LEVEL SET METHOD IN STRUCTURAL OPTIMIZATION

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1. Introduction

The paper is concerned with the numerical solution of a structural optimization problem for an elastic body in unilateral contact with a rigid foundation. Shape optimization of contact problems is considered, among others, in [3, 7] where necessary optimality conditions and numerical results are provided. The material derivative method is employed in monograph [7] to calculate the sensitivity of solutions to contact problems as well as the derivatives of domain depending functionals with respect to variations of the boundary of the domain occupied by the body. Topology optimization deals with the optimal material distribution within the body resulting in its optimal shape [1, 5, 8]. The topological derivative [8] is employed to account variations of the solutions to state equations or shape functionals with respect to emerging of small holes in the interior of the domain occupied by the body. The notion of topological derivative and results concerning its application in optimization of elastic structures are reported in many papers (see references in [8]).

2. Problem formulation

In the paper the elastic contact problem with a given friction, described by Coulomb law, is considered. The displacement field of the body in unilateral contact is governed by an elliptic variational inequality of the second order [2]. The structural optimization problem for the elastic body in contact consists in finding such topology of the domain occupied by the body and the shape of its boundary that the normal contact stress along the boundary of the body is minimized. The volume of the body is assumed to be bounded.

3. Necessary optimality condition

Introducing an adjoint system and using the material derivative method we calculate shape derivative of the cost functional in direction of the velocity field V with respect to perturbations of the boundary of the domain occupied by the body. Asymptotic expansion method is used to calculate topological derivative of this cost functional with respect to the inserting of a small ball at a point inside the optimized domain. Formulae of these derivatives are provided [4]. These derivatives are employed to formulate a necessary optimality condition for simultaneous shape and topology optimization problem and to calculate descent direction in the numerical algorithm.

4. Level set method

In structural optimization the level set method [6] is employed in numerical algorithms for tracking the evolution of the domain boundary on a fixed mesh and finding an optimal domain. This method is based on an implicit representation of the boundaries of the optimized structure, i.e., the position of the boundary of the body is described as an isocountour of a scalar function of a higher dimensionality. While the shape of the structure may undergo major changes the level set function remains to be simple in its topology. The evolution of the domain boundary is governed by Hamilton - Jacobi equation. The speed vector field driving the propagation of the level set function is given by the Eulerian derivative of the cost functional with respect to the variations of the free boundary. Applications of the level set methods in structural optimization can be found, among others, in [1].

Recently [9], different numerical improvements of the level set method employed for the numerical solution of the structural optimization problems are proposed and numerically tested.

5. Numerical Methods and Results

The structural optimization problem is solved numerically as the simultaneous shape and topology optimization problem. Contact system as well as the adjoint system are discretized and numerically solved using finite element method and primal - dual algorithm with active set strategy. Lagrange multiplier method is used to solve the structural optimization problem. First this problem is solved as topology optimization problem and at grid points where the topology derivative is negative the holes are created. Next shape optimization problem is solved. During this step in Hamilton - Jacobi equation velocity field V is set equal to the calculated shape gradient of the cost functional. Finite difference method and explicit up - wind scheme are used to solve Hamilton - Jacobi equation. Numerical examples indicating that the proposed numerical algorithm allows for significant improvments of the structure from one iteration to the next are provided and discussed.

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