# A NONASYMPTOTIC MODELLING OF HEAT CONDUCTION IN SOLIDS REINFORCED BY SHORT FIBRES WITH FUNCTIONALLY GRADATION OF FEATURES

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In the paper, the model of heat conduction in solids reinforced by short fibres with a functionally gradation of effective features (functionally graded materials - FGM) will be constructed. The proposed model will contain a set of equations for an averaging temperature (describing macroscopic changes) and functions called fluctuations (describing the influence of a microstructure on the heat flow). The model will be tested on the examples of numerical solutions.

## 1. Conductors reinforced by fibres

We are going to consider the conductors, whose configuration in the physical space with the Cartesian orthogonal coordinate system $(x_1, x_2, x_3) \in R^3$ will be an area $\Omega = \Pi \times (-h_3, h_3)$, where $\Pi = (-h_1, h_1) \times (-h_2, h_2)$. Those conductors will be strengthened with thin fibres. We assume that the fibres are distributed parallel to the axes $x_\alpha$, $\alpha = 1,2$.

The conductors need not to be periodical, however in the area $\Pi$ is possible to section off a part $I_1 \times I_2$, where $I_\alpha = [-\lambda_\alpha / 2, \lambda_\alpha / 2]$, $\alpha = 1,2$ and $\lambda_\alpha$ are the quantities characterizing the structure of the reinforcement.

For any $(x_1, x_2) \in \Pi$, let us determine the functions of a saturation by the fibres $v_1 = v_1(x_1, x_2)$, $v_2 = v_2(x_2, x_1)$, such that $v_1(\cdot, x_2)$ is for every $x_2$ a $\lambda_1$-periodic function, while $v_2(\cdot, x_1)$ is for every $x_1$ a $\lambda_2$-periodic function.

An essential assumption from the point of view of a method of modeling which we are going to use below, is a demand, that the functions $v_1$, $v_2$ were slowly varying with respect to the coordinates $x_2$, $x_1$, respectively, what we denote in a form: $v_1(x_1, \cdot) \in SV_\varepsilon^1(I_2)$, $v_2(x_2, \cdot) \in SV_\varepsilon^1(I_1)$. The introduced way of gradation can be a consequence of a slow variability of the length of the fibres or a slow variability of the saturation of these fibres.

In the case under consideration, the functions $v_1$, $v_2$ are not periodical and the direct application of methods of asymptotical homogenization is not effective. Therefore, we are going to use a nonasymptotic method, known as the tolerance averaging technique [1].

## 2. Equations of a heat conduction

For the described conductors is assumed that the heat conduction is held according to the Fourier's law, i.e. is described in the area $\Omega \times \langle t_0, t_1 \rangle$ by the equation

(1) $\qquad \partial_k (K_{kl} \partial_l \theta) - c\dot{\theta} = f$, $\quad k,l = 1,2,3$

where $\theta = \theta(x_1, x_2, x_3, t)$ is a temperature, $f = f(x_1, x_2, x_3, t)$ – a source of heat, $c$, $\mathbf{K} = (K_{kl})$ – a specific heat and a tensor of a heat conduction in $\Omega$, respectively, which assume the constant values $c^M$, $K_{kl}^M$ in the matrix of the conductor, $c^F$, $K^F$ in the fibres. We also assume that in the fibres a one-directional heat flow is held and $K_{kl}^M << K^F$.

The thermophysical features of the medium will be described with functions: $c: \Pi \to R$, $\mathbf{K}: \Pi \to R$ of the form as below:

$$c(x_1,x_2) = c^M + c^F (v_1(x_1,x_2) + v_2(x_2,x_1)),$$

$$K_{11}(x_1,x_2) = K_{11}^M + K^F v_1(x_1,x_2), \qquad K_{22}(x_1,x_2) = K_{22}^M + K^F v_2(x_2,x_1),$$

(2) $\qquad K_{12}(x_1,x_2) = K_{21}(x_1,x_2) = K_{12}^M$

where $v_\alpha$ are $\lambda_\alpha$-periodic functions with respect to $x_2$ and $x_1$, respectively, $0 \le v_\alpha \le 1$, $1 - v_\alpha \cong 1$. The functional coefficients (2) are oscillating and non-continuous. They describe a micrononhomogeneity of the considered conductors.

### 3. Averaged model

According to the tolerance averaging technique, we carry out a micro-macro-decomposition of a temperature in a form:

(3) $\qquad \theta(x_1,x_2,t) = \vartheta(x_1,x_2,t) + \varphi^1(x_1)\psi^1(x_1,x_2,t) + \varphi^2(x_2)\psi^2(x_1,x_2,t)$

where

(4) $\qquad \vartheta(\cdot,t), \quad \psi^A(\cdot,t) \in SV_\varepsilon^1(\Delta) \quad A=1,2$

for every $t$.

The function $\vartheta(\cdot)$, occurring in (3), can be interpreted as the macroscopic part of a temperature field $\theta(\cdot)$, whereas the functions $\psi^A(\cdot)$ describe the microfluctuations of the temperature $\theta(\cdot)$ in any part $I_1 \times I_2$. These functions are the new functions to be sought. The functions $\varphi^A(\cdot, t)$ (shape functions) are linearly independent and must be known.

According to the tolerance averaging technique, the equations for the sought functions $\vartheta(\cdot)$, $\psi^A(\cdot, t)$, $A = 1, 2$, have the form [2]:

$$-<c>\dot{\vartheta}+<K_{\alpha\beta}\partial_\alpha\varphi^A>\partial_\beta\psi^A+<K_{\alpha\beta}>\partial_\alpha\partial_\beta\vartheta =< f >$$

(5) $\qquad -<c\varphi^A\varphi^B>\dot{\psi}^B-<K_{\alpha\beta}\partial_\alpha\varphi^A\partial_\beta\varphi^B>\psi^B-<K_{\alpha\beta}\partial_\alpha\varphi^A>\partial_\beta\vartheta =< f\varphi^A >$

The equations (5) have a sense only if the functions $\psi^A(\cdot,t) \in SV_\varepsilon^1(\Delta)$, i.e. they are slowly varying functions. This condition can be proved only *a posteriori* – when these functions are already known.

### 4. Concluding remarks

In the constructed nonasymptotic model occurs $n + 1$ equations for the temperature and fluctuations, whose determination depends on certain periodic and oscillating functions called shape functions, which must be known. In the paper will be presented such functions, according to the tolerance averaging technique. Moreover, the model will be numerically tested.

### 5. References

[1] Woźniak Cz., Wierzbicki E.: *Averaging Techniques in Thermomechanics of Composite Solids*, Wyd. Politechniki Częstochowskiej, Częstochowa, 2000.

[2] Nagórko W.: Przepływ ciepła w przewodnikach wzmocnionych siatką włókien z funkcjonalną gradacją własności materiałowych, *I Kongres Mechaniki Polskiej*, Warszawa, 2007.