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Investigation of surface wave operators in a nonhomogeneous anisotropic elastic half-space.

Abstract. In the paper new alternative approach to surface wave problem in a nonhomogeneous anisotropic elastic semi-space in terms of stress tensor components vanishing on semi-space boundary is presented. This approach alows to perform analysis of surface wave using Green's function theory similar as is it done for Sturm-Liouville operators in the space of unbounded measure.

Key words. stress surface wave, anisotropic nonhomogeneous elastic semi-space, nonlinear eigenvalue problem.

1. Introduction.

The surface wave propagation problem a nonhomogeneous anisotropic elastic semi-space can be formulated in terms of stresses (see [1]) and reduced to generalized eigenvalue problem (see [2]) of the form:

$$\boldsymbol{A}(s)\boldsymbol{\alpha} - \lambda \boldsymbol{B}\boldsymbol{\alpha} = \boldsymbol{0} \tag{1.1}$$

where $\boldsymbol{\alpha} = [\alpha_{11} \;\; \alpha_{22} \;\; \alpha_{12}]^T$ is a wave amplitude vector from the space $C^2[0,\infty) \times C^3[0,\infty) \times C^2[0,\infty)$ satisfying the condition:

$$\begin{aligned} \alpha_{22}(0) &= \alpha_{12}(0) = 0 \\ \alpha_{22}(\infty) &= \alpha_{12}(\infty) = \alpha_{11}(\infty) = 0 \end{aligned} \tag{1.2}$$

The symbol s in (1.1) denotes a wave number ($s > 0$), the operators $\boldsymbol{A}(s)$ and $\boldsymbol{B}$ are 3×3 symmetric – matrices. More precisely $\boldsymbol{A}(s)$ is a selfadjoint matrix – operator and $\boldsymbol{B}$ positively definite matrix – operator, depending on six elastic parameters (see [1]).

In the paper the case of a nonhomogeneous anisotropic semi-space is analysed. Applying the method propose by Najmark (see [3]) is shown, that the problem can be reduced to equation

$$\ddot{\gamma} + \boldsymbol{Q}\gamma = \boldsymbol{O} \tag{1.3}$$

where $\gamma = [\gamma_1 \ \ \gamma_2]^T \overset{df}{:=} [\alpha_{22} \ \ \alpha_{12}]^T$ is a vector from the space $C^3[0,\infty) \times C^2[0,\infty)$ satysfying compatibility conditions and the relations:

$$\begin{aligned} \gamma_1(0) = \gamma_2(0) = 0 \\ \gamma_1(\infty) = \gamma_2(\infty) = 0 \end{aligned} \tag{1.4}$$

The dot over any letter denote differentiation with respect semi-space depth. Problem (1.3)–(1.4) can be analysed with the help of Green's function theory in the similar way as can be done for Sturm-Liouville and Dirac operators (see [7]).

2. The alternative formulation of surface wave propagation problem in a nonhomogeneous anisotropic elastic semi-space.

We are consider nondimensional stress equation for a nonhomogeneous anisotropic elastic semi-space $U = \Big\{(x_1, x_2): \quad |x_1| < \infty, x_2 \geqslant 0\Big\}$ (the plane displacement state and arbitrary anisotropy)

$$2\kappa_{ijkl}(x)\ddot{\tau}_{kl}(x,t) = \left[\varrho^{-1}(x)\tau_{ik,k}(x,t)\right]_{,j} + \left[\varrho^{-1}(x)\tau_{jk,k}(x,t)\right]_{,i} \tag{2.1}$$

where $\tau_{kl} = \tau_{kl}(x,t)$, $(i,j,k,l = 1,2)$, $x = (x_1, x_2)$ is stress field to be found (see [1]).
The tensor κ_{ijkl} is 4-th order tensor of compliance moduli as a function of a spatial coordinate, satysfing condition:

a) symmetry

$$\kappa_{ijkl} = \kappa_{jikl} = \kappa_{ijlk} = \kappa_{lkij} \tag{2.2}$$

b) strong elipticity condition

$$\kappa_{ijkl}\, a_i\, b_k\, a_j\, b_l > 0 \tag{2.3}$$

for arbitrary vectors $\boldsymbol{a}$, $\boldsymbol{b}$. Medium density is defined by the function $\varrho(x)$.

