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New technique of vibration control

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A new semi-active control strategy for damping out vibrations of structures is presented in this paper. This control law consists basically of releasing at every local vibration peak the previously accumulated prestress effect as instantaneously as possible (on-off). This approach is known as PAR (Prestress Accumulation-Release) strategy. Both theoretical studies as well as numerical simulations and experimental results are presented to show the efficiency of the proposed approach. Two types of structures are considered: statically redundant truss beams and multi-layer beams. In the first case, actuators are semi-active joints allowing sudden member's disconnection while in the second one actuators are devices able to stick and unstick adjacent layers (delamination effect). Observed reduction of the response is fast showing a significant increase in the structural damping. Agreement between numerical and experimental results is good. Sensitivity to the main control parameters (instants for beginning and end of the prestress effect release) is investigated. The so-called Virtual Distortion Method (VDM) is used to select the optimal locations for actuators. Because of the simplicity of the required actuators the proposed strategy can be implemented in many other situations.

1. Introduction

The main objective of this chapter is to present and to discuss a new, semi-active technique for damping out vibrations of structures (after [6]). This technique is based on controlling of the cyclic accumulation/release of prestress effect, due to switching off/on of specially designed structural interfaces. Let us demonstrate on the simple example of numerically simulated vibration of the mass supported by two elastic elements (one of them switchable) shown in Fig. 1, that the controlled prestress effect can give a substantial overall damping result. The first element is the actively controlled member (*the active interface).* Assume cross-sections $A_I = A_{II}$ and the material properties $E_I = E_{II}$ for both elements of the structure, excited to free vibration shown in Fig. 2 as the line $t^{-1}, 0, t^1$.

The presented concept of damping control is based on the *Prestress Accumulation-Release* (PAR) strategy realising the following actions:

- switch off the interconnection when $\dot{u} = 0$ (the turning point with extremal deflection)
- reactivate interconnection just after the full release of the accumulated prestress effect.

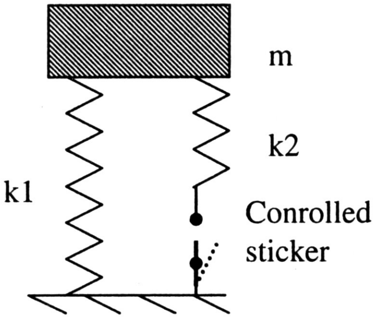


FIGURE 1. Spring-mass system and applied notation.

According to the above control strategy, let us provoke disconnection of the first structural element in the instant $t = t^1$ of the extreme deflection of the mass m (Fig. 2). In the consequence, the stress in the element I drops to zero and its elastic strain energy is released (as in the brittle fracture), what corresponds to the point $t^{1'}$ in Fig. 2. The side effect of this operation is instantaneous generation of incompatible strain (so called *virtual distortions*) equal to the current strain of the member $\varepsilon_I^o = \varepsilon_I(t)$ causing accumulation of *prestress effect* in the next phase of vibration. The first two phases of movement can be described by the following equations:

$$\begin{aligned} (A_I + A_{II})\, E\varepsilon(t) + m\ddot{u}(t) = 0 \quad \text{for} \quad t \in [t^{-1}, t^1], \\ A_I E\left(\varepsilon_I(t) - \varepsilon(t^1)\right) + A_{II} E\varepsilon(t) + m\ddot{u}(t) = 0 \quad \text{for} \quad t \in [t^1, t^2] \end{aligned} \tag{1}$$

