ON A SURFACE-RELATED SHELL FORMULATION FOR THE NUMERICAL SIMULATION OF TEXTILE REINFORCED CONCRETE LAYERS

R. Schlebsuch and B. Zastrau
Institute of Mechanics an Shell Structures, Dresden, Germany

1. Introduction

The numerical simulation of thin textile reinforced concrete (TRC) strengthening layers is the object of this research. Its mechanical description is implemented by a shell formulation demanding an efficient numerical solution strategy. The shell model is formulated with respect to one of the outer surfaces, i.e. the shell formulation is surface-related. The discretization and interpolation of the associated variational formulation are sources of several locking phenomena. Extensions and/or adjustments of well-known techniques to prevent or at least to reduce locking like the assumed natural strain (ANS) method and the enhanced assumed strain (EAS) method have to be made.

2. Governing Equations

Since shells are three-dimensional bodies the field equations of continuum mechanics are the starting point for the mechanical model. They can be found in many textbooks, e.g. [1]. This set of partial differential equations with pertinent boundary conditions has to be solved for the TRC strengthening layer. An efficient numerical solution of this problem becomes easier if the problem is reformulated using a background of variational calculus.

3. Variational Formulation

The weak formulation of the governing equations is gained by the standard procedure and leads for hyperelasticity to the well-known generalized HU-WASHIZU functional:

$$\Pi_{HW}(\mathbf{U},\tilde{\mathbf{E}},\mathbf{S},\mathbf{t}_0) = \int_{\mathcal{B}_{\underline{t}}} (\rho_{\underline{t}} f(\mathbf{E}(\mathbf{U})+\tilde{\mathbf{E}}) - \operatorname{sym}\mathbf{S} : \tilde{\mathbf{E}}) dV + \int_{\mathcal{B}_{\underline{t}}} \rho_{\underline{t}} (\ddot{\mathbf{U}} - \mathbf{f}) \cdot \mathbf{U} dV$$

$$(1) \qquad - \int_{\partial_{\mathbf{t}}\mathcal{B}_{\underline{t}}} \hat{\mathbf{t}}_0 \cdot \mathbf{U} dA + \int_{\partial_{\mathbf{U}}\mathcal{B}_{\underline{t}}} \mathbf{t}_0 \cdot (\widehat{\mathbf{U}} - \mathbf{U}) dA \rightarrow \text{stat.},$$

whereby a re-parametrization following the suggestion of [5] was made:

$$(2) \qquad \mathbf{E} = \mathbf{E}(\mathbf{U}) + \tilde{\mathbf{E}} \qquad \Leftrightarrow \qquad \mathbf{E}^{\mathbf{U}} - \mathbf{E} = -\tilde{\mathbf{E}}$$

introducing the residuum of the kinematical field equation $\tilde{\mathbf{E}}$. The demand for stationarity of this functional is equivalent with the field equations and the pertinent boundary conditions. But now the residuum of the kinematical field equation: $\tilde{\mathbf{E}} = \mathbf{0}$ appears as EULER-LAGRANGE equation. Further following the suggesting of [5] a L_2-orthogonality between the second PIOLA-KIRCHHOFF stress tensor $\mathbf{S}$ and the residuum $\tilde{\mathbf{E}}$ of the kinematical field equation is enforced. Therefore the second term in the first integral on the right-hand side of equation (1) vanishes. This results in a modified stationarity condition that represents the following abstract variational formulation:

Find

$$(\mathbf{U},\tilde{\mathbf{E}}) \in \mathcal{X}_1 \times \mathcal{X}_2 = \mathbb{H}^1(\mathcal{B}_{\underline{t}},\mathcal{E}^3) \times L_2(\mathcal{B}_{\underline{t}},\mathcal{E}^3 \otimes \mathcal{E}^3)$$
$$(\mathbf{t}_0,\mathbf{S}) \in \mathcal{M}_1 \times \mathcal{M}_2 = L_2(\mathcal{B}_{\underline{t}},\mathcal{E}^3) \times L_2(\mathcal{B}_{\underline{t}},\mathcal{E}^3 \otimes \mathcal{E}^3),$$

such that

$$\begin{aligned} a(\mathbf{U}, \tilde{\mathbf{E}}; \boldsymbol{\eta}, \delta\tilde{\mathbf{E}}) + b_1(\boldsymbol{\eta}, \mathbf{t}_0) &= \mathcal{F}(\boldsymbol{\eta}, \delta\tilde{\mathbf{E}}) \quad \forall(\boldsymbol{\eta}, \delta\tilde{\mathbf{E}}) \in \mathcal{X}_1 \times \mathcal{X}_2 \\ b_1(\mathbf{U}, \delta\mathbf{t}_0) &= \mathcal{G}_1(\delta\mathbf{t}_0) \quad \forall \delta\mathbf{t}_0 \in \mathcal{M}_1 \end{aligned}$$

and the orthogonality condition $\int_{\mathcal{B}_t} \mathbf{S} : \tilde{\mathbf{E}} \, dV = 0$ *is fulfilled.*

This abstract mathematical formulation allows to investigate the problem from a mathematical point of view and shows the structure of the three-dimensional problem.

4. Surface-Related Shell Formulation

The displacement field $\mathbf{U}$ representing the motion of the shell continuum, i.e. of the TRC strengthening layer, is restricted by a kinematical assumption:

(3) $\mathbf{U} = \mathbf{V} + \Theta^3 \mathbf{W},$

Corresponding to the particular position of the reference surface it follows for the normal coordinate $\Theta^3 \in [0, H]$. The disadvantage of this shell kinematics is that it causes POISSON thickness locking.

Starting from the kinematics (3) a surface-related shell formulation is derived, i.e. surface-related strain tensors, surface-related stress resultant tensors etc. are defined. Further details can be found in [2, 3].

5. Finite Element Formulation and Further Locking Phenomena

The discretization of the functional is one source of locking phenomena that can be reduced or even avoided by an enhancement of the strain tensor, cp. [5], or of the finite element formulation, cp. [4]. Since we are dealing with a surface-related shell formulation extensions and/or adjustments of these techniques have to be made and are presented. This procedure finally leads to a very efficient surface-related finite volume shell element that can be used in its respective framework of application, i.e. the simulation of TRC strengthening layers.

6. Acknowledgement

The authors gratefully acknowledge financial support of this research from Deutsche Forschungsgemeinschaft DFG within the Sonderforschungsbereich SFB 528 ”Textile Reinforcement for Structural Strengthening and Repair” at the Technische Universitt Dresden.

[1] P. Haupt (2002). *Continuum Mechanics and Theory of Materials*, 2nd ed. Springer, Berlin.

[2] R. Schlebusch (2005). *Theorie und Numerik einer oberflächenorientierten Schalenformulierung*, Ph.D. thesis, Institute of Mechanics and Shell Structures, Dresden.

[3] R. Schlebusch, J. Matheas and B. Zastrau (2003). On Surface-Related Shell Theories for the Numerical Simulation of Contact Problems, *J. Theor. Appl. Mech.*, **41(3)**,623-642.

[4] J.-C. Simo and J. C. Hughes (1986). On Varaitional Foundation of Assumed Strain Methods. *J. Appl. Mech.*, **53**,52-54.

[5] J.-C. Simo and M. S. Rifai (1990). A Class of Mixed Assumed Strain Methods and the Method of Incompatible Modes. *Int. J. Num. Meth. Engng.* **29**,1595-1638.