INTERFACIAL THERMAL STRESS ANALYSIS OF AN ELLIPTICAL INCLUSION WITH AN IMPERFECT INTERFACE IN ANISOTROPIC PLANE

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1. General introduction

At the early stage on developing the analysis of inclusion problems, the bonding condition between the inclusion and the matrix is always considered perfectly bonded. However in the most real situation, the inclusion interfaces are not perfectly bonded at all especially as the temperature of the composite is in relatively higher level. This study provides the interfacial thermal stress analysis for the problems of an elliptical inclusion embedded in an anisotropic plane with imperfect interface. The thermal load we consider here is that the inclusion is subjected to a uniform temperature change. The analytical results which give the distributions of interfacial stresses are derived base on Stroh formalism [1] in conjunction with the techniques of using mapping functions. As to the imperfect interface, a spring-type model with vanishing thickness is applied such that we consider the interfacial tractions are continuous and the displacement jumps across the interface layer are in proportion to the traction components in their respective direction [2]. The non-negative interfacial parameter h_j, n, t or z, which is the ratio of the interfacial stress and the interfacial displacement jump in the normal, tangential or anti-plane direction, varies from zero to infinite value. The limiting value of interfacial parameters, i.e. h_j=0 or h_j=∞, imply a particular case which represents a completely debonded interface or a perfectly bonded interface, respectively. Therefore, our results can be applied to the most problems of all possible kinds of interfaces. Among the derivations of this study, due to the fact that the interfacial displacement jumps proportionally relate to the corresponding interfacial stresses, only using analytical continuation could lead to an unsolvable situation with expanding the solution on the inclusion domain into a complex Fourier series. To overcome this awkward situation, an idea of semi-inverse manipulation is introduced by virtue of applying the exact expression for a Fourier series, which is the multiplication of two different Fourier series [3]. According to the obtained distribution curves for the interfacial thermal stresses of an elliptical inclusion problem with an imperfect interface, the results of this research indicate that the extreme values and distributions of the interfacial stresses strongly depend on the values of interfacial parameters.

2. Basic formulations

In a coordinate system x_i, i=1, 2, 3, the temperature, displacement vector $\mathbf{u} = [u_1, u_2, u_3]^T$ and stress function vector $\phi = [\phi_1, \phi_2, \phi_3]^T$ on an anisotropic plane can be expressed as follows [1]

(1) $$T = 2\mathrm{Re}\{g'(z_\tau)\}$$

(2) $$\mathbf{u} = 2\,\mathrm{Re}\{\mathbf{Af}(z_\alpha) + \mathbf{c}g(z_\tau)\}$$

(3) $$\phi = 2\mathrm{Re}\{\mathbf{Bf}(z_\alpha) + \mathbf{d}g(z_\tau)\}$$

where **A** and **B** are Stroh matrices, **c** and **d** are heat eigenvectors, and $g(z_\tau)$ and $\mathbf{f}(z_\alpha)$ are arbitrary functions of their arguments. According to the assumptions for a spring-type model for an imperfect interface, the interfacial conditions at the elliptical interface are given by

(4) $$\|\sigma_{ni}(a_0, \theta)\| = 0,$$

(5) $$\sigma_{ij}(a_0, \theta) = h_j(\|u_i(a_0, \theta)\| - u_i^*);\, i,\, j = n, t, z.$$