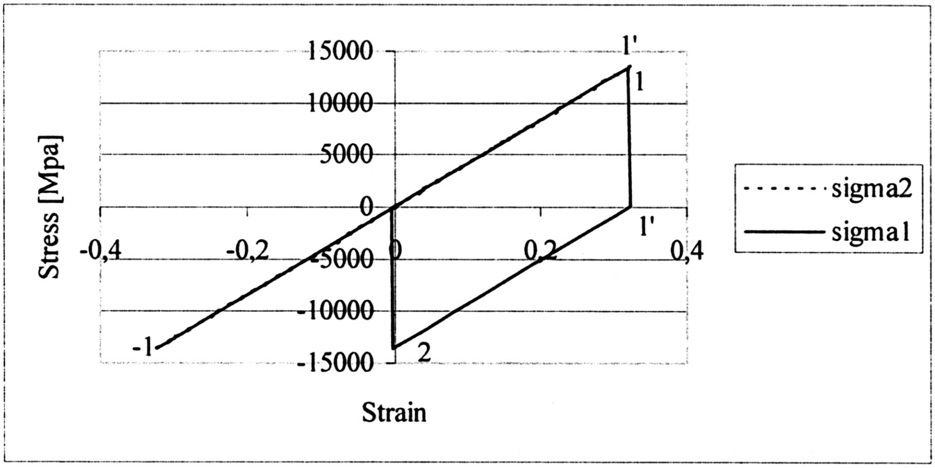
where common deformations $\varepsilon_I(t) = \varepsilon_{II}(t) = \varepsilon(t)$ for both elastic elements are guaranteed when the *active interface I* is *switched on*. However, in the "turning point" $t = t^1$ the virtual distortion $\varepsilon_I^o(t) = \varepsilon_I(t')$ is generated instantaneously in the active member.

It follows from the above formula that the second phase of the movement (from the instant $t^{1'}$ to t^2) generates prestress accumulation effect due to incompatibility between structural elements. The corresponding equation of motion $(1)^2$ can be rearranged as follows:

$$ml\frac{A_{II}}{A_I + A_{II}}\ddot{\varepsilon} + E\frac{A_{II}}{A_I + A_{II}}\alpha\dot{\varepsilon} + E\varepsilon = 0 \tag{2}$$

where: $\ddot{\varepsilon} = \ddot{u}/l$ and the coefficient $\alpha = \frac{\varepsilon_I^o}{\dot{\varepsilon}} = \frac{\varepsilon(t^1)}{\dot{\varepsilon}} - \frac{(A_I + A_{II})\varepsilon^L}{A_{II}\dot{\varepsilon}}$ is positive and increasing up to the instant $t^{1'-2}$, when velocity $\dot{\varepsilon}$ takes its maximum and acceleration $\ddot{\varepsilon}$ vanishes. In the consequence we can observe the damping effect (corresponding to the damping coefficient α) related to the accumulated prestress. It follows from Eq. 1 that internal forces between both members are self-equilibrated in the instant, when $\varepsilon = \frac{A_{II}}{A_I + A_{II}}\varepsilon_I^o$ (or, substituting ε_I^o, in the instant when $\varepsilon = \frac{A_{II}}{A_I + A_{II}}\varepsilon(t^1)$). In our case $(A_I = A_{II})$ this point corresponds to the deformation $\varepsilon(t^1)/2$ of elastic member. The point 2 corresponds to the second turning point (with velocity reduced to zero) which appears to be the finally stabilised position. In this way, switching once the interconnection*off* an *on* the vibration-damping-control strategy PAR can be fully realised. Results shown in Fig. 2 are obtained numerically, integrating equations of motion (2).

(a) Stress-strain histeretic loop.



(b) Mass displacement and velocity history.

