FREE VIBRATIONS OF ORTHOTROPIC SHALLOW SHELLS OF VARIABLE THICKNESS ON BASIS OF SPLINE-APPROXIMATION METHOD

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Plates and shells with a complex shape made of inhomogeneous anisotropic materials are widely used for construction of structure elements in modern engineering. The present report proposes an efficient approach to solving the free vibrations problems of shallow shells with the variable thickness within the framework of the classic models. The object of investigation is the class of free vibration problems for orthotropic rectangular in a plane shallow shells of variable thickness in two coordinate directions. The problems are described by the system of linear partial differential equations with the variable coefficients [1]:

$$
\begin{aligned}
&C_{11}\frac{\partial^2 u}{\partial x^2}+\frac{\partial C_{11}}{\partial x}\frac{\partial u}{\partial x}+C_{66}\frac{\partial^2 u}{\partial y^2}+\frac{\partial C_{66}}{\partial y}\frac{\partial u}{\partial y}+\left(C_{12}+C_{66}\right)\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial C_{66}}{\partial y}\frac{\partial v}{\partial x}+\\
&+\frac{\partial C_{12}}{\partial x}\frac{\partial v}{\partial y}+\left(C_{11}k_1+C_{12}k_2\right)\frac{\partial w}{\partial x}+\frac{\partial\left(C_{11}k_1+C_{12}k_2\right)}{\partial x}w=0,\\
&C_{66}\frac{\partial^2 v}{\partial x^2}+\frac{\partial C_{66}}{\partial x}\frac{\partial v}{\partial x}+C_{22}\frac{\partial^2 v}{\partial y^2}+\frac{\partial C_{22}}{\partial y}\frac{\partial v}{\partial y}+\left(C_{12}+C_{66}\right)\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial C_{12}}{\partial y}\frac{\partial u}{\partial x}+\\
&+\frac{\partial C_{66}}{\partial x}\frac{\partial u}{\partial y}+\left(C_{12}k_1+C_{22}k_2\right)\frac{\partial w}{\partial y}+\frac{\partial\left(C_{12}k_1+C_{22}k_2\right)}{\partial y}w=0,\\
&D_{11}\frac{\partial^4 w}{\partial x^4}+D_{22}\frac{\partial^4 w}{\partial y^4}+2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w}{\partial x^2\partial y^2}+2\frac{\partial D_{11}}{\partial x}\frac{\partial^3 w}{\partial x^3}+2\frac{\partial D_{22}}{\partial y}\frac{\partial^3 w}{\partial y^3}+\\
&+2\frac{\partial}{\partial y}\left(D_{12}+2D_{66}\right)\frac{\partial^3 w}{\partial x^2\partial y}+2\frac{\partial}{\partial x}\left(D_{12}+2D_{66}\right)\frac{\partial^3 w}{\partial x\partial y^2}+\left(\frac{\partial^2 D_{11}}{\partial x^2}+\frac{\partial^2 D_{12}}{\partial y^2}\right)\frac{\partial^2 w}{\partial x^2}+\\
&+\left(\frac{\partial^2 D_{12}}{\partial x^2}+\frac{\partial^2 D_{22}}{\partial y^2}\right)\frac{\partial^2 w}{\partial y^2}+4\frac{\partial^2 D_{66}}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}+\left(C_{11}k_1^2+2C_{12}k_1k_2+C_{22}k_2^2\right)w+\\
&+\left(C_{11}k_1+C_{12}k_2\right)\frac{\partial u}{\partial x}+\left(C_{12}k_1+C_{22}k_2\right)\frac{\partial v}{\partial y}+\rho h\frac{\partial^2 w}{\partial t^2}=0.
\end{aligned}
\tag{1}
$$

Here u, v and w are the unknown displacements of shell midsurface points ; t is the time; $h=h(x,y)$ is the shell thickness; $\rho=\rho(x,y)$ is the density of the material; $C_{ij}=B_{ij}h$, $D_{ij}=B_{ij}h^3/12$ $(\{i,j\}=\{1,2,6\})$ are the strain and bending stiffness of the shell.

