# Risk acceptability and optimization

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Optimization of technical facilities involving risk for human life and limb require suitable assessments of life saving cost and discount rates both for the public and the operator. Discount rates $\gamma$ must be long term averages and net of inflation and taxes. While the operator may use rates from the financial market for his cost-benefit analysis the assessment of interest rates for investments of the public into risk reduction is difficult. Most authors in economics propose values of 4 to 6%. The classical Ramsey model for economic growth decomposes the output growth rate (= interest rate) into the rate of time preference of consumption and the rate of economical growth multiplied by the elasticity of marginal utility of consumption. It is found that the rate of time preference of consumption should be a little larger than the long term population growth rate. The output growth rate should be smaller than the sum of the population growth rate and the long term growth rate of a national economy which is around 2% for most industrial countries. Accordingly, the rate of time preference of consumption is about 0.8%, which is also intergenerationally acceptable from an ethical point of view. Given a certain output growth rate there is a corresponding maximum financial interest rate in order to maintain non-negativity of the objective function.

Key words: *reliability, optimum technical facilities, life quality index, risk acceptability, discounting*

## 1. Optimal technical facilities

A technical facility is optimal if the following objective is maximized [42]:

$$Z(\mathbf{p}) = B(\mathbf{p}) - C(\mathbf{p}) - D(\mathbf{p}). \tag{1.1}$$

For the purpose of this paper it is assumed that all quantities in Eq. (1.1) can be measured in monetary units. $\mathbf{p}$ is the vector of all safety relevant parameters. $B(\mathbf{p})$ is the benefit derived from the existence of the facility, $C(\mathbf{p})$ is the cost of design and construction and $D(\mathbf{p})$ is the cost in case of failure. While $B(\mathbf{p})$ and $C(\mathbf{p})$ can be considered as nearly deterministic, $D(\mathbf{p})$ generally is uncertain. Then, statistical decision theory dictates that expected

values are to be taken [28]. In the following it is assumed that $B(\mathbf{p}), C(\mathbf{p})$ and $D(\mathbf{p})$ are differentiable in each component of $\mathbf{p}$. It is reasonably assumed that $C(\mathbf{p})$ increases whereas $D(\mathbf{p})$ decreases in each component of $\mathbf{p}$. The cost may differ for the different parties involved, e.g. the owner, the builder, the user and society. The erection of a facility makes sense only if $Z(\mathbf{p})$ is positive within certain parameter ranges for all parties involved. Their intersection defines reasonable facilities (public or other subsidizing excluded).

The facility has to be optimized during design at the decision point, i.e. at time $t = 0$. Therefore, all cost need to be discounted. A continuous discounting function is assumed which is accurate enough for all practical purposes

$$\delta(t) = \exp\left[-\gamma t\right] \qquad (1.2)$$

where $\gamma$ is the interest rate.

In general, one has to distinguish between two replacement strategies, one where **the facility is given up after service or failure** and one where **the facility is systematically replaced after failure**. Further we distinguish between facilities which **fail upon completion or never** and facilities which **fail at a random point in time** much later due to service loads, extreme external disturbances or deterioration. The first option implies that loads on the facility are time-invariant. Reconstruction times are assumed to be negligibly short. At first sight there is no particular preference for either of the replacement strategies. For infrastructure facilities the second category is a natural strategy. Facilities used only once, e.g. special auxiliary construction structures, boosters for space transport vehicles or devices exploiting limited deposits, might fall into the first category.

## 2. Failure under Poissonian disturbances and systematic reconstruction

For simplicity, the objective function is only derived for a special case. Assume random events (storms, earthquakes, explosions, etc.) in time forming a renewal process and systematic reconstruction. The times between events have distribution function $F(t, \mathbf{p})$ with probability density $f(t, \mathbf{p})$. For constant benefit per time unit $b(t) = b$ and $f_n(t, \mathbf{p})$ the density of the time to the $n$-th renewal with Laplace-transform $f_n^*(\gamma, \mathbf{p})$ an objective function can be derived by making use of the convolution theorem for Laplace transforms

$$Z(\mathbf{p}) = \int\limits_0^\infty b\,e^{-\gamma t}dt - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{n=1}^\infty \int\limits_0^\infty e^{-\gamma t} f_n(t, \mathbf{p})\,dt = \dots \quad (2.1)$$

$$\ldots = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{f^*(\gamma, \mathbf{p})}{1 - f^*(\gamma, \mathbf{p})} \quad (2.1)$$
$$= \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\, h^*(\gamma, \mathbf{p})$$