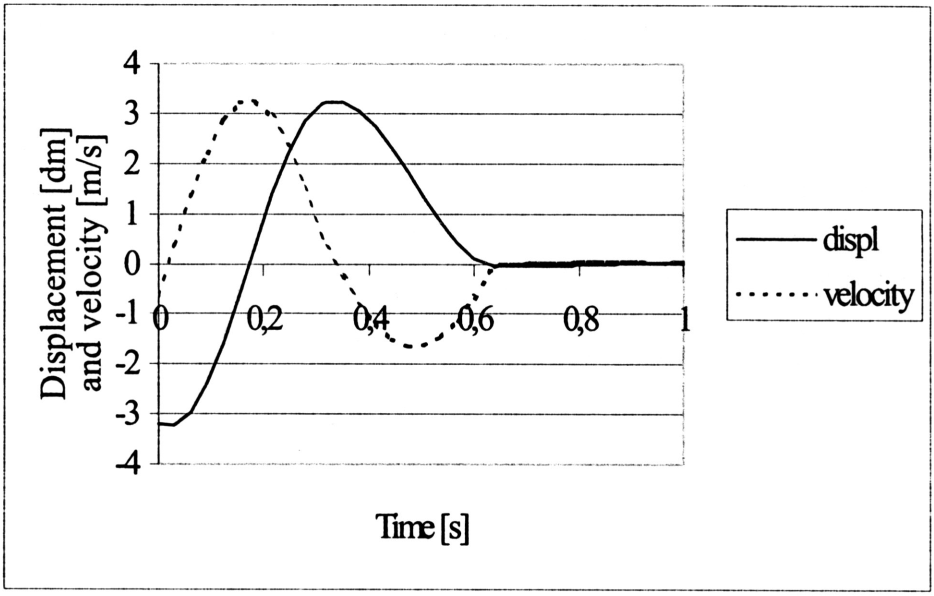
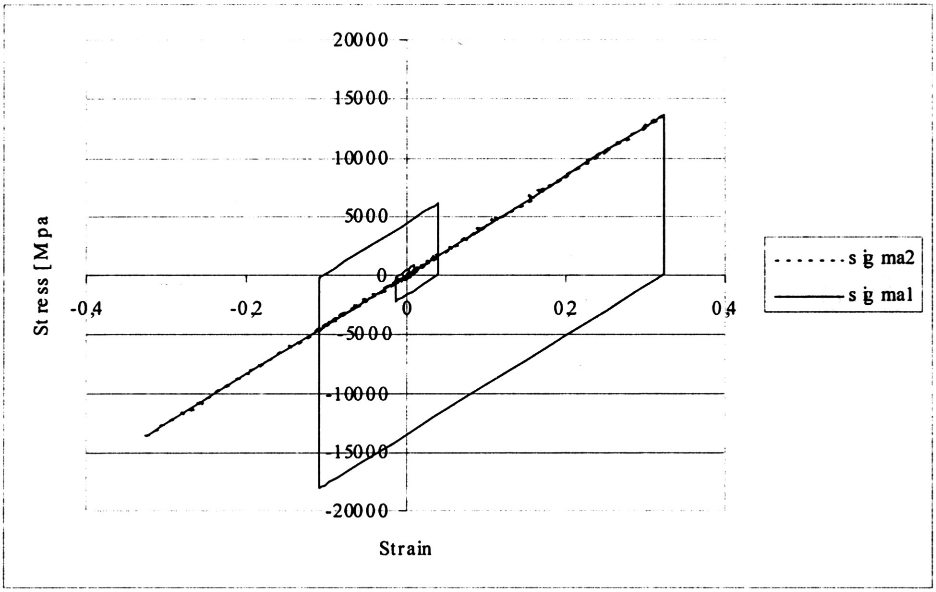


FIGURE 2. Instantaneous switching case ($\Delta t = 0, E_I = E_{II}$).

(a) Stress-strain histeretic loop.



(b) Mass displacement and velocity history.

