ON THE LIMIT INTERNAL PRESSURE OF HOLLOW CYLINDERS OF STRAIN HARDENING VISCOPLASTIC MATERIALS

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1. Introduction

The paper presents the analytical and finite-element efforts of limit loads of thick-walled hollow circular cylinders. The internally pressurized structures are of strain hardening viscoplastic materials. It is appropriate to evaluate the limit loads by limit analysis sequentially to illustrate the interesting interaction between strengthening and weakening behavior reflecting the properties of strain hardening and strain-rate sensitivity during the deformation process. Particularly, the related analytical solutions are also derived for rigorous validation of the numerical results.

2. Problem Statement and numerical investigation

We consider a plane-strain viscoplastic problem of the von Mises-type material with nonlinear isotropic hardening. The problem domain D consists of the kinematic boundary ∂D_k. The problem statement leads naturally to the lower bound formulation. By duality theorems [1], the corresponding upper bound formulation can be stated in the form of a constrained minimization problem as

$$\begin{aligned} &\text{minimize} \quad \bar{q}(\vec{u}) \\ &\text{subject to} \quad \bar{q}(\vec{u}) = \frac{\sigma_Y}{G}\int_D \|\dot{\varepsilon}\|_{-\vee}\, dA \\ &\qquad \nabla\cdot\vec{u} = 0 \qquad \text{in } D \\ &\qquad \text{kinematic boundary conditions on } \partial D_k \end{aligned} \tag{1}$$

where $\|\dot{\varepsilon}\|_{-\vee}$ is the dual norm of the primal norm $\|\sigma\|_{\vee}$ based on the flow rule associated with the von Mises yield criterion. σ_Y is a material constant denoting the yield strength. G is a constant relating to the velocity control in each step but may be of various values in a process. $\nabla\cdot\vec{u} = 0$ is the incompressibility constraint inherent in the von Mises model. On the other hand, the behavior of viscoplastic, nonlinear isotropic hardening is described in the form as

$$\sigma_Y = \left[\sigma_\infty - (\sigma_\infty - \sigma_0)\exp(-h\bar{\varepsilon})\right]\ (\dot{\bar{\varepsilon}}/\dot{\bar{\varepsilon}}_0)^m \tag{2}$$

where σ_0 is the initial yield strength, σ_∞ is the saturation value of σ_0 and h is the hardening exponent. $\bar{\varepsilon}$ is the equivalent strain and $\dot{\bar{\varepsilon}}$ is the equivalent strain rate. $\dot{\bar{\varepsilon}}_0$ and m are the reference strain rate and strain-rate sensitivity, respectively. We conduct a sequence of limit analysis problems with updating the configuration of the deforming structures and the current yield strength. In each step and therefore the whole deforming process, rigorous upper bound solutions are solved iteratively by a combined smoothing and successively approximation (CSSA) algorithm [2].

3. Analytical investigation

For rigorous comparisons, we also derive the corresponding analytical solutions with the hardening exponent $h = \sqrt{3}$. The initial interior and exterior radii of the cylinder are denoted by a_0 and b_0. Also, its current interior and exterior radii are denoted by a and b. With the boundary conditions $\sigma_r(r = a) = P_i$, $\sigma_r(r = b) = 0$, we have the limit load expressed as [3]

$$P_i/\sigma_0 = \left(1/\sqrt{3}\right)^{m+1}\left(2\dot{a}a/\dot{\bar{\varepsilon}}_0\right)^m\left[\left(1/b^{2m} - 1/a^{2m}\right)/m + \left(\sigma_\infty/\sigma_0 - 1\right)\left(a_0{}^2 - a^2\right)\left(1/a^{2m+2} - 1/b^{2m+2}\right)/(m+1)\right] \tag{3}$$

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where $\dot{a}$ is the velocity of the innermost edge. In addition, it is interesting to reveal the interaction between strengthening and weakening behavior during the deformation process. Therefore, we come to consider the condition of stability, namely the existence of a hardening phenomenon before the weakening behavior. Mathematically, it is expressed as $\partial(P_i/\sigma_0)/\partial a > 0$. If we apply the velocity control to simulate the action of internal pressure, we get the stability condition as

$$\sigma_\infty/\sigma_0 > (m+3)/2 + [(m+1)/2][(b_0/a_0)^2 - 1]/[(b_0/a_0)^{2m+2} - 1] \tag{4}$$

4. Comparison and validation

We adopt the dimensional consistently parameters: $a_0 = 5.0$, $b_0 = 10.0$, $h = \sqrt{3}$, $\dot{a} = 1.0$, $\dot{\bar{\varepsilon}}_0 = 1.0$, and a constant step size $\Delta t = 0.01$. Figure 1 showns the effect of the strain-rate sensitivity m on the limit internal pressure P_i/σ_0 with the yield strength ratio $R = \sigma_\infty/\sigma_0 = 2.05$. As shown, the computed upper bounds agree well with the analytical solutions. Table 1 lists the analytical results of the stability condition showing the effects of the strain-rate sensitivity m.

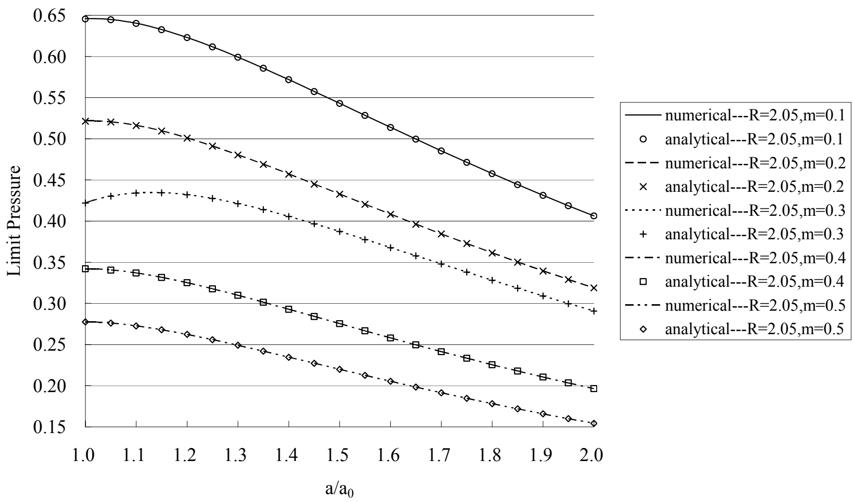

Figure 1. Effect of the strain-rate sensitivity m on the limit internal pressure P_i/σ_0

m=0.1	m=0.2	m=0.3	m=0.4	m=0.5
2.009	2.201	2.035	2.052	2.071

Table 1. Effect of the strain-rate sensitivity m on the stability condition in terms of the yield strength ratio σ_∞/σ_0

5. References

[1] W.H. Yang (1993). Large deformation of structures by sequential limit analysis, *Int. J. Num. Meth. Eng.*, **30**,1001-1013.

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