It is assumed that the functions $\kappa_{ijkl}(x)$ and $\varrho(x)$ $(i,j,k,l=1,2)$ depend on x_2 and are bounded function of class $C^2[0,\infty)$.

In the elastic semispace:

$$U=\Big\{(x_1,x_2):\quad |x_1|<\infty,\ \ x_2\geqslant 0\Big\} \tag{2.4}$$

we are looking for the functions $\tau_{kl}(x,t)$ being the solution of equation (2.1) in the form:

$$\begin{aligned}\tau_{11} &= \alpha_{11}(x_2)\exp\Big[i(sx_1-t\sqrt{\lambda})\Big] \\ \tau_{22} &= \alpha_{22}(x_2)\exp\Big[i(sx_1-t\sqrt{\lambda})\Big] \\ \tau_{22} &= i\alpha_{12}(x_2)\exp\Big[i(sx_1-t\sqrt{\lambda})\Big] \ \text{ for } t\in\langle 0,+\infty),\end{aligned} \tag{2.5}$$

where $i=\sqrt{-1}$, $s>0$, $\lambda>0$ and the functions $\tau_{kl}(x,t)$ satisfy the conditions

$$\tau_{22}(x_1,0,t)=\tau_{12}(x_1,0,t)=0 \quad \text{ for } x_1\in\boldsymbol{R},\ \ t\in\langle 0,+\infty) \tag{2.6}$$

$$\tau_{22}(x_1,\infty,t)=\tau_{12}(x_1,\infty,t)=\tau_{11}(x_1,\infty,t)=0 \ \text{ for } x_1\in\boldsymbol{R},\ \text{ and } t\in\langle 0,+\infty) \tag{2.7}$$

After substitution (2.5) to (2.1) and applying conditions (2.6), (2.7) and the symmetry of tensor τ the problem of the surface wave propagation can be reduced to generalised eigenvalue problem

$$\boldsymbol{A}(s)\boldsymbol{\alpha}-\lambda\boldsymbol{B}\boldsymbol{\alpha}=\boldsymbol{0} \tag{2.8}$$

where $\boldsymbol{\alpha}=[\alpha_{11}\ \ \alpha_{12}\ \ \alpha_{22}]^T$,

$$\boldsymbol{A}(s)=\begin{bmatrix}\dfrac{s^2}{\varrho} & \dfrac{s}{\varrho}D & 0\\ -sD\dfrac{1}{\varrho} & \dfrac{s^2}{\varrho}-D\dfrac{1}{\varrho}D & \dfrac{1}{\varrho}D\\ 0 & -sD\dfrac{1}{\varrho} & -D\dfrac{1}{\varrho}D\end{bmatrix} \tag{2.9}$$

$$\boldsymbol{B}=\begin{bmatrix}C_1 & C_2 & C_4\\ C_2 & C_3 & C_5\\ C_4 & C_5 & C_6\end{bmatrix} \tag{2.10}$$

$$\begin{aligned} C_1 &= \kappa_{1111}(x_2), & C_2 &= 2\kappa_{1112}(x_2),\\ C_3 &= 4\kappa_{1212}(x_2), & C_4 &= \kappa_{1122}(x_2),\\ C_5 &= 2\kappa_{1222}(x_2), & C_6 &= \kappa_{2222}(x_2).\\ D &= \frac{d}{dx_2}. \end{aligned} \tag{2.11}$$