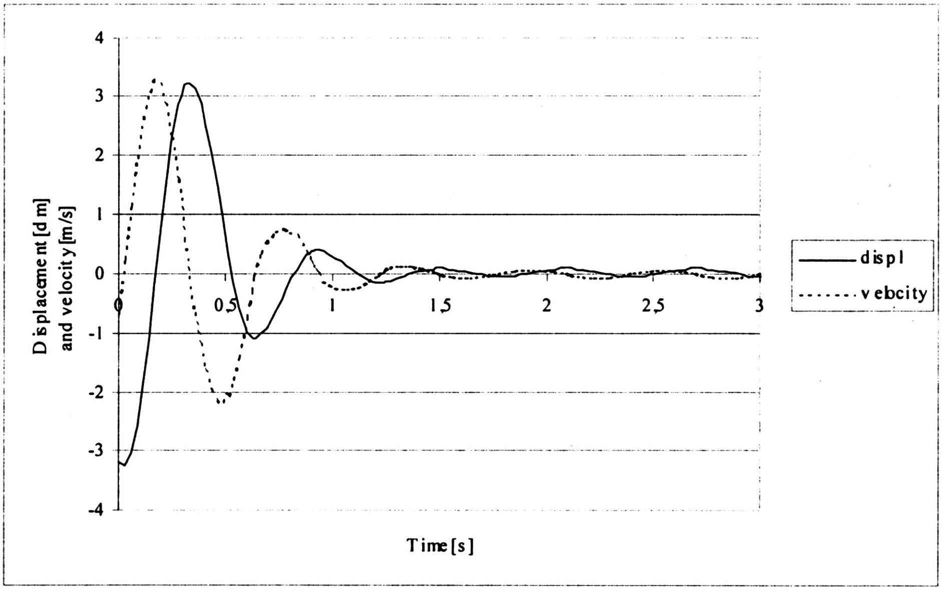
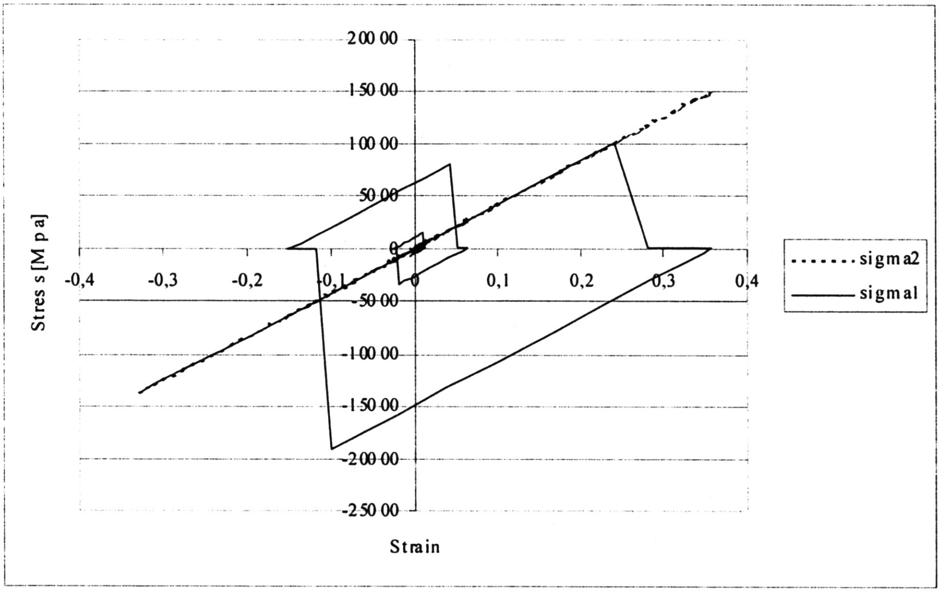


FIGURE 3. Instantaneous switching case ($\Delta t = 0$, $E_I = 2E/3$, $E_{II} = 4E/3$).

(a) Stress-strain histeretic loop.



(b) Mass displacement and velocity history.

