APPLICATION OF THE MULTISCALE FEM TO THE MODELING OF COMPOSITE MATERIALS

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The multiscale FEM is a numerical method based on the theory of homogenization, with the specific principle that real material properties have to be replaced by effective ones obtained by the examination of a RVE [6]. The terminology ”macro” relates to the examined body, while ”micro” relates to the RVE describing the material structure thereby the macroquantities are defined using the concept of the volume average and the coupling of the scales requires Hill's macrohomogeneity condition to be satisfied. Transformation of the latter condition leads to the definition of the boundary conditions at microscale and in that way to the closed formulation of the boundary value problem related to this level. The contribution examines materials with periodic and random microstructure, explaining three examples in detail.

The first example simulates the behavior of microporous nonlinear material. Here, a tension test of a plate is considered at macroscale, while a square RVE with an elliptical pore is chosen to describe the material properties. Given a random microstructure, the RVE is assumed to have a different orientation in each Gauss' point. The material investigation is illustrated by three groups of tests with different lengths of the semi-major axis of the pore. Each time, ellipticity is changed in an interval $[0, 1]$ where the lower limit corresponds to pores with zero thickness and the upper limit to circular ones. The results show that in the moment when pores appear, even if their thickness can be neglected, the material parameters decrease at once; furthermore, with pore growth, Young's and bulk modulus undergo a monotonous decrease while Poisson's ratio increases. Calculations also show that voids with elongated shape have a more significant influence on material weakening than voids whose shape is close to the circular one.

The second example looks at modeling solution-precipitation creep, which is a diffusional process occurring in polycrystals if pressure and temperature are in the specific range [3, 4, 5]. For this problem, firstly a continuum-mechanical model is proposed where the deformation is decomposed into an elastic and an inelastic part and the total power is written as a superposition of total elastic power and dissipation. The elastic energy is chosen in the standard form, dependent on the Helmholtz free energy, while the dissipated energy is formulated particularly for the process of solution-precipitation creep. It depends on the normal velocity of the crystal boundary due to precipitation or solution of material and on the velocity of material transport within the crystal interfaces. One of the main properties of this model is that the difference between the normal component of the Eshelby stress tensor and its smooth approximation becomes the driving force of the process. Such behavior is already endorsed by the experiments showing that under homogeneous pressure acting on one side of a rectangular crystal, solution-precipitation creep occurs only in edge zones of the sample. Another advantage of the proposed model is that in contrast to other procedures, continuity of stress in triple points is not required. Preliminary results for the behavior of polycrystals are obtained using the Taylor model and show that solution-precipitation creep leads to the elongation of the crystal shape. FEM-based methods are used for more realistic simulations and to estimate the change in effective material parameters over time. Here the most important simulations are those concerning materials with completely random structure and materials with regular structure consisting of hexagonal crystals.

The motivation behind developing the model for the RVE of cancellous bone, which is the last example presented in this contribution, is to investigate the process of osteoporosis, whose main indicators are the decrease and partial disappearance of the solid phase [1, 2]. The important feature of the model is that the presence of the fluid phase necessitates dynamic interrogation and analysis in the complex domain. According to the geometry of the microstructure it is assumed that the RVE has a cubic form and that it consists of the solid frame and of viscous blood marrow filling the core of the frame. The effective elasticity tensor and the parameters of materials with different microstructure are calculated as the final results at microscale. Comparison of the real parts of material parameters with the experimental results shows good agreement. The calculations at macrolevel are illustrated by simulating the ultrasonic test where the attenuation coefficient is calculated as a final result, using the ratio of amplitudes of particle oscillations. The obtained numerical values are much smaller than the experimental ones so that an improvement of the model of the RVE is envisaged. Two main ideas for overcoming the problem consist of assuming a new geometry of the solid phase of the RVE, and introducing wave scattering on the interface of the phases.

From the previous overview it can be seen that, although limited by the requirements concerning the size of the RVE, the multiscale FEM can still be applied to modeling composite materials with very diverse microstructures. The examples presented here confirm in particular that the method can be applied efficiently in modeling nonlinear materials with a regular structure and a random structure, which mostly exceeds the abilities of analytical solutions and other numerical methods.

1. References

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