FORMULATION OF THE INITIAL INVARIANT–BASED SHELL FINITE ELEMENT MODEL USING THE PLANE CURVE GEOMETRY

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A new approach is proposed for formulating a triangular finite-element of the Kirchhoff-Love thin elastic shells undergoing arbitrarily large displacements and rotations. The starting point of the approach is to represent the strain energy of the shell as a function of the invariants of the strain and curvature-change tensors of the shell middle surface. Given elongations and curvature changes of any three fibers lying on the middle surface along three independent directions, one can readily calculate these invariants. For a triangular element, it is a natural choice to take the element sides as these fibers. Thus, the strain energy is written as (summation over m=1,2,3)

$$\Pi = \frac{1}{2}\int_F B\left[I_\varepsilon^2 - 2(1-\nu)I_{\varepsilon\varepsilon}\right]dF + \frac{1}{2}\int_F D\left[I_\kappa^2 - 2(1-\nu)I_{\kappa\kappa}\right]dF,$$

$$I_\varepsilon = \frac{1}{8F^2}\left(\varepsilon_m l_m^2 l^2 - 2\varepsilon_m l_m^4\right),\ I_{\varepsilon\varepsilon} = \frac{1}{16F^2}\left[\left(\varepsilon_m l_m^2\right)^2 - 2\varepsilon_m^2 l_m^4\right],$$

$$I_\kappa = \frac{1}{8F^2}\left(\kappa_m l_m^2 l^2 - 2\kappa_m l_m^4\right),\ I_{\kappa\kappa} = \frac{1}{16F^2}\left[\left(\kappa_m l_m^2\right)^2 - 2\kappa_m^2 l_m^4\right],$$

$$B = Eh/\left(1-\nu^2\right),\ D = Bh^2/12,\ F = \frac{1}{4}\left(l^4 - 2l_m^2 l_m^2\right)^{1/2},\ l^2 = l_m l_m .$$

where E, ν, and h are Young's modulus, Poisson's ratio, and wall thickness of the shell, respectively, I_ε and $I_{\varepsilon\varepsilon}$ (I_κ and $I_{\kappa\kappa}$) are the first and second invariants of the strain (curvature-change) tensor of the middle surface, respectively, F is the area of the middle surface of the finite element, and l_m, ε_m, and κ_m are the length, strain, and curvature change of the mth element side, respectively.

Since the normal components ε_m and κ_m have clear physical meaning of normal elongation and normal curvature change, respectively, they can be approximated without using shape functions for the displacement fields over the element. Namely, the strain and curvature-change fields are obtained by superposing approximations of ε_m and κ_m for three independent directions. For this purpose, combinations of the beam solutions can be used.

The use of the invariants allows one to avoid constructing of the local coordinate systems related to finite elements, calculate the stiffness matrix of the finite element straightforwardly for the lengths of the element sides, simplify the formulation of the shell finite-element model, and reduce the computational work.

For the shell element presented, the question of description of finite rotations is solved by associating the element with a certain geometrical object called the kinematic group [1,2] that consists of the nodal position vectors and normal vectors to the shell middle surface and possesses the property of geometrical variability. The finite element–kinematic group association implies that the strains and curvature changes of an element are related to strain parameters that characterize changes in the kinematic group configuration.

An attempt is undertaken to improve nonlinear bending capabilities of the finite element by taking into account finite curvature changes within the element. To this end, the following assumptions are used:

(a) each side of the triangular element is a plane nearly circular curve which remains plane and nearly circular during the deformation;

(b) for each element side, the normal vector to the middle surface of the shell does not deviate from the curvature plane of this side;

(c) the strains of the element sides are constant.

It should be noted that the assumption (a) imposes no restriction on the magnitude of the curvature changes of the triangle sides. The assumption (c) implies that the strain-tensor components are constant within the element. It follows that the membrane behavior of the element is modeled in a simple manner and similar to that of the constant-strain triangle.

A three-node curved triangular element with five degrees of freedom per node (three translations of the node and two rotations of the normal vector to the middle surface) is developed. The accuracy of the shell element is studied using typical geometrically nonlinear benchmark problems of thin elastic plates and shells [3]. Numerical results obtained show that the finite element provides high accuracy and convergence rate with respect to the number of finite elements thus supporting the validity of underlying assumptions. The solutions presented are in good agreement with numerical data available in the literature. Namely, the element performs very well under pure bending loading conditions: it can be rolled up so that the mutual rotation of the normal vectors within the element can be as large as 90°.

References

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