The domain of the operators $\boldsymbol{A}$ and $\boldsymbol{B}$ are:

$$\begin{aligned}\operatorname{dom}\boldsymbol{A} &= \Big\{\boldsymbol{\alpha}:\ \boldsymbol{\alpha}\in C^2[0,\infty)\times C^3[0,\infty)\times C^2[0,\infty);\quad \boldsymbol{\alpha}_{12}(0)=\alpha_{22}(0)= \\ &\qquad\qquad = \alpha_{12}(\infty)=\alpha_{22}(\infty)=\alpha_{11}(\infty)=0\Big\} \\ \operatorname{dom}\boldsymbol{B} &= \Big\{\boldsymbol{\alpha}:\ \boldsymbol{\alpha}\in C^2[0,\infty)\times C^3[0,\infty)\times C^2[0,\infty)\Big\}\end{aligned} \tag{2.12}$$

The formulation covers the cases(1) of

a) isotropy

b) transverse isotropy

c) generalised anisotropy (see [4], [5], [6]).

Since $\kappa_{ijkl}(x_2)$ are bounded functions of the class $C^2[0,\infty)$, the functions $C_1(x_2)$, $C_2(x_2)$, ..., $C_6(x_2)$ are also bounded functions of the class $C^2[0,\infty)$. Similary medium density $\varrho(x_2)$ is also bounded function of class $C^2[0,\infty)$.

In order to obtain alternative form of the problem (2.8) – (2.12) we consider eigenvalue problem (2.8) written in the form

$$\begin{aligned} \frac{s^2}{\varrho}\alpha_{11}+\frac{s}{\varrho}D\alpha_{12} &= \lambda\big(C_1\alpha_{11}+C_2\alpha_{12}+C_4\alpha_{22}\big) \\ -sD\Big(\frac{1}{\varrho}\alpha_{11}\Big)+\Big(\frac{s^2}{\varrho}-D\frac{1}{\varrho}D\Big)\alpha_{12}-\frac{s}{\varrho}D\alpha_{22} &= \lambda\big(C_2\alpha_{11}+C_3\alpha_{12}+C_5\alpha_{22}\big) \\ -sD\Big(\frac{1}{\varrho}\alpha_{12}\Big)-D\frac{1}{\varrho}D\alpha_{22} &= \lambda\big(C_4\alpha_{11}+C_5\alpha_{12}+C_6\alpha_{22}\big)\end{aligned} \tag{2.13}$$

From the first equation of the system (2.13) we obtain

$$\alpha_{11}=\frac{\lambda\varrho(x_2)C_2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}\alpha_{12}-\frac{s}{s^2-\lambda\varrho(x_2)C_1(x_2)}\dot{\alpha}_{12}+\frac{\lambda\varrho(x_2)C_4(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}\alpha_{22} \tag{2.14}$$

and after differentiation with respect x_2 we get

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(1)if $C_1=\frac{1-\nu}{2\mu}$, $C_2=\frac{-\nu}{2\mu}$, $C_4=0$, $C_3=\frac{1-\nu}{2\mu}$, $C_5=0$, $C_6=\frac{1}{\mu}$, and $C_1=-B_1(x_2)$, $C_2=0$, $C_3=2B_2(x_2)$, $C_4=-4f(x_2)$, $C_5=0$, $C_6=B(x_2)f(x_2)g(x_2)$ we obtain an isotropy and a transverse isotropy, respectively (see T. Rojnowski [4]).

The parameters: $B(x_2)$, $B_1(x_2)$, $B_2(x_2)$, $f(x_2)$, $g(x_2)$ are defined by T. Rojnowski in the paper [4].

$$\begin{aligned}\dot{\alpha}_{11} &= \left(\frac{\lambda\varrho(x_2)C_2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}\right)^{\cdot}\alpha_{12}(x_2)+\frac{\lambda\varrho(x_2)C_2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}\dot{\alpha}_{12}(x_2)+\\ &-\left(\frac{s}{s^2-\lambda\varrho(x_2)C_1(x_2)}\right)^{\cdot}\alpha_{12}(x_2)-\frac{s}{s^2-\lambda\varrho(x_2)C_1(x_2)}\ddot{\alpha}_{12}(x_2)+ \\ &+\left(\frac{\lambda\varrho(x_2)C_4(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}\right)^{\cdot}\alpha_{22}(x_2)+\frac{\lambda\varrho(x_2)C_4(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}\dot{\alpha}_{22}(x_2)\end{aligned} \tag{2.15}$$