<div style="text-align:right">[cont.]</div>

where $h^*(\gamma, \mathbf{p})$ is the Laplace transform of the renewal density (renewal intensity) $h(t, \mathbf{p})$. $H$ is the monetary loss in case of failure including direct failure cost, loss of business and, of course, the cost to reduce the risk to human life and limb. If at an extreme loading event (e.g. flood, wind storm, earthquake, explosion) failure occurs with probability $P_f(\mathbf{p})$ one obtains [41, 16]:

$$h^*(\gamma, \mathbf{p}) = \sum_{n=1}^{\infty} f^*(\gamma) P_f(\mathbf{p}) \left[ f^*(\gamma) R_f(\mathbf{p}) \right]^{n-1} = \frac{P_f(\mathbf{p}) f^*(\gamma)}{1 - R_f(\mathbf{p}) f^*(\gamma)}$$

with $R_f(\mathbf{p}) = 1 - P_f(\mathbf{p})$. If, in particular, the events follow a stationary Poisson process with intensity $\lambda$ we have

$$h^*(\gamma, \mathbf{p}) = \frac{\lambda P_f(\mathbf{p})}{\gamma + \lambda P_f(\mathbf{p})}. \quad (2.2)$$

The precise details of the renewal model can be found in [35]. Many other objective functions can be formulated. For example, serviceability failure, obsolescence, aging, deterioration and inspection and maintenance and finite service times can be dealt with [35]. Also, multiple mode failures (series systems) can be considered.

In accordance with economic theory benefits and (expected) cost, whatever types of benefits and cost are considered, should be discounted by the same rate as done above. Different parties, e.g. the owner, operator or the public, may, however, use different rates. While the owner or operator may take interest rates from the financial market the assessment of the interest rate for an optimization in the name of the public is difficult. The requirement that the objective function must be non-negative leads immediately to the conclusion that the interest rate must have an upper bound $\gamma_{\max}$ depending on the benefit rate $\beta$ ($b = \beta C(\mathbf{p})$) (see [17, 36]). For the model in Eq. (2.2) we have

$$\frac{\beta C(\mathbf{p})}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{\lambda P_f(\mathbf{p})}{\gamma + \lambda P_f(\mathbf{p})} = 0 \quad (2.3)$$

and, therefore, by solving for $\gamma$:

$$\gamma < \gamma_{\max} < \frac{\beta}{2} - \lambda P_f(\mathbf{p})\left(1 + \frac{H}{2C(\mathbf{p})}\right)$$
$$+ \sqrt{\frac{\beta^2}{4} - \frac{\beta \lambda P_f(\mathbf{p}) H}{2C(\mathbf{p})} + (\lambda P_f(\mathbf{p}))^2 \left(1 + \frac{H}{C(\mathbf{p})} + \frac{H^2}{4C(\mathbf{p})^2}\right)} \quad (2.4)$$

implying $\gamma < \beta$ for $\lambda P_f(\mathbf{p}) \ll \beta$. The right hand side of inequality (2.4) depends on $\mathbf{p}$ and, therefore, we could solve for a maximum interest rate $\gamma_{\max}$ by maximizing it. It turns out that the solution vector $\hat{\mathbf{p}}$ is very close or numerically identical to the solution vector $\mathbf{p}^*$ for the task (2.1) with Eq. (2.2) so that $\hat{\mathbf{p}} \approx \mathbf{p}^*$. It follows that the benefit rate $\beta$ must be slightly larger than $\gamma_{\max}$. From Eq. (2.3) one also concludes that there must be $\gamma > 0$.

