE, in short) has been developed [1], where state variables are a function of the referential coordinates (not connected to material points). In the ALE method, the mesh is neither connected to the material nor fixed to the spatial coordinate system (nodal displacements are uncoupled from material displacements) but it can be prescribed in an arbitrary manner. As a result, a mesh velocity has to be computed in order to compute the mesh. Grid points on the surface move with the mesh velocity, but these points must remain on the free surface. Since the mesh is not connected to the material, a remap of state variables has to be performed. The freedom in the mesh movement helps to handle greater distortions than would be allowed by a Lagrangian method with more resolution than that afforded by an Eulerian approach.

In our paper, a so-called uncoupled ALE-method was used [2], [3], where the deformation process was split into a pure Lagrangian and a pure Eulerian phase combined with a smoothing approach. This approach [2], [3] has some advantages with respect to the full coupled ALE-approach [1], where both nodal point and material values are calculated by solving a global assembled set of equations. The uncoupled approach simplifies this problem since the Lagrangian approach can be used and the stiffness matrix does not contain any convective terms. Thus, it is not necessary to describe the mesh velocity in a set of equations.

2. FE-analysis

Our dynamic FE-analysis was carried out with a non-local hypoplastic constitutive model, which is able to describe the essential properties of granular bodies during shear localization in a wide range of pressures and densities [4], [5]. Due to the presence of a characteristic length of micro-structure (by application of non-local terms), the model can simulate the formation of shear zones with a certain thickness and related size effects. It includes barotropy (dependence on pressure level), pycnotropy (dependence on density), dilatancy and contractancy and material softening during shearing of a dense material. This law describes the evolution of effective stress tensor with the evolution of rate of deformation tensor by isotropic linear and non-linear tensorial functions. In contrast to elasto-plastic models, a decomposition of deformation components into elastic and plastic parts, the formulation of a yield surface, plastic potential, flow rule and hardening rule is not needed. A characteristic length was taken into account in hypoplasticity by means of a non-local theory. It is enough to treat non-locally the local modulus of deformation rate to obtain mesh-independent FE-results [4], [5]. The constitutive relationship requires totally 9 material constants.

The calculations of shear localization were carried out for 2 different dynamic problems using an explicit FE-formulation: plane strain compression and confined granular flow in silos. First, the calculations were carried out for a plane strain compression test. The following parameters were varied: loading velocity, initial void ratio, characteristic length of micro-structure and specimen size. Attention was paid to a deterministic dynamic size effect and thickness of a shear zone inside the deformed granular specimen. Some comparative analysed were also carried out using a pure Lagrangian approach.

The results show that inertial forces influence the shear zone formation. The calculations with a slow loading velocity were compared with corresponding quasi-static laboratory tests performed at Karlsruhe University [6]. A satisfactory agreement was achieved between numerical and experimental results.

Second, the FE-studies were performed for granular silo flow in a bin and hopper with a controlled or free outlet velocity [7]. Both, the initial void ratio and wall roughness were varied. The wall roughness was described by different Coulomb wall friction. The FE-results were compared with corresponding laboratory tests [8], [9]. A good agreement with respect to the shape of propagating internal shear zones inside the flowing solid was obtained between experiments and calculations. An uncoupled ALE-approach enabled us to avoid large mesh distortions during flow at the silo outlet.

4. References

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