ESTIMATION OF MATERIAL EFFORT DURING DRYING PROCESSES

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ABSTRACT

One of the main problems accompanying drying of saturated porous materials (e.g. ceramics, wood, and others) is the problem of cracking initiated by the drying induced stresses. This destructive effect appears very often at the surface of dried products, but not always. Sometimes cracks occur in a strange place inside the material. The reason for that is of different nature as, for example, the pre-existing flaws, the stress reverse phenomenon, or the accumulation of energy coming from several components of the stress tensor.

The aim of this paper is to discuss in more detail the problem of mechanical energy accumulation as well as the effort of material under drying according to the energetic hypothesis. This hypothesis allows calculating the overall stress, which is necessary to formulate the strength condition for a given material. Such an approach is always necessary when more components of the stress tensor appear in a dried sample.

The problem of energy effort in dried materials is very complex as the mechanical properties of such materials change themselves during the process. In order to grasp adequately this problem one has to use a mechanistic model of drying, in which the mechanical coefficients have to be depended on the moisture content. Only such a model may allow to obtain the adequate values of the stress components and to calculate properly the overall stress.

On the other hand the admissible stress, which has to be determined for the purpose of the strength condition, also changes itself along with the moisture content variation. This stress has to be determined in separate experimental tests for given material, similarly as the mechanical coefficients that are involved in the drying model.

The objective of the present consideration is the analysis of the stress state in a cylindrically shaped sample made of kaolin-clay and subjected to convective drying. The distribution of stress components throughout the cylinder and their evolution in time is determined. These stress components allow calculating the overall stress as a function of place and moisture content. The map of the cylinder space presenting the points of possibly violated strength condition is given.

Distributions and time evolutions of liquid content X (dry basis) and temperature T are determined for the first and second period of drying using the differential equations that include the phase transitions of liquid into vapour and the diffusive and thermodiffusive moisture transport, [1]:

$$\dot{X} = D_X \nabla^2 (c_T T + c_X X) - \Omega (c_T T + c_X X), \qquad \dot{T} = D_T \nabla^2 T - l \frac{\Omega}{\rho^s c_v} (c_T T + c_X X) \tag{1}$$

where D_X and D_T denote the mass and thermal diffusivity, c_X and c_T express the ratio of diffusion and thermodiffusion, Ω expresses the intensity of phase transition of liquid into vapour, l is the latent heat of evaporation, c_v is the specific heat, ρ^s is the density of dry body, and ∇^2 denotes the Laplace operator in cylindrical coordinates.

The boundary conditions for the heat and mass transfer express the convective exchange of heat and vapour between cylinder and the ambient air, and the symmetry conditions with respect to the middle of the cylinder. The initial conditions assume the uniform distribution of moisture and temperature.

The following system of two coupled equations is used for determination of radial and longitudinal displacements u_r and u_z

$$M\nabla^2 u_r + \frac{\partial}{\partial r}\left[(M+A)\varepsilon - \gamma_T T - \gamma_X X\right] = M\frac{u_r}{r^2}, \quad M\nabla^2 u_z + \frac{\partial}{\partial z}\left[(M+A)\varepsilon - \gamma_T T - \gamma_X X\right] = 0 \tag{2}$$

where $\gamma_T = (2M + 3A)\kappa^{(T)}$, $\gamma_X = (2M + 3A)\kappa^{(X)}$, $\kappa^{(T)}$ and $\kappa^{(X)}$ are the coefficients of linear thermal and humid expansion, ε is the volumetric strain, $M(X)$ and $K(X)$ are the elastic shear and bulk modules dependent on moisture content.

Since no any external surface forces acting on the cylindrical sample the radial and longitudinal stresses on the external surfaces equal zero. The other two boundary conditions assume zero-valued radial and longitudinal displacements at cylinder axis and at the bottom of the cylinder, that is

$$\sigma_{rr}\big|_{r=R} = 0, \qquad \sigma_{zz}\big|_{z=H} = 0, \qquad u_r|_{r=0} = 0 \quad \text{and} \quad u_z|_{z=0} = 0 \tag{3}$$

The state of stress in the cylinder is fully described by the components σ_{rr}, $\sigma_{\varphi\varphi}$, σ_{zz}, σ_{rz}, where

$$\sigma_{ij} = 2M\varepsilon_{ij} + \left(A\varepsilon - \gamma_T T - \gamma_X X\right)\delta_{ij}, \qquad \varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad \varepsilon = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \tag{4}$$

The overall (reduced) stress and admissible stress [2] read.

$$\sigma_{red} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{rr} - \sigma_{zz})^2 + (\sigma_{rr} - \sigma_{\varphi\varphi})^2 + (\sigma_{\varphi\varphi} - \sigma_{zz})^2 + 6\sigma_{rz}^2}, \quad \sigma_{adm} = \sigma_0 + \sigma_X \exp(-C_\sigma X) \tag{4}$$

Figure 1 presents the mapping of stress difference between σ_{adm} and σ_{red} in quarter plane of the cylinder

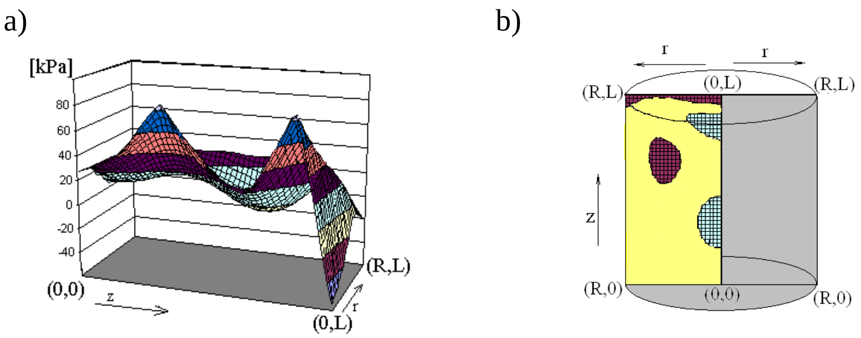

Fig. 1. Difference ($\sigma_{adm} - \sigma_{red}$) in a quarter plane of the cylinder: a) spatial mapping, b) flat visualization of the places with violated strength condition.

The places in which ($\sigma_{adm} - \sigma_{red}$) < 0 denote violation of the material strength (dark area in Fig. 1b).

References

[1] Kowalski S.J., Rybicki A., Residual Stresses in Dried Bodies, *Drying Technology*, **25** (4), 2007.
[2] Musielak G., Modelling and numerical simulations of transport phenomena and drying stresses in capillary-porous materials, Ed. Poznań University of Technology, 2004, (in Polish.)