COMPOSITE SHELLS IN 6-FIELD NONLINEAR SHELL THEORY

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1. General

This communication addresses the application of the general nonlinear 6-field shell theory in the analysis of layered composite shells. The FEM model is constructed basing on the statically and kinematically exact theory of shells that accommodates naturally finite (unlimited) translations and rotations [1]. Kinematics of the shell is described by the field of generalized displacements composed of the translation field and the proper rotation field. Due to the presence of rotation tensor the elements have naturally six degrees of freedom at each node, including the so-called drilling dof.

A typical composite shell made of an orthotropic fiber-reinforced material can be analyzed as a layered structure, with the fibers of the reinforcement in each lamina placed in the surfaces parallel to the shell mid-surface. It is assumed that the shell is composed of a finite number of individually homogeneous layers. Each layer is made of linearly elastic and orthotropic material. The layers are perfectly bonded and no slip between them is possible. Assuming an Equivalent Single Layer (ESL) model the entire laminate is represented by a single-layer panel with macro-mechanical properties estimated as a weighed average of the mechanical properties of each lamina [2, 3].

2. Formulation

The rigorous treatment of the shell theory and its various FEM implementations for isotropic shells has been already extensively dealt with, see e.g. [4] and references given there. Since the current formulation incorporates the drilling dofs, the strain measures are generally not symmetric ($\varepsilon_{12} \neq \varepsilon_{21}$ and $\kappa_{12} \neq \kappa_{21}$). As a consequence, the constitutive relations for the layered composite panel are assumed in the following forms:

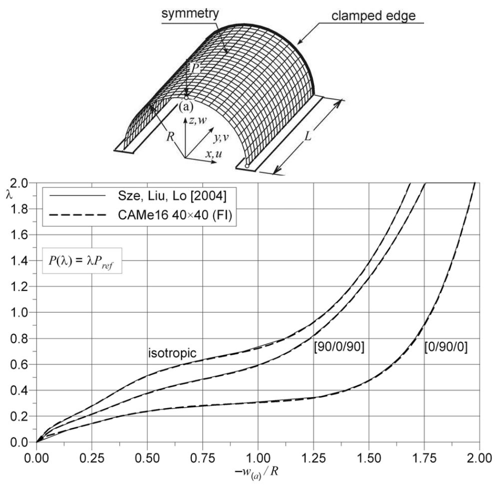
$$\begin{Bmatrix} n^{11} \\ n^{22} \\ n^{12} \\ n^{21} \end{Bmatrix} = \begin{bmatrix} C^{1111} & C^{1122} & C^{1112} & C^{1121} \\ C^{2211} & C^{2222} & C^{2212} & C^{2221} \\ C^{1211} & C^{1222} & C^{1212} & C^{1221} \\ C^{2111} & C^{2122} & C^{2112} & C^{2121} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \\ \varepsilon_{21} \end{Bmatrix}, \quad \begin{Bmatrix} n^{23} \\ n^{13} \end{Bmatrix} = \begin{bmatrix} C^{2323} & C^{2313} \\ C^{1323} & C^{1313} \end{bmatrix} \begin{Bmatrix} \gamma_{23} \\ \gamma_{13} \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} m^{11} \\ m^{22} \\ m^{12} \\ m^{21} \end{Bmatrix} = \begin{bmatrix} \Sigma^{1111} & \Sigma^{1122} & \Sigma^{1112} & \Sigma^{1121} \\ \Sigma^{2211} & \Sigma^{2222} & \Sigma^{2212} & \Sigma^{2221} \\ \Sigma^{1211} & \Sigma^{1222} & \Sigma^{1212} & \Sigma^{1221} \\ \Sigma^{2111} & \Sigma^{2122} & \Sigma^{2112} & \Sigma^{2121} \end{bmatrix} \begin{Bmatrix} \kappa_{11} \\ \kappa_{22} \\ \kappa_{12} \\ \kappa_{21} \end{Bmatrix}, \quad \begin{Bmatrix} m^{23} \\ m^{13} \end{Bmatrix} = \begin{bmatrix} \Sigma^{2323} & \Sigma^{2313} \\ \Sigma^{1323} & \Sigma^{1313} \end{bmatrix} \begin{Bmatrix} \kappa_{23} \\ \kappa_{13} \end{Bmatrix}, \tag{2}$$

where the components C^{ijkl} and Σ^{ijkl} resulted from an appropriate integration of the 3-D anisotropic constitutive relations through the thickness of the whole shell (assuming zero value of the transverse normal stress components).

3. Example

To illustrate the performance of the considered model, the results for one of the most demanding benchmark tests for large rotation shell analysis, the semi-cylindrical shell under a point load are presented in Fig. 1. This example was introduced by Stander et al. [5], who, however, considered only the case of an isotropic shell. The case of a layered composite shell was proposed by Sze et al. [6] who investigated two different cross-ply lamination schemes: [90/0/90] and [0/90/0]. Quite recently, the same composite panel was examined by Arciniega and Reddy [7]. One should notice, that none of the authors of the papers [5-7] bothered with a proper physical meaning of the applied input data; to correct that issue we assume the following dimensions: $L = 304.8$ mm, $R = 101.6$ mm and $h = 3$ mm; together with material parameters for boron-epoxy type composites: $E_a = 20.685$ kN/mm^2, $E_b = 5.17125$ kN/mm^2, $G_{ab} = G_{ac} = 7.956$ kN/mm^2, $G_{bc} = 1.989$ kN/mm^2 and $\nu_{ab} = 0.25$. For isotropic case the following material properties are used: $E = 20.685$ kN/mm^2, $\nu = 0.25$. The load is assumed as the proportional $P(\lambda) = \lambda P_{ref}$, where $P_{ref} = 1000$kN. To avoid discussions about mesh convergence or spurious zero-energy forms the computations were carried out using 40×40 CAMe16 elements with full integration. The discretizations in the Fig.1 are given for the whole structure.



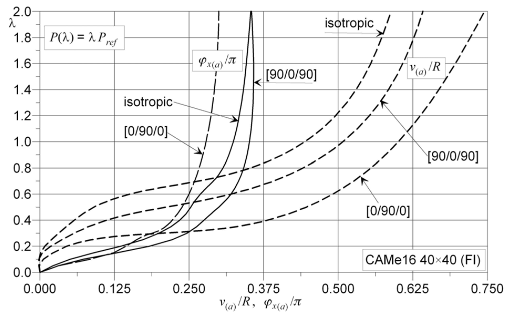


Figure 1. Semi-cylindrical shell under a point load, comparison of solutions

The obtained results, as presented in Fig. 1, show a very good agreement with the reference solutions [6], what demonstrates a big potential of the proposed formulation and encourages to further research.

5. References

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