After substituting (2.15) and (2.14) to the second and thrid equation of the system (2.13) we obtain

$$\begin{aligned}&\left[\frac{s^2}{(s^2-\lambda\varrho(x_2)C_1(x_2))\varrho(x_2)}-\frac{1}{\varrho(x_2)}\right]\ddot{\alpha}_{12}(x_2)+\left[\frac{-s\lambda C_2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}+\right.\\ &\left.\left(\frac{s^2}{(s^2-\lambda\varrho(x_2)C_1(x_2))\varrho(x_2)}\right)^{\cdot}-\left(\frac{1}{\varrho(x_2)}\right)^{\cdot}+\frac{s\lambda C_2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}\right]\dot{\alpha}_{12}(x_2)+\\ &+\left[-\left(\frac{s\lambda C_2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}\right)^{\cdot}+\frac{s^2}{\varrho(x_2)}-\frac{\lambda^2\varrho(x_2)C_2^2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}+\right.\\ &\left.-\lambda C_3(x_2)\right]\alpha_{12}(x_2)+\left[-\frac{s\lambda\varrho(x_2)C_4(x_2)}{(s^2-\lambda\varrho(x_2)C_1(x_2))\varrho(x_2)}-\frac{s}{\varrho(x_2)}\right]\dot{\alpha}_{22}(x_2)+\\ &+\left[-\frac{s\lambda\varrho(x_2)C_4(x_2)}{(s^2-\lambda\varrho(x_2)C_1(x_2))\varrho(x_2)}-\frac{\lambda^2\varrho(x_2)C_2(x_2)C_4(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}-\lambda C_5(x_2)\right]\alpha_{22}(x_2)=0\end{aligned} \tag{2.16}$$

$$\begin{aligned}&\left[-\frac{s}{\varrho(x_2)}+\frac{\lambda sC_4(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}\right]\dot{\alpha}_{12}(x_2)+\left[-s\left(\frac{1}{\varrho(x_2)}\right)^{\cdot}+\right.\\ &\left.-\frac{\lambda^2\varrho(x_2)C_2(x_2)C_4(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}-\lambda C_5(x_2)\right]\alpha_{12}(x_2)+\left(\frac{-1}{\varrho(x_2)}\right)\ddot{\alpha}_{22}(x_2)+\\ &+\left[-\left(\frac{1}{\varrho(x_2)}\right)^{\cdot}\right]\dot{\alpha}_{22}(x_2)+\left[\frac{-\lambda^2\varrho(x_2)C_4^2(x_2)}{s^2-\lambda C_1(x_2)\varrho(x_2)}-\lambda C_6(x_2)\right]\alpha_{22}(x_2)=0\end{aligned} \tag{2.17}$$

The equation (2.16) and (2.17) can be written in more compact way

$$\boldsymbol{R}_0\ddot{\boldsymbol{\beta}}+\boldsymbol{R}_1\dot{\boldsymbol{\beta}}+\boldsymbol{R}_2\boldsymbol{\beta}=\boldsymbol{0} \tag{2.18}$$

where

$$\boldsymbol{\beta}(x_2) = \left[\alpha_{22}(x_2) \;\; \alpha_{12}(x_2)\right]^T \tag{2.19}$$

$$\boldsymbol{R}_0(s,\lambda) = \begin{bmatrix} -\dfrac{1}{\varrho(x_2)} & 0 \\ 0 & \dfrac{s^2}{(s^2-\lambda\varrho(x_2)C_1(x_2))\varrho(x_2)} - \dfrac{1}{\varrho(x_2)} \end{bmatrix} \tag{2.20}$$

