APPLICATION OF THE STABILIZATION METHOD FOR ANALYSIS OF GEOMETRICALLY NON-LINEAR FORCED VIBRATIONS OF ELASTIC BEAMS ON UNILATERAL WINKLER FOUNDATION

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The computational model of geometrically non-linear elastic beam is frequently used for analysis of dynamics of one-dimensional distributed systems on unilateral elastic foundation. The problems of forced vibrations of deformable solids under unilateral constraints are nonlinear problems with conditions expressed in the form of inequalities. Besides, in most cases it is necessary to take into account the dissipation of energy that is caused by external viscous resistance.

As a rule, capabilities of analytical methods for solving this type of problems are limited to discrete systems with a small number of freedoms. That's why the principal role in solving forced vibrations of deformable solids under unilateral constraints belongs to numerical methods. A stabilization method for computational modeling of geometrically non-linear forced vibrations of elastic beams on unilateral Winkler foundation is presented. This method was used earlier for computational modeling of forced vibrations of viscoelastic solids under unilateral contact [1] and geometrically non-linear forced vibrations of elastic beams without unilateral constraints [2].

It is well known, that if damping is present in a system then initial conditions have considerable effect on forced vibrations of the deformable solids only during a limited period of time after which the system moves to a steady-motion state. Therefore, the main idea of this approach is that the T-periodical solution of the original problem can be found as a solution of the Cauchy problem when damping is present in the system. In this case initial conditions can be chosen arbitrarily.

Beam deformation is described by the Timoshenko model. It is proposed that a value of resistance forces is proportional to velocity. Geometrically nonlinear equations of motion for Timoshenko beam on unilateral Winkler foundation are as follows

$$\frac{\partial Q}{\partial x}+\frac{\partial}{\partial x}\left(N\frac{\partial w}{\partial x}\right)+q-c\cdot H(w-\phi)\cdot(w-\phi)-\varepsilon\frac{\partial w}{\partial t}-\rho F\frac{\partial^2 w}{\partial t^2}=0\,,$$

$$\frac{\partial M}{\partial x}-Q-\rho J\frac{\partial^2\gamma}{\partial t^2}=0\,,\qquad \frac{\partial N}{\partial x}-\rho F\frac{\partial^2 u}{\partial t^2}=0\,,$$

where w is the transverse displacement; γ is the angle of rotation of the normal relatively to the axis of the beam; u is the longitudinal displacement; M is the bending moment; Q is the shear force; N is the normal force; ρ is the mass per unit of length; F is the area of the cross section; J is the moment of inertia of the cross section; q is T-periodic transversal distributed load; c is the foundation stiffness; ϕ is the clearance between the beam and the foundation; $H(\cdot)$ is Hevyside function; ε is the viscous damping coefficient per unit of length.

The forces Q, N and the bending moment M are related to the displacements w, u and the angle of rotation γ by the constitutive relations

$$M=EJ\frac{\partial\gamma}{\partial x},\quad Q=k^2GF\left(\frac{\partial w}{\partial x}+\gamma\right),\quad N=EF\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right),$$

where E is Young's modulus of the beam material; G is the shearing modulus; $k^2 = 5/6$ for a rectangular cross section.

For definiteness the boundary conditions have the following form:

$$w(0) = w(l) = 0; \quad M(0) = M(l) = 0; \quad u(0) = u(l) = 0 .$$

Through a standard variational procedure, we obtain the following variational equation:

$$\int_0^l \left(\rho F \frac{\partial^2 w}{\partial t^2}(\tilde{w} - w) + \rho J \frac{\partial^2 \gamma}{\partial t^2}(\tilde{\gamma} - \gamma) + \rho F \frac{\partial^2 u}{\partial t^2}(\tilde{u} - u) + \varepsilon \frac{\partial w}{\partial t}(\tilde{w} - w) \right) dx +$$

$$+ \int_0^l \left(k^2 GF \left(\frac{\partial w}{\partial x} + \gamma \right) \left(\frac{\partial \tilde{w}}{\partial x} + \tilde{\gamma} - \frac{\partial w}{\partial x} - \gamma \right) + EJ \frac{\partial \gamma}{\partial x} \left(\frac{\partial \tilde{\gamma}}{\partial x} - \frac{\partial \gamma}{\partial x} \right) + EF \frac{\partial u}{\partial x} \left(\frac{\partial \tilde{u}}{\partial x} - \frac{\partial u}{\partial x} \right) \right) dx +$$

$$+ \int_0^l EF \left(\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) \frac{\partial w}{\partial x} \left(\frac{\partial \tilde{w}}{\partial x} - \frac{\partial w}{\partial x} \right) dx + \frac{1}{2} \int_0^l EF \left(\frac{\partial w}{\partial x} \right)^2 \left(\frac{\partial \tilde{u}}{\partial x} - \frac{\partial u}{\partial x} \right) dx -$$

$$- \int_0^l q(\tilde{w} - w)\, dx + \int_0^l c \cdot H(w - \phi) \cdot (w - \phi)(\tilde{w} - w) dx = 0 , \quad \forall (\tilde{w}, \tilde{\gamma}, \tilde{u}) \in K .$$

where K is the set of the kinematically admissible beam displacements.

This equation expresses the principle of admissible displacements for the elastic beam on unilateral Winkler foundation and includes only first spatial derivatives of the displacements.

The finite difference method is used for time semi-discretization of the variational equation. The second and the first time derivatives are approximated with three-point central differences. As a result an explicit three-layer scheme is used for numerical time integration. The minimization problem which is equivalent to the obtained variational problem on each time step is derived.

The finite element method is used for the spatial discretization of the minimization problem. Two-nodal and three-nodal Lagrange finite elements are used.

A software package based on the described computational algorithm was developed. Numerical solutions of a number of problems were obtained and convergence of the computational algorithms was investigated. The influence of foundation compliance on the solution behavior was investigated. Specifics of amplitude-frequency dependencies of stresses and displacements were researched. It is known that an amplitude-frequency dependency for the elastic beam is not unique due to geometrical nonlinearity, i.e. a few values of the amplitude can correspond to the one value of the frequency near a resonance. It is a difficulty for numerical solving. In this study the continuation method is used to derive the amplitude-frequency curves. The frequency of forced vibrations is chosen as a continuation parameter. The calculation was performed in two stages. On the first stage the frequency of forced vibrations was increased in the range under investigation. On the second stage the frequency was decreased from the maximum to the minimum values. The solution for the previous value of the frequency was used for the initial condition.

Performed computational experiments confirmed effectiveness of suggested methods for solving problems of geometrically non-linear forced vibrations of elastic beams on unilateral Winkler foundation.

[1] A. Bobylov and E. Suturin (2005). Application of the stabilization method for analysis of forced vibrations of viscoelastic solids under unilateral constraints. *8th Conference on Dynamical Systems – Theory and Applications, Lodz, Poland,* 269-276.

[2] A. Bobylov and A. Zubko (2007). Application of the stabilization method for analysis of geometrically nonlinear forced vibrations of an elastic beam. 9th *Conference on Dynamical Systems – Theory and Applications, Lodz, Poland,* 553-560.