The different boundary conditions (rigid fixing, hinge supporting and their combinations) for displacements are specified on the shell contours.

System (1) is solved in two steps. At the first step, we approximate the unknown displacements in one of the coordinate directions (for example OY) by the segments of series consisting the linear combinations of the B-splines of the third and fifth power [2, 3]:

$$
u=e^{j\omega t}\sum_{i=0}^{N}u_i(x)\psi_{1,i}(y),\; v=e^{j\omega t}\sum_{i=0}^{N}v_i(x)\psi_{2,i}(y),\; w=e^{j\omega t}\sum_{i=0}^{N}w_i(x)\psi_{3,i}(y),
\tag{2}
$$

where, $v_i(x)$ and $w_i(x)$ ($i=\overline{0,N}$) are the unknown functions, $\psi_{1,i}(y)$ and $\psi_{2,i}(y)$ are the linear combinations of the B-splines of the third power, $\psi_{3,i}(y)$ are the linear combinations of the B-splines of the fifth power, which exactly satisfied boundary conditions for displacements on the contours $y = const$; ω is the unknown frequency of free vibrations; $j = \sqrt{-1}$; $0 \le y \le b$.

Substituting (2) into (1) with allowance for the boundary conditions and requiring expansion (2) would be the coincident with the exact solution in the number of points of collocation, which are the roots of the Legendre second-order polynomial on segment [0,1], we arrive at the one-dimensional eigen-value problem . This problem can be written down in the normalized Cauchie form as:

(3) $$\frac{d\overline{Y}}{dx} = \mathbf{A}(x,\omega)\overline{Y} \quad (0 \le x \le a),$$

(4) $$\mathbf{B}_1\overline{Y}(0) = \overline{0},\ \mathbf{B}_2\overline{Y}(a) = \overline{0},$$

where $\overline{Y} = [\overline{u},\overline{u}',\overline{v},\overline{v}',\overline{w},\overline{w}',\overline{w}'',\overline{w}''']^T$ is the vector-column of the unknown functions and theirs derivatives with the dimensionality $8(N+1)$; $\mathbf{A}(x,\omega)$ is the specified square matrix of the order $8(N+1)$; $\mathbf{B}_1$ and $\mathbf{B}_2$ are specified rectangular matrices with the dimensionality $4(N+1)\times 8(N+1)$.

At the second step, the one-dimensional eigen-value problem (3) – (4) is solved by the stable numerical method of discrete orthogonalizaiton in combination with the method of incremental-step search [2, 3].

On the basis of the method proposed, the spectrum of frequencies and modes of free vibrations of orthotropic plates and shallow rectangular in a plane shells of different shapes, whose thickness varies in one or two coordinate directions, is studied. It is supposed that the shells contours are rigidly fixed or hinged supported. Theirs combinations are possible.

The following cases of free vibrations are considering: the orthotropic rectangular plate with linearly-variable thickness:

(5) $$h = h_0\left[1+\alpha\left(2\,x/a-1\right)\right];$$

orthotropic shallow rectangular in plane cylindrical shell of the thickness varying by the law :

(6) $$h = h_0\left[1+\alpha\left(6x^2-6x+1\right)\right];$$

cylindrical orthotropic panel with the thickness varying in two directions by the law:

(7) $$h = h_0\left(1+\alpha\cos(\pi x/a)\right)\left(1+\beta\cos(\pi y/b)\right).$$

Here h_0 is the thickness of plates (shallow shells) with an equivalent mass and constant thickness, $|\alpha| \le 0.5$, $|\beta| \le 0.5$ are the coefficients, which determine the type of the shell in the given coordinate direction.

The numerical-analytical approach proposed makes it possible to analyze frequencies and modes of free vibrations of rectangular in plane shallow anisotropic shells with different boundary conditions and the thickness varying in two directions in wide range of varying geometrical and mechanical parameters. rectangular planes rectangular planes rectangular planes rectangular planes .

References

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