DYNAMIC CONTACT OF THE ELASTIC IMPACTOR AND SPHERICAL SHELL

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1. General

The dynamic contact of the solid body through elastic buffer on a circular sector of spherical shell is studied. The intender is simulated by mass and the elastic cylindrical element, which strikes the target. Interaction is considered to be elastic with constant contact area. Dynamic behaviour of the shell is described by without moment moving equations taking the rotary inertia into account and, therefore, are wave equations. These equations allow to assume, that in the plate the transient longitudinal wave, because of which there is a deformation of a shell material outside of contact area, is generated with final velocity. In the present work, the procedure similar to the one proposed in [1] for the analysis of transverse impact of a solid sphere upon an elastic buffer positioned on an elastic orthotropic plate, is used to the case of shock interaction of a solid body with an elastic circular sector of spherical shell which hinge-supported on the perimeter.

During the interaction of the body with the shell, a quasilongitudinal wave representing the surfaces of strong discontinuity begin to propagate. In a spherical shell of the surface of strong discontinuity represent spherical surfaces – strip, whose generators are parallel to the normal to the median surfaces and guides locating in the median surface are circumferences extending with the normal velocity G. Behind the wave fronts, the solution is constructed in terms of ray series representing power series, whose coefficients are the different order discontinuities in the time-derivatives of the required functions, and the variable is the time passed from the moment of arrival of a wave to the given points of the shell:

$$Z(\varphi,t)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[Z_{,(k)}\right]_{t=R_1\varphi/G}\left(t-\frac{R_1(\varphi-\varphi_0)}{G}\right)^k H\left(t-\frac{R_1(\varphi-\varphi_0)}{G}\right), \tag{1}$$

where $[Z_{,(k)}]=Z^{+}_{,(k)}-Z^{-}_{,(k)}=[\partial^k Z/\partial t^k]$ are the leaps of the derivatives of k-degree by the time t from the equation Z on the wave surface Σ, i.e. if $t=R_1(\varphi-\varphi_0)/G$, r_0 is the initial radius of the contact area, indexes "+" and "–" mean that the value is found directly in front of and behind the wave front respectively, $H(t)$ – the one-term Heviside's function, R_1 – shell's radius, φ - coordinate directed on the meridian, φ_0 – angle coordinate for boundary of contact area.

To determine the ray series coefficients for the desired functions, it is necessary to differentiate the governing equations for shell with respect to time, to take their difference on the different sides of the wave surface, and to apply the condition of compatibility [2]

$$G\left[\frac{\partial Z_{,(k)}}{\partial r}\right]=-\left[Z_{,(k+1)}\right]+\frac{\delta\left[Z_{,(k)}\right]}{\delta t}\ , \tag{2}$$

where $\delta/\delta t$ is the δ- derivative with respect to time.

As a result of the procedure described, we are led to the system of recurrent differential equations, which solution gives us the discontinuities in time-derivatives of the desired values within arbitrary constants

$$\left(1-\frac{\rho\left(1-\sigma^2\right)G^2}{E}\right)U_{(k+1)}=2\frac{\delta U_{(k)}}{\delta t}-Gctg\varphi U_{(k)}+(1+\sigma)G\sin\varphi X_{(k)}+F_{1(k-1)}, \tag{3}$$

$$X_{(k+1)}=\frac{(1+\sigma)}{G\sin\varphi}U_{(k)}+F_{2(k-1)}, \tag{4}$$

where $X_{(k)}=\left[w_{,(k+1)}\right]$, $U_{(k)}=\left[u_{\varphi,(k+1)}\right]$, $\varphi=\varphi_0+R_1Gt$,

$$F_{1(k-1)}=\frac{\delta U_{(k-1)}}{\delta t}(Gctg\varphi-1)-G^2(1-\sigma)U_{(k-1)}-(1+\sigma)G\sin\varphi\frac{\delta X_{(k-1)}}{\delta t},$$

$$F_{2(k-1)}=-(1+\sigma)\left(\frac{1}{G\sin\varphi}\frac{\delta U_{(k-1)}}{\delta t}+2X_{(k-1)}\right).$$

where E is the modulus of elasticity, σ is Poisson's ratio, ρ is the density shell's material, w and u_φ are the normal and tangential along meridian displacement respectively.

The arbitrary constants are determined at splicing on border of contact area of the solution for required function inside a contact disk and outside of it from following equations

$$m\left(\ddot{\alpha}+\ddot{w}\right)=-P(t),\ \ \rho h\pi r_0^2\ddot{w}=2\pi r_0\left(N_\varphi+N_\theta\right)\Big|_{\varphi=\varphi_0}\sin\varphi+P(t), \tag{5}$$

where α is the displacements of the impactor's upper end, $P(t)$ is the contact force proportional to the buffer's deformation, m is the mass of the impactor, h is the thickness of the target, r_0 is the impactor's radius, N_φ and N_θ are the longitudinal forces on the boundary of the contact region.

The compact analytical expressions for contact force and dynamical normal displasement are defined.

$$P(t)=E_1\left[V_1t-\frac{1}{6}E_1V_1\left(\frac{1}{m}+\frac{1}{\pi r_0\left(\rho hr_0-E\sin\varphi_0\right)}\right)t^3-\frac{E(1+\sigma)(\sin\varphi_0)^{1/2}}{6G\rho hr_0}\times\right.$$

$$\left.\times\left\{\left[\frac{ctg\varphi_0\,\rho hr_0G}{\rho hr_0-E\sin\varphi_0}+\frac{\ln\sin\varphi_0}{4}+\frac{ctg\varphi_0G}{4}-\varphi_0\left[\frac{G}{4}+\frac{(1+\sigma)^2}{2G}\right]\right]c_1+c_2\right\}t^4\right], \tag{6}$$

$$w(t)=\frac{1}{6\pi r_0\left(\rho hr_0-E\sin\varphi_0\right)}t^3+\frac{E(1+\sigma)(\sin\varphi_0)^{1/2}}{6G\rho hr_0}\left\{\left[\frac{ctg\varphi\,\rho hr_0G}{\rho hr_0-E\sin\varphi_0}+\right.\right.$$

$$\left.\left.+\frac{\ln\sin\varphi_0}{4}+\frac{ctg\varphi_0G}{4}-\varphi_0\left[\frac{G}{4}+\frac{(1+\sigma)^2}{2G}\right]\right]c_1+c_2\right\}t^4, \tag{7}$$

here E_1 is the impactor's elastic modulus, V_1 is the initial velocity of contact, c_1, c_2 are the constants.

The carried out numerical researches allow to make the conclusion about influence of parameters of a construction on dynamic characteristics of interaction.

References

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