GEOMETRIC SENSITIVITY ANALYSIS OF TRUSS AND FRAME STRUCTURES

D. Bojczuk, M. Jabłoński
Kielce University of Technology, Faculty of Management and Computer Modelling, Kielce, Poland

1. Introduction

The problem of manufacturing imperfections and their influence on structural configuration is discussed in the paper. When a structure is composed of members with dimension imperfections, states of stresses, strains and displacements induced by external load will vary and additionally in hyperstatic structures self-equilibrated states of stresses will arise. The influence of these effects on the optimal design was analyzed in [3] and [4]. However, length imperfections even in unloaded structure may induce variations of geometry. The problem of determination of these changes for selected points of a structure, which can be called *geometric sensitivity analysis*, is studied in this paper. It is extension of the previous considerations presented in [2], and here also geometrically nonlinear case is analyzed. It is important to notice, that the cumulative change in point position may be far greater then it would appear from the value of members tolerances and it even may cause that the structure will reach unstable state corresponding to the limit point.

2. Geometric sensitivity analysis – linear case

Let us consider a truss or a frame composed of members, which have nominal lengths $l_1, l_2, ..., l_n$, where n is the number of members. It is assumed that member lengths may deviate from their nominal values by tolerances $t_1^{(l)}, t_2^{(l)}, ..., t_n^{(l)}$. The change of *i*-th member length arising directly from the tolerance can be expressed in the form $\Delta l_i^{(t)} = \alpha_i t_i^{(l)}$, where $|\alpha_i| \le 1$, $i = 1, 2, ..., n$.

The first problem analyzed here is to determine maximal translation w in fixed direction of certain point (node) induced by considered tolerances. Let us introduce the adjoint structure without any dimension imperfections and with the same boundary conditions as the primary structure but subjected to force λP^a applied in direction of the analyzed displacement w, where λ is the load parameter and P^a denotes the reference load. Now, the virtual work equation for the primary and adjoint structures can be written in the form

$$\lambda P^a w = \sum_{i=1}^{n} \left(N_i^a \Delta l_i^{(c)} + \int_{l_i} M_g^a \kappa^{(c)} dx_i \right), \tag{1}$$

where $\Delta l_i^{(c)}$, $\kappa^{(c)}$ are the total elongations of members and the total curvatures in the primary structure, while N_i^a, M_g^a denote the normal forces and the bending moments in the adjoint structure. Let us notice, that analyzed tolerances induce self-equilibrated state of stress with elastic elongations $\Delta l_i^{(el)} = \Delta l_i^{(c)} - \Delta l_i^{(t)}$ and elastic curvatures $\kappa^{(el)} = \kappa^{(c)}$. Using for this state and for the adjoint structure the virtual work principle, we get

$$\sum_{i=1}^{n} \left(N_i^a \Delta l_i^{(el)} + \int_{l_i} M_g^a \kappa^{(el)} dx_i \right) = 0. \tag{2}$$

In order to obtain relationship for the translation w in the selected direction we should substitute equation (2) into (1). Now, it is easy to notice, that the maximal translation occurs, when $\Delta l_i^{(t)}$ attain extreme values i.e. when $\alpha_i = 1$ or $\alpha_i = -1$ and when for all members it has simultaneously the same (or simultaneously the opposite) sign as corresponding force N_i^a. So, taking $\lambda P^a = 1$, the maximal translation in fixed direction can be written as follows

(3) $$\left|w\right|_{max} = \max_{\alpha_i}\left|\sum_{i=1}^{n} N_i^a \alpha_i t_i^{(l)}\right| = \sum_{i=1}^{n}\left|N_i^a t_i^{(l)}\right|.$$

Now, let us consider problem of determination of maximal translation w_c of certain point (node) in arbitrary direction. We assume, that this direction forms unknown angle β with axis x of the rectangular coordinate system x, y. The translation w_c can be determined using translations in x and y directions. Then, we have

(4) $$\lambda P^a w_c = \lambda P^a \cos\beta \cdot w_x + \lambda P^a \sin\beta \cdot w_y = \sum_{i=1}^{n}\left(N_i^{a(x)}\cos\beta + N_i^{a(y)}\sin\beta\right)\alpha_i t_i^{(l)},$$

where $N_i^{a(x)}, N_i^{a(y)}$ denote forces in i-th member induced by unit loads $\lambda P^a = 1$ applied respectively in x and y direction. Finally, maximal translation can be determined as the solution of the following problem

(5) $$\left|w_c\right|_{max} = \max_{\beta}\left[\sum_{i=1}^{n}\left|\left(N_i^{a(x)}\cos\beta + N_i^{a(y)}\sin\beta\right)t_i^{(l)}\right|\right].$$

3. Geometric sensitivity analysis – non-linear case

In this case the adjoint problem is introduced by analogy to the incremental problem for the primary structure related to perturbation of the current equilibrium state. Then, using finite element notation, equilibrium equation for the adjoint structure can be written in the form $\mathbf{K}^t\mathbf{u}^a = \dot{\lambda}P^a$ (cf. [1]), where $\mathbf{K}^t$ is the tangent stiffness matrix and $\mathbf{u}^a$ denotes vector of displacements. The load $\dot{\lambda}P^a$ is applied, as previously, in direction of the analyzed displacement w, where point over the symbol denotes increment of the quantity. Using approach analogous as for the linear case, we get

(6) $$\dot{\lambda}P^a w = \sum_{i=1}^{n} N_i^a \alpha_i t_i^{(l)}.$$

In order to obtain the maximal translation, non-linear equation (6) should be solved with respect to w, where values α_i are chosen analogously as in (3). It is important to notice that during incremental process of solving the problem (6), situation when $\dot{\lambda} = 0$ may appear and it corresponds to structure geometric instability related to the limit point.

4. Concluding remarks

The problem of maximal translation determination for certain points (nodes) of truss or frame structures induced by length imperfections of their members, is discussed in the paper. Apart from the linear analysis, in order to reveal possible geometric instabilities of the limit point type, the considerations are also developed for the non-linear case.

5. References

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