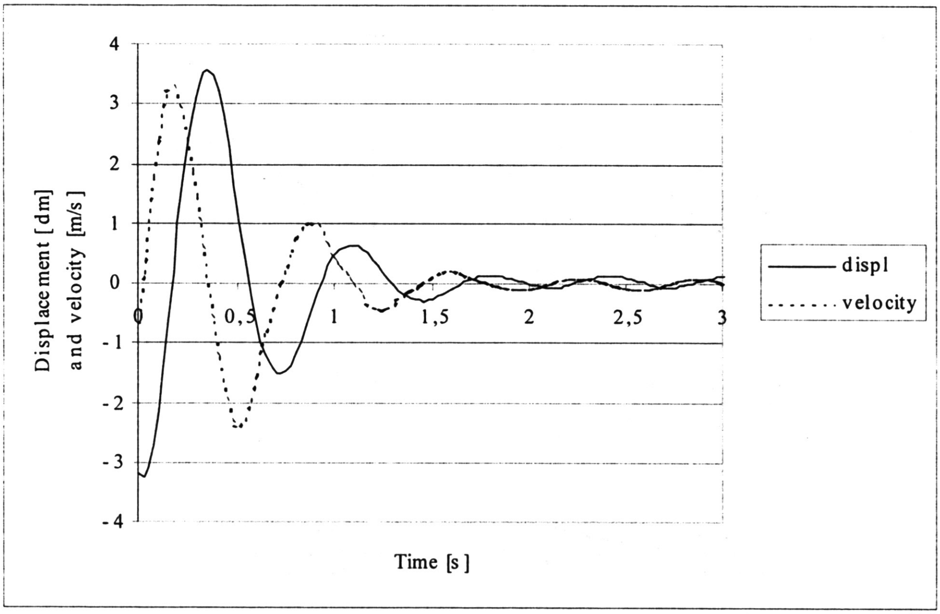


FIGURE 4. Finite switching time case ($\Delta t_1 = \Delta t_2 = 0.1$ sec., $E_I = 2E/3$, $E_{II} = 4E/3$).

Of course, the above example is quite idealised. If the stiffness proportion between the active and the passive members are different, the overall damping process requires several switching actions (see Fig. 3 for the case: $E_I = 2E/3$, $E_{II} = 4E/3$). Also, the instantaneous switching process should be modelled more adequately in real applications. Assuming that the minimal time necessary for full opening of the sticking device is Δt_1 we can search for the most effective instant for the switching device activation (e.g. Δt_2 before the turning point t^1) to get the most effective overall damping. The corresponding results (for the case: $\Delta t_2 = \Delta t_1$ and $E_I = 2E/3$, $E_{II} = 4E/3$) are demonstrated in Fig. 4.

Note the angle $\gamma E = (1 - D)E/D$ (cf. Figs. 4a and 6) of inclination for the effective line describing stress reduction in active member, to be discussed in the next section. Note also, that in more complex systems sudden release of a portion of strain energy (e.g. related to the first mode of vibration) has the following consequences:

- reduction of feeding up the kinetic energy of the first mode in the next phase of movement,
- accumulation of prestress energy slowing down movement of the first mode in the next phase of vibration,
- spill-over effect transporting vibration to higher (easily suppressed due to natural damping) modes.

Actuators have to be fast and modifying structural characteristics rather than introducing extra energy into the system. Because of the simplicity of the control rules the proposed strategy can be implemented in many situations. The only requirement is that the structure is statically redundant and that a significant part of the prestress energy (self-accumulated due to generated in the controlled way strain incompatibilities ε^o) generated during vibration can be released by freeing instantaneously some attached

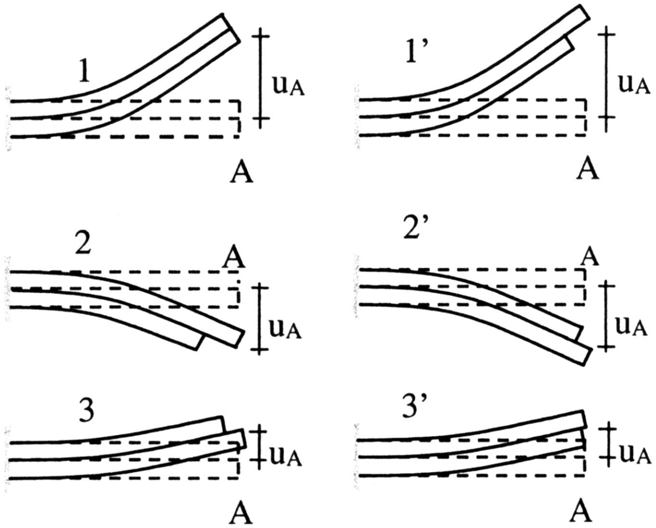


FIGURE 5. PAR approach for a two-layer beam.

parts of the structure. In the case described in Fig. 5 this is accomplished by separating the two adjacent layers. Other situations might be: plates or shells (composed of several layers) or trusses. In this last case the actuators can be either devices placed in the mid of the members or in the joints. In the first possibility the energy is released by uncoupling longitudinally (or rotationally) the two halves of each member and in the second one by allowing longitudinal movement (or rotations) of the rods connected to the joint. Usually, the more actuators are installed the better results are obtained.

