# SLIGHT IN-PLANE PERTURBATION OF A SYSTEM OF TWO COPLANAR PARALLEL TENSILE SLIT-CRACKS

***N. Pindra*, *V. Lazarus* and *J.B. Leblond***
*Institut Jean Le Rond d'Alembert, Université Pierre et Marie Curie (Paris VI), Tour 65-55, 4 place Jussieu, 75252 Paris Cedex 05, France*

## 1. Introduction

A number of recent papers have studied the evolution in time of the shape of the front of planar cracks propagating in brittle materials with heterogeneous fracture properties. The ultimate goal of such studies is to get a better understanding of the path of propagation of cracks in composite materials and geological faults. Favier *et al.* [1] considered for instance the case of a tensile slit-crack propagating in fatigue in an infinite body with spatially varying Paris constant.

The aim of the present work is to lay the grounds for an extension of Favier *et al.*'s [1] work to a system of *two coplanar parallel slit-cracks*. The aim of this extension will be to study the evolution in time of the shape of the fronts of the cracks during their coalescence.

## 2. Presentation of the problem

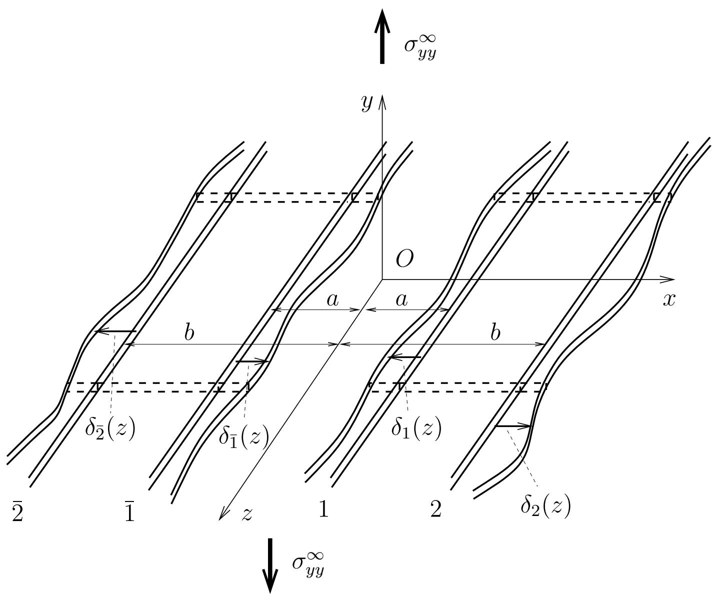


**Figure 1.** A system of two coplanar parallel slit-cracks with slightly perturbed crack fronts

The geometry of the problem is represented in Figure 1. The two slit-cracks lie in the plane $Oxz$. The unperturbed fronts $1$ and $2$ of the first crack are located at $x = a$ and $x = b$ respectively, and the fronts $\bar{1}$ and $\bar{2}$ of the symmetric crack at $x = -a$ and $x = -b$. All fronts are slightly perturbed within the plane $Oxz$; the local perpendicular distance between the unperturbed and perturbed positions of the front $\alpha$ ($\alpha = 1, 2, \bar{1}, \bar{2}$) is denoted $\delta_\alpha(z)$. The cracks are loaded through some uniform tensile stress $\sigma_{yy}^\infty$ exerted at infinity.

The discussion of crack propagation of course demands detailed knowledge of the distribution of the (mode I) stress intensity factors $K_\alpha(z)$ along the perturbed crack fronts. The variations $\delta K_\alpha(z)$ of the $K_\alpha(z)$ are given, to first order in the perturbation, by the following formula (Rice [2]):

$$\begin{aligned} \delta K_\alpha(z) &= C_\alpha(z)\delta_\alpha(z) + PV \int_{-\infty}^{+\infty} f_\alpha\left(\frac{z-z'}{b}\right) K_\alpha(z') \frac{\delta_\alpha(z') - \delta_\alpha(z)}{(z-z')^2} dz' \\ &\quad + \sum_{\beta \neq \alpha} \int_{-\infty}^{+\infty} g_{\alpha\beta}\left(\frac{z-z'}{b}\right) K_\beta(z') \frac{\delta_\beta(z')}{b^2} dz'. \end{aligned} \tag{1}$$

In this expression the functions $C_\alpha$ depend on both the unperturbed geometry and the loading, but the functions $f_\alpha$ and $g_{\alpha\beta}$, which are tied to Bueckner-Rice's fundamental weight functions, depend only on the unperturbed geometry, that is on the ratio $k \equiv a/b$.

Although the work of Rice [2] does establish the *existence* of the functions $f_\alpha$, $g_{\alpha\beta}$, it does not provide their *actual values* for the specific geometry considered, which are of course required for the discussion of crack propagation. The present paper is therefore devoted to the calculation of these functions.

## 3. Method of analysis

The method of calculation of the functions $f_\alpha$, $g_{\alpha\beta}$ is similar to that already used by Leblond *et al.* [3] in the case of a single slit-crack. Another formula of Rice [2] provides the variation of the functions $f_\alpha$, $g_{\alpha\beta}$ arising from an arbitrary perturbation of the fronts. This equation is applied to special perturbations preserving the shape and relative dimensions of the cracks while modifying their size and orientation. Since for such perturbations, the unperturbed and perturbed geometries are identical up to a change of scale combined with a rotation, the variations of the functions $f_\alpha$, $g_{\alpha\beta}$ are tied to these functions themselves. Rice's formula then yields a system of nonlinear integrodifferential equations on the functions $f_\alpha$, $g_{\alpha\beta}$, which are transformed *via* Fourier transform in the direction $z$ of the crack fronts into nonlinear ordinary differential equations on the Fourier transforms $\bar{f}_\alpha$, $\bar{g}_{\alpha\beta}$. These differential equations are solved numerically once and for all for all values of the parameter $k$.

The case $a \to 0$ or equivalently $k \to 0$ is of special interest for the future study of the coalescence of the cracks. Taking this limit is a non-trivial task because it raises a problem of singular perturbation in Fourier's space, implying the presence of a boundary layer for small values of the wavenumber (large values of the wavelength). This problem is solved through matched asymptotic expansions. The output consists of a system of two nonlinear differential equations on the sole functions $\bar{f}_1$, $\bar{g}_{1\bar{1}}$, which is again solved numerically.

It is thus possible to obtain the functions $f_\alpha$, $g_{\alpha\beta}$, at least numerically, for both finite and infinitesimal values of the parameter $k$, and this opens the way to the study of the evolution in time of the shape of the fronts during the propagation of the cracks, including their coalescence.

## 4. References

[1] E. Favier, V. Lazarus and J.B. Leblond (2006). Statistics of the deformation of the front of a tunnel-crack propagating in some inhomogeneous medium, *J. Mech. Phys. Solids*, **54**, 1449-1478.

[2] J.R. Rice (1989). Weight function theory for three-dimensional elastic crack analysis, in: *Fracture Mechanics: Perspectives and Directions (Twentieth Symposium), ASTM STP 1020*, Philadelphia, USA, pp. 29-57.

[3] J.B. Leblond, S.E. Mouchrif, G. Perrin (1996). The tensile tunnel-crack with a slightly wavy front, *Int. J. Solids Structures*, **33**, 1995-2022.