$$\boldsymbol{R}_1(s,\lambda) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{2.21}$$

$$\begin{aligned}
a_{11} &= -\left(\frac{1}{\varrho(x_2)}\right)^{\cdot} \\
a_{12} &= -\frac{s}{\varrho(x_2)} + \frac{\lambda s C_1(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)} \\
a_{21} &= \left(\frac{-s\lambda\varrho(x_2)C_4(x_2)}{(s^2-\lambda C_1(x_2)\varrho(x_2))\varrho(x_2)} - \frac{s}{\varrho(x_2)}\right) \\
a_{22} &= \frac{-s\lambda C_2(x_2)}{s^2-\lambda C_1(x_2)\varrho(x_2)} + \left(\frac{s^2}{(s^2-\lambda C_1(x_2)\varrho(x_2))\varrho(x_2)}\right)^{\cdot} - \left(\frac{s}{\varrho(x_2)}\right)^{\cdot} + \\
&\quad + \frac{\lambda s C_2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}
\end{aligned}$$

$$\boldsymbol{R}_2(s,\lambda) = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{2.22}$$

$$\begin{aligned}
b_{11} &= \frac{-\lambda^2\varrho(x_2)C_4^2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)} - \lambda C_6(x_2) \\
b_{12} &= -s\left(\frac{1}{\varrho(x_2)}\right)^{\cdot} - \frac{\lambda^2\varrho(x_2)C_2(x_2)C_4(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)} - \lambda C_5(x_2) \\
b_{21} &= -\frac{s\lambda C_4(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)} - \frac{\lambda^2\varrho(x_2)C_2(x_2)C_4(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)} - \lambda C_5(x_2) \\
b_{22} &= -\left(\frac{s\lambda C_2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)}\right)^{\cdot} + \frac{s^2}{\varrho(x_2)} - \frac{\lambda^2\varrho(x_2)C_2^2(x_2)}{s^2-\lambda\varrho(x_2)C_1(x_2)} - \lambda C_3(x_2)
\end{aligned}$$

The equation (2.18) shut by supplemented by compatibility conditions written for the field $\beta = \begin{bmatrix} \alpha_{22} \\ \alpha_{12} \end{bmatrix}$.

Compability conditions has the form:

$$\left(C_1\alpha_{11} + C_2\alpha_{12} + C_3\alpha_{22}\right)^{\cdot\cdot} - s^2\left(C_4\alpha_{11} + C_5\alpha_{12} + C_6\alpha_{22}\right) + \\ + s\left(C_2\alpha_{11} + C_3\alpha_{12} + C_5\alpha_{22}\right)^{\cdot} = 0^{(2)} \quad \text{for } x_2 \in [0, +\infty). \tag{2.23}$$

Tensor α_{11} is given by (2.14).

By means of the substitution

$$\boldsymbol{\beta} = \boldsymbol{U}\boldsymbol{\gamma} \tag{2.24}$$

where $\boldsymbol{U}$ is any solution of differential matrix equation

$$\dot{\boldsymbol{U}} + \frac{1}{2}\boldsymbol{P}_1\boldsymbol{U}, \quad \det \boldsymbol{U} \neq 0 \quad \text{(see [3] p. 118)} \tag{2.25}$$

the equation

$$\boldsymbol{P}_0\ddot{\boldsymbol{\beta}} + \boldsymbol{P}_1\dot{\boldsymbol{\beta}} + \boldsymbol{P}_2\boldsymbol{\beta} = \boldsymbol{0} \tag{2.26}$$

$$\left(\boldsymbol{P}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{P}_1 = \boldsymbol{R}_0^{-1}\boldsymbol{R}_1, \quad \boldsymbol{P}_2 = \boldsymbol{R}_0^{-1}\boldsymbol{R}_2\right)$$

can be reduced to the following form

$$\ddot{\boldsymbol{\gamma}} + \boldsymbol{Q}\boldsymbol{\gamma} = \boldsymbol{0} \tag{2.27}$$

