COMPUTATIONS OF EFFECTIVE ELASTIC PROPERTIES OF SOLIDS WITH MICROCRACKS USING THE BOUNDARY ELEMENT METHOD

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1. Introduction

One of the most important aims of micromechanical modeling is the determination of effective material properties of bodies containing microcracks [1, 2]. There are several theoretical approaches to analyze these problems. In the *non-interacting method*, it is assumed that the microcracks are isolated in the initial undamaged material. In the *self-consistent method*, the microcracks are embedded in the effective medium. The influence of interactions is simulated by reducing stiffness of the surrounding material. In the *differential method*, similar to the self-consistent method, one isolated crack is considered in the effective material. The interactions are taken into account through an incremental increase of crack densities and the effective modulus of the matrix is recalculated at each iteration. The interactions cause softening and effective moduli are lower than predicted by the non-interacting method. These theoretical methods usually give accurate results for low crack densities.

More general cases can be considered using numerical methods. Renaud et. al. [3] applied the indirect boundary element method (BEM) – the displacement discontinuity method to compute effective moduli of brittle materials weakened by microcracks. Structures with microcracks of different size, location and orientation were investigated. The structures were subjected to tensile and compressive loadings. In the last case an iterative algorithm was used to analyze cracks in contact with friction. The numerical results were compared with theoretical approximations. For microcracks in finite bodies good agreement with the differential method was obtained. Contact with friction decreases the effect of randomly distributed cracks on effective compliance. Huang et. al. [4] used the boundary element method and the unit cell method to calculate effective properties of solids with randomly distributed and parallel microcracks. In the BEM the modified fundamental solutions were used. The method does not require integration along the crack surfaces. Traction free cracks were considered. The results agree well with the differential method for low crack densities and with the generalized self-consistent method for high crack densities. Zhan et. al. [5] applied a series expansions of complex potentials and the superposition technique. The governing equations were solved numerically using the boundary collocation procedure. The effective Young moduli were calculated for randomly distributed and parallel cracks. The results were compared with various micromechanical models and experimental results. For randomly oriented cracks the results agree with differential method and for parallel cracks the moduli are below non-interacting solution and above the differential results.

The effective moduli were also determined experimentally by Carvalho and Labuz [6]. Artificially cracked aluminum plates with randomly distributed slots were subjected to tension. The experimental results agree well with the non-interacting approximation even for high density of cracks, where interactions are expected to occur. The authors found that the number of slots should be about 20 to guarantee the randomness of distribution.

Structures with high density of cracks have small Young moduli, however there is no direct quantitative correlation between the stiffness and damage [1]. Local positions of microcracks have strong influence on stress intensity factors while the effective modulus, which is a volume average quantity, is insensitive to such distributions.

2. The boundary element method for static and dynamic crack problems

In the present work microcracks in two-dimensional, linear-elastic, isotropic and homogenous solids will be analyzed using the dual boundary element method (DBEM) [7]. In this approach only boundaries of the body and crack surfaces are divided into boundary elements. The variations of boundary coordinates, displacements and tractions are interpolated using shape functions and nodal values. In the DBEM the relations between boundary displacements and tractions are expressed by the displacement and traction boundary integral equations. The displacement equation is applied for boundary nodes and both equations for nodes on crack surfaces. For dynamic problems, additionally the time of analysis is divided into time steps [8]. In this method boundary displacements and tractions are computed directly. For crack problems stress intensity factors (SIF) can be calculated very accurately using crack opening displacements or path independent integrals.

3. Numerical computations of effective elastic properties of solids with microcracks

The DBEM is very efficient method for analysis of effective properties of solids with multiple microcracks. It is very easy to generate solids with randomly distributed microcracks because in this approach only crack surfaces are discretized.

The method will be used to compute effective Young modulus and Poisson ratio for randomly distributed and parallel microcracks in bodies subjected to static loadings. The influence of crack density on effective properties of cracked materials will be investigated. The numerical results will be compared with available theoretical and experimental results. Additionally, stress intensity factors will be analyzed for various structures. The influence of microcracks densities on velocities of waves will be investigated for cracked bodies subjected to dynamic loadings.

4. Conclusions

Computations of effective material properties of damaged structures require analysis of many structures having different configurations of defects. The boundary element method is very efficient for such problems because modification of defects is simpler than in methods, which need discretization of the whole domain of the body. Contrary to analytical methods, solids with high density of cracks, curved and intersecting cracks can be considered.

5. References

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