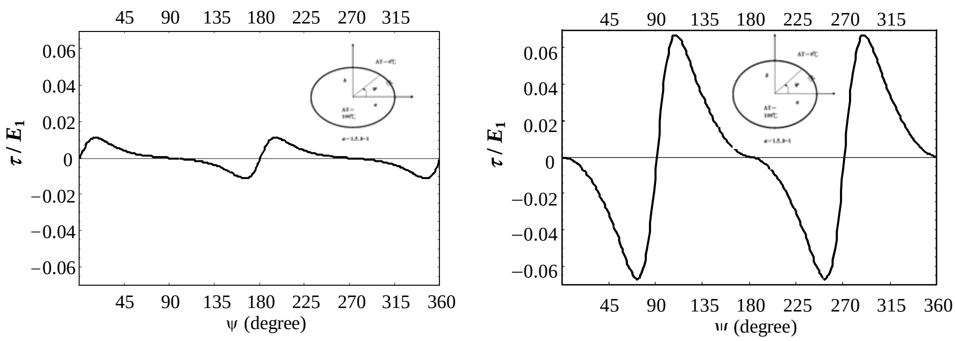
where the notation $\|*\| = (*)_2 - (*)_1$ denotes for the function value jump across the interface layer and u_i^* represents the displacements in direction i associated with the eigenstrain. The subscript indices "1" and "2" stand for the associate quantities on the matrix and the inclusion domains, respectively. The values of the three non-negative interfacial parameters h_n, h_t, h_z, in Eq. (5) can represent the bonding condition at the interface. By using Eqs. (1)~(3) Eqs. (4) and (5) lead to a set of simultaneous equations in terms of $\mathbf{f}_1(z_\alpha)$ and $\mathbf{f}_2(z_\alpha)$ pertaining to the exact solutions on the domains of matrix and inclusion, respectively. After mapping the elliptical interface into a unit circle and then expanding $\mathbf{f}_2(z_\alpha)$ into a Laurent series, the exact forms are successfully solved by virtue of introducing a semi-inverse approach in conjunction with the analytical continuation method.

3. Numerical results

The results of this research are presented by the interfacial shear stress curve for an orthotropic inclusion problem. Consider a temperature change of 100°C on the inclusion and the half length of the axes a=1.5, b=1 and that the material properties of the composite system are as

$(E_1)_1 = 10\,GPa$, $(E_2)_1 = 5\,GPa, (E_3)_1 = 5\,GPa$, $(\nu_{ij})_1 = 0.4, i; j = 1 \sim 3, i \neq j$, $(G_{12})_1 = 1\,GPa$,
$(G_{13})_1 = 2\,GPa$, $(G_{23})_1 = 1\,GPa$, $(\alpha_{11})_1 = 70 \times 10^{-6}(1/°\mathrm{C})$, $(\alpha_{22})_1 = 50 \times 10^{-6}(1/°\mathrm{C})$,
$(\alpha_{33})_1 = 50 \times 10^{-6}(1/°\mathrm{C})$, $(k_{11})_1 = 1\,W/m°C$, $(k_{22})_1 = 0\,W/m°C$, $(k_{12})_1 = 0\,W/m°C$,
$(E_1)_2 = 17\,GPa$, $(E_2)_2 = 7\,GPa, (E_3)_2 = 5\,GPa$, $(\nu_{ij})_2 = 0.3, i; j = 1 \sim 3, i \neq j$, $(G_{12})_2 = 3\,GPa$,
$(G_{13})_2 = 2\,GPa$, $(G_{23})_2 = 1\,GPa$, $(\alpha_{11})_2 = 70 \times 10^{-6}(1/°\mathrm{C})$, $(\alpha_{22})_2 = 50 \times 10^{-6}(1/°\mathrm{C})$,
$(\alpha_{33})_2 = 60 \times 10^{-6}(1/°\mathrm{C})$, $(k_{11})_2 = 1\,W/m°C$, $(k_{22})_2 = 0\,W/m°C$, $(k_{12})_2 = 0\,W/m°C$,

Figure 1 shows the comparison of the interfacial shear stress distribution curves for two different cases: the perfectly bonded interface and the frictional sliding interface. It is obvious that the interfacial shear stress distribution can change evidently and the extreme value of shear stress notably magnifies as the interface changes from perfectly bonded to frictional sliding.



perfectly bonded interface **frictional sliding interface**

Fig. 1. The distribution curves of the interfacial shear stress along the elliptical interface.

4. References

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