where

$$\boldsymbol{\gamma}(0) = \left(\boldsymbol{U}^{-1}\boldsymbol{\beta}\right)(0) = \boldsymbol{0}$$

$$\boldsymbol{\gamma}(\infty) = \left(\boldsymbol{U}^{-1}\boldsymbol{\beta}\right)(\infty) = \boldsymbol{0}$$

$$\boldsymbol{Q} = -\frac{1}{2}\dot{\boldsymbol{P}}_1 - \frac{1}{4}\boldsymbol{P}_1^2 + \boldsymbol{P}_2 \tag{2.28}$$

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(2) Compatibility theorem for plane displacement (c.f. [8]). Assume that B is a simply-connected open region in the (x_1, x_2)-plane. Let u_i be a class C^3 field on B, and let

$$\varepsilon_{ij} = \tfrac{1}{2}\left(u_{i,j} + u_{j,i}\right) \quad i, j = 1, 2 \tag{a}$$

Then

$$2e_{12,12} = e_{11,22} + e_{22,11} \tag{b}$$

Conversely, let $e_{ij} (= e_{ji})$ be a class C^N $(N \geqslant 2)$ field on B that satisfies (b). Then there exists a class C^{N+1} field u_i on B such that (a) holds.

The problem (2.27) can be investigated with the help of the Green's function theory or iterative method used for Sturm-Liouville and Dirac operators in the space of unbounded measure (see [7]). Compatibility theorem for plane displacement implies that the problem of surface wave propagation in a nonhomogeneous, anisotropic semispace can be reduced to finding a solution Eq. (2.27), which satisfying plane compatibility condition in zero:

$$\begin{aligned} &\Big\{\Big[C_1(x_2)\alpha_{11} + C_2(x_2)\alpha_{12} + C_3(x_2)\alpha_{22}\Big]^{\cdot} + \\ &-s^2\Big[C_4(x_2)\alpha_{11} + C_5(x_2)\alpha_{12} + C_6(x_2)\alpha_{22}\Big] + \\ &+s\Big[C_2(x_2)\alpha_{11} + C_3(x_2)\alpha_{12} + C_5(x_2)\alpha_{22}\Big]^{\cdot\cdot}\Big\}(0) \quad = 0 \end{aligned} \tag{2.29}$$

Tensor α_{11} is defined by (2.14)

Remark 1. *In the case of isotropic and homogeneous semi-space equation (2.26) has the following solution:*

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_1(x_2) \\ \beta_2(x_2) \end{bmatrix} := C \begin{bmatrix} e^{-x_2 h_2} - e^{-x_2 h_1} \\ -\frac{2}{s}\frac{h_1}{2-\omega}\left(e^{-x_2 h_2} - e^{-x_2 h_1}\right) \end{bmatrix} \tag{2.30}$$

Where

$$h_1 = s\sqrt{1-\omega\kappa}, \quad h_2 = s\sqrt{1-\omega},$$
$$\kappa = \frac{1-2\nu}{2-2\nu}, \quad 0 < \omega < 1.$$

From (2.30) and from the plane compatibility condition one obtains

$$\begin{aligned} &\frac{Cs^2}{2\mu(2-\omega)} e^{-x_2 s\sqrt{1-\omega}}\Big[(2-\omega)^2 - 4\sqrt{(1-\omega)(1-\omega\kappa)}\Big] + \\ &\qquad + \frac{Cs^2(1-\kappa)}{2\mu(2-\omega)} e^{-x_2 s\sqrt{1-\omega\kappa}}[0] = 0 \end{aligned} \tag{2.31}$$

Since $C \neq 0$ one obtains the classical equation determining velocity of the surface wave in homogeneous isotropic elastic semi-space

$$(2-\omega)^2 - 4\sqrt{(1-\omega)(1-\omega\kappa)} = 0. \tag{2.32}$$

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