# A PIEZOELECTRIC SOLID SHELL ELEMENT ACCOUNTING FOR MATERIAL AND GEOMETRICAL NONLINEARITIES

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## 1. Introduction

This contribution is concerned with a piezoelectric solid shell finite element formulation. In recent years several new elements have been proposed. Some of these model a reference surface of the shell structure. Here, a surface oriented piezoelectric solid shell element is developed. With respect to the laminated structure of piezoelectric devices a more or less sophisticated laminate theory is necessary. The so-called solid shell elements circumvent laminate theories by modelling each ply with one element, see e.g. [3] and the references therein.

The most piezoelectric shell formulations assume a geometrically linear theory. In [4] it is pointed out that nonlinear characteristics can significantly influence the performance of piezoelectric systems. In particular this holds for buckling of plates. A geometrically non-linear theory allows large deformations and includes stability problems. Typical materials for the utilization of the piezoelectric effect are ferroelectric ceramics like barium titanate (BaTiO3) and lead zirconate titanate (PbZrTiO3) abbreviated as PZT. Ferroelectric ceramics show a strongly nonlinear behavior, which is caused by so-called domain switching effects, see e.g. [2] and the references therein. The present shell formulation incorporates a material model accounting for the physical nonlinearities. The model is thermodynamically consistent and determined by two scalar valued functions: the Gibb's free energy and a switching criterion.

Usually the electric potential inside the piezoelectric model is assumed to be linear through the shell thickness. To fulfill the electric charge conservation law exactly a quadratic electric potential through the thickness is necessary. In this paper the finite element formulation is based on a variational principle including six independent fields: displacements, electric potential, strains, electric field, mechanical stresses and dielectric displacements. To obtain correct results in bending dominated situations a linear distribution through the thickness of the independent electric field is assumed. The element has 8 nodes; the nodal degrees of freedom are displacements and the electric potential. The presented finite shell element is able to model arbitrary curved shell structures and incorporates a 3D-material law.

## 2. Numerical simulation

Telescopic actuators consist of concentric shells interconnected by end caps which alternate in placement between the two axial ends of the shells, see Fig. 1. The diameters in Fig. 1 refer to the outside of the cylindric shells. The telescopic actuators are designed to accomplish for a high displacement actuation at the cost of force, see [1]. The cascading shells are polarized in radial direction. The transversal isotropic elastic material constants are given as $E_1 = E_2 = 60.61 \cdot 10^9\,\mathrm{N/m^2}$, $E_3 = 48.31 \cdot 10^9\,\mathrm{N/m^2}$, $\nu_{23} = \nu_{13} = 0.41$, $\nu_{12} = 0.29$, $G_{23} = G_{13} = 22.99 \cdot 10^9\,\mathrm{N/m^2}$, $G_{12} = 23.47 \cdot 10^9\,\mathrm{N/m^2}$. The piezoelectric modulus is described by $\mathbb{e}_{13} = \mathbb{e}_{23} = -29.878\,\mathrm{C/m^2}$, $\mathbb{e}_{33} = 10.631\,\mathrm{C/m^2}$, $\mathbb{e}_{51} = \mathbb{e}_{62} = 17.034\,\mathrm{C/m^2}$ and the permittivity is defined as $\epsilon_{11} = \epsilon_{22} = 15.09 \cdot 10^{-9}\,\mathrm{C^2/N\,m^2}$, $\epsilon_{33} = 14.16 \cdot 10^{-9}\,\mathrm{C^2/N\,m^2}$. The piezoelectric constants $\mathbb{e}_{13}$ and $\mathbb{e}_{33}$ are taken from [1] and all other quantities are assumed. The nonlinear material model is described in [2]. The orientation of the local basis system $\boldsymbol{t}_i$ is defined by $\boldsymbol{t}_1$ in circumferential direction, $\boldsymbol{t}_2$ in axial direction and $\boldsymbol{t}_3$ in radial direction. With respect to symmetry only a quarter of the system is modeled by finite elements, see Fig. 1.

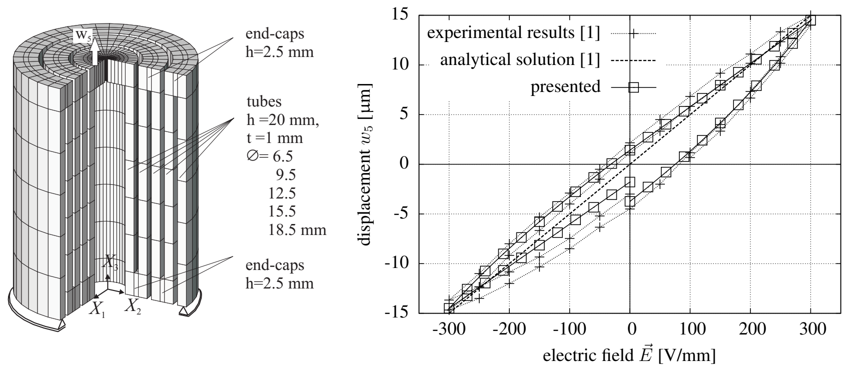


**Figure 1.** Left: Telescopic actuator: system, boundary conditions and finite element model. Right: axial deflection versus electric field

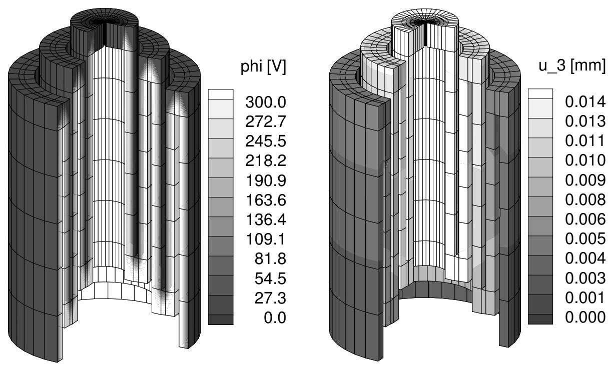


**Figure 2.** Telescopic actuator: deformed configuration with a plot of electric potential and axial displacement

The system is supported in $X_3$-direction at the lower bottom at the outside edge and it is is loaded by applying an electric potential to the piezoelectric cylindrical shells shown in Fig. 2. The load deflection behavior of the axial displacements, see Fig. 1, is highly nonlinear due to the occurring domain switching effects. A comparison to the experimental data in [1] shows good agreement, which is very promising for further calculations.

## 3. References

[1] P. Alexander, D. Brei, W. Miao, J. Halloran (2001), Fabrication and experimental characterization of $d_{31}$ telescopic piezoelectric actuators. *Journal of Material Science*, **36**, 4231–4237.

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[4] H.S. Tzou HS and Y.H. Zhou (1997). Nonlinear piezothermoelasticity and multi-field actuations, part 2: Control of nonlinear deflection, buckling and dynamics. *Journal of Vibration and Acoustics*, **119**, 382–389.