The main concept has been already formulated and discussed on the classical elastic beam model [2]. The numerical simulation of the active damping process has been discussed in [3]. Similar approach to the problem of strain energy dissipation, with use of controllable rotational dissipaters in joints of skeletal structures, has been also proposed [1]. The general problem formulation and the robustness of the PAR strategy for the active damping of vibration will be demonstrated through numerical simulations as well as experimental verification.

Figure 5 shows the application of PAR approach for a two-layer beam. This situation is similar to the one described in Figs. 1 and 2.

The control strategy of dissipaters in this cantilever beam (to reduce the first mode of vibration) can be formulated as follows: instantaneous full opening of dissipaters and full locking after a short time interval whenever $\dot{u}_A = 0$ (extremal deflection of the beam) where u_A denotes the vertical movement of point A. In that Figure, releases are generated at instants 1, 2 and 3. It will be demonstrated through numerical simulation as well as experimental verification that this controlled *delamination effect* is very effective due to high portion of prestress energy to be accumulated and released in the PAR strategy.

The crucial point in PAR approach is the selection of the most effective location of the dissipaters, what is discussed in the next section. For simplicity of presentation

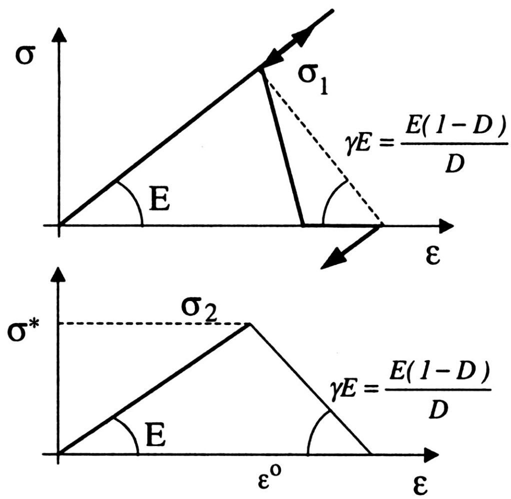


FIGURE 6. Characteristics of adaptive member.

the concept is demonstrated on ideal quasi-static model of dissipative effect in the second section and then, fully dynamic simulation of the adaptive structure behaviour is demonstrated on numerical examples in the next one. Finally, experimental verification and general comments are presented.

2. Optimal location of active members

The quasi-static strains and stresses in adaptive truss structures (equipped with *adaptive* members able to generate virtual distortions ε^o modelling brittle fracture like effect) can be expressed as follows (see so called VDM method [1], [2]):

$$\begin{aligned} \sigma_i &= E_i\left(\varepsilon_i - \varepsilon\right) = E_i\left(\varepsilon_i^* + \sum_k \left(D_{ik} - \delta_{ik}\right)\varepsilon_k^o\right) \\ \varepsilon_i &= \varepsilon_i^* + \sum_k D_{ik}\varepsilon \end{aligned} \tag{3}$$

where ε^* denotes linear structure response (without adaptive members) and D_{ij} (so called *influence matrix*) denotes deformations caused in the members i by the unit virtual distortions ε^o generated in members j. The subscripts k in the above formulas run through all self-adapted members.

The piece-wise linear modification of the constitutive law for an adaptive member (due to generated virtual distortions) can be expressed through the following requirement of brittle-fracture-like behaviour

$$\sigma_i = 0 \tag{4}$$

Substituting $(3)_1$ to (4) we obtain:

$$\sum_k [D_{lk} - \delta_{lk}]\varepsilon_k^o = -\left(\frac{\sigma_l^*}{E}\right) \tag{5}$$

and

$$\varepsilon_k^o = \sum_l [D_{lk} - \delta_{lk}]^{-1}\left[-\left(\frac{\sigma_l^*}{E}\right)\right] \tag{6}$$

where k and l runs through self-adaptive members.

The prestress effect due to virtual distortions determined by Eq. (6) can be defined by the following measure:

$$E^o = \frac{1}{2E}\sum_l A_l l_i \left(\sigma_l^R\right)^2 \tag{7}$$

where

$$\sigma_i^R = E_i\sum_k (D_{ik} - \delta_{ik})\varepsilon_k^o \tag{8}$$

Looking for the optimal location of adaptive members we can search for the configuration maximising the objective function (7). Note, that the contribution of an element to this function depends not only on the local stress σ^* value but also on its stiffness and compliance (described by elements of the matrices $[D-I]$ and $[D-I]^{-1}$, respectively). Note also, that the rank k of the main matrix of the set of equations (8) is equal to the structural redundancy and therefore, the solution (9) exists only if the set of adaptive members is not bigger than k and the main matrix is not singular.

Simplifying the problem of optimal location for one active member we can search for the location "k" with maximal expression:

$$\varepsilon_k^o = \frac{-\varepsilon_k^L}{D_{kk}-1} \tag{9}$$

and then, the prestress energy can be calculated from Eqs. 7 and 8.

Determining angels γE (cf. Figs. 4 and 6) corresponding to dynamic structural responses to disconnection of each active member (treated separately), an auxiliary components of quasi-static influence matrix D can be defined. Then, the above procedure for optimal location of active members can be performed.

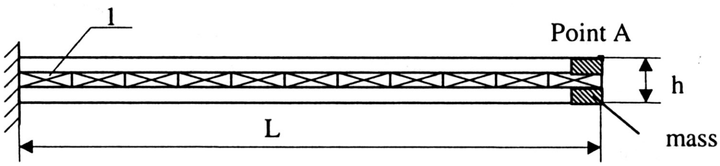
A structure can be called an *adaptive structure* if there is no need for external energy sources to generate movements of actuators, what takes place in our case. It means that actuators are purely dissipative (semi-active interfaces allowing controlled dissipation of stored strain energy) what can be formally postulated by the following condition:

$$\Delta\varepsilon_i^o \geqslant 0 \tag{10}$$

The numerical example demonstrated in the next section (Fig. 7) corresponds to two-layer beam with active interface giving high contribution to the damping effect thanks to high shear stress accumulated along active surface during bending process.

Constraining our control activity to the first mode of vibration the corresponding PAR control strategy is the following:

- Instantaneous full opening of valves in $k$dissipaters and full locking (after a short time interval Δt) when $u \cdot \ddot{u} < 0$ and $\dot{u} = 0$ (extremal deflection of the beam) where u denotes vertical movement of the node A.
- Smooth unloading of the introduced distortions ε whenever $\sigma_i\varepsilon < 0$ (release of stored prestress after reaching the equilibrium point) if $\ddot{u} = 0$.



$E = 42\,\text{GPa}$ $\quad\nu = 0.3$

$l = 0.8\,\text{m}$ $\quad h = 0.012\,\text{m}$ $\quad d = 0.02\,\text{m}$

$\rho = 2000\,\text{kg/m}^3$ $\quad m = 20\,\text{kg}$

FIGURE 7. Adaptive Double Layer Beam Model.

3. Numerical example

Let us demonstrate the effect of the *impulse-release* strategy for damping of free vibration on the following example of adaptive structures.

The double layer beam (Fig. 7) with the third thin layer composed of switchable elements able to release instantly accumulated shear stresses has been applied to model a controllable *delamination effect.* The free vibration response to the force type of impulse applied to the tip point A has been demonstrated in Fig. 8 (line uA). The structure response after application of the *impulse-release* strategy of damping is shown in Fig. 8 (line uAd). The strain-stress hysteretic loop for the interface element has been shown in Fig. 9.

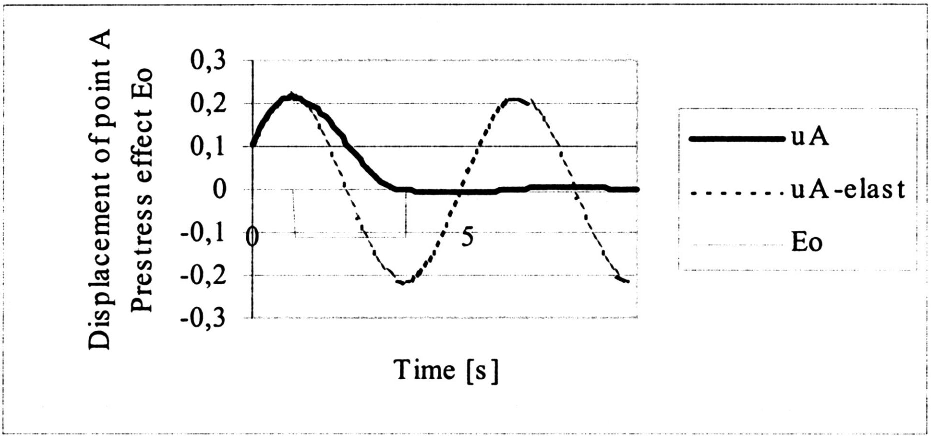


FIGURE 8. Tip point A oscillations.

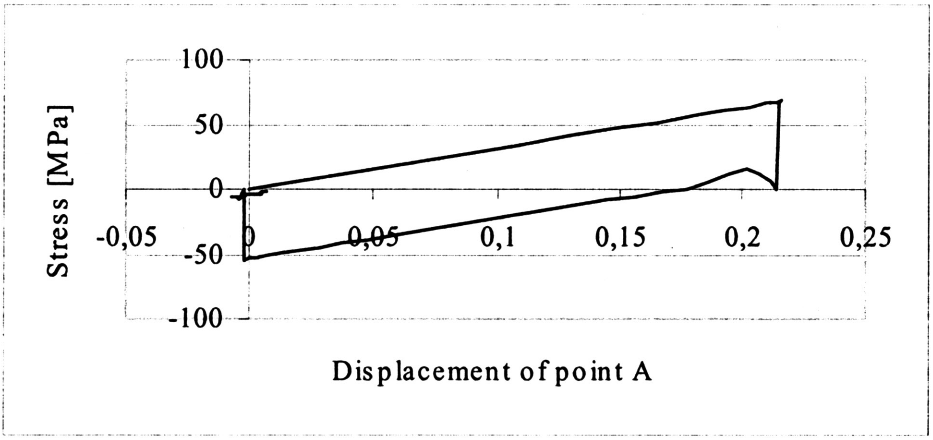


FIGURE 9. Histeretic loops.

4. Conclusions and further steps

High efficiency of the proposed semi-active vibration control has been demonstrated. It has been shown that actively controlled, low-energy consuming actuators, realising switchable behaviour of structural inter-connections, allow effective vibration energy dissipation. It has been demonstrated also that double-layer beams (and similarly, plates and shells) with switchable delamination effect along the active interface are good examples of effectively controlled adaptive structures.

In the further research an experimental model will be presented. The realise "amplified piezoelectric actuators" (cf. [5]) will controll *delamination* effect. The calibrated numerical model of this tested experiment will be also elaborated. Then, farther improvements of the active damping control strategy will be discussed end numerically simulated. This simulation will include the strategy for active damping of vibration composed of the first two modes. In this case the following two possible strategies can be applied:

1. the local control strategy (described above) switching actuators due to locally detected extremes of vibration amplitudes,
2. the global control strategy making use of the numerical model of the structure and performing (in real time) identification of the current eigenmodes' composition.

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