## 3. Rational socio-economic risk acceptability – the life quality index

Modern approaches to the question of risk to human lives do not speak of a monetary value of the human life but rather speak of the cost to reduce the risk to life, that is to "*reduce the probability of premature death by some intervention changing the behavior and/or technology of individuals or organizations*" [48]. Any argumentation must be within the framework of our moral and ethical principles as laid down in our constitutions and elsewhere including everyone's right to life, the right of a free development of her/his personality and the democratic equality principle. It is clear that only involuntary risks, i.e. risks to which the public is exposed involuntarily and anonymously from its technical and natural environment, can reasonably be discussed here.

Cantril [11] and similar more recent studies conclude from empirical studies that long life and wealth are among the primary concerns of humans in a modern society. Life expectancy at birth (mean time from birth to death) $e$ is the area under the survivor curve (survival function) $\ell(a) = \exp[-\int_0^a \mu(t)dt]$, i.e. $e = e(0) = \int_0^{a_u} \ell(a)da$ ($a_u$ = largest age considered, $\mu(a)$ = age dependent mortality). It makes sense to adjust it for times in poor health, times in hospital or homes for elderly people so that the "quality adjusted" (disability adjusted) life expectancy $e_{\mathrm{QALY}}$ is about 90% of $e$ [55]. However, this adjustment is not done in this paper. Another suitable indicator of the quality of life is the gross domestic product (GDP) per capita and year $g_{\mathrm{total}}$. The GDP is roughly the sum of all incomes created by labor and capital (stored labor) in a country during a year. It provides the infrastructure of a country, its social structure, its cultural and educational offers, its ecological conditions among others but also the means for the individual enjoyment of life by consumption. In most developed countries about 60±5% of the GDP is used privately, 20±5% by the state (e.g. for military, police and jurisdiction) and the rest for investments. Most importantly in our context, it creates the possibilities to "buy" additional life years through better medical care, improved safety in road traffic, more safety in or around building facilities, more safety from hazardous technical activities, more safety from natural hazards, etc.

In our context it does not matter whether those investments into "life saving" are carried out individually and voluntarily or enforced by regulation, or by the state via taxes. If it is assumed that neither the share for the state nor the investments into depreciating production means can be reduced, only the part for private use is available for risk reduction. Therefore, the part available for risk reduction is $g \approx 0.6\, g_{\text{total}}$.

Lind [23] sets out from a composite social indicator:

$$L = L(a, b, \ldots, e, \ldots) \tag{3.1}$$

with $a, b, \ldots, e, \ldots$, as certain social indicators. Let it be differentiable so that:

$$dL = \frac{\partial L}{\partial a} da + \frac{\partial L}{\partial b} db + \ldots + \frac{\partial L}{\partial e} de + \ldots . \tag{3.2}$$

If only the two factors mentioned before, that is $g$ and $e$, are considered $dL$ vanishes for:

$$\frac{dg}{de} = -\frac{\frac{\partial L}{\partial e}}{\frac{\partial L}{\partial g}} \tag{3.3}$$

implying that a change in $e$ should be compensated for by an appropriate change in $g$. Some elegant derivation in Nathwani/Lind/Pandey [26] then lead to the traditional form of the **Life Quality Index (LQI)** [1]

$$L = \frac{g^q}{q} e \tag{3.4}$$

where $q = \frac{w}{1-w}$ and $w$ the fraction of time of $e$ necessary for paid work. $q$ varies between 0.15 and 0.25 and equals $q \approx 0.19$ on average (see [38] for an assessment of $w$ for different countries). Using Eq. (3.3) yields a general

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[1] Nathwani et al. [26] assume that $L = f(g)h(t)$ with $t = (1 - w)e$ and where $t$ is the fraction of life devoted to leisure and $we$ the fraction of life devoted to paid work. Thus, the LQI is a product of a function $f(g)$ measuring life quality and a function $h(t)$ measuring the duration of enjoyment of life. Defining relative changes in the LQI by $\frac{dL}{L} = \frac{g}{f(g)} \frac{df(g)}{dg} \frac{dg}{g} + \frac{t}{h(t)} \frac{dh(t)}{dt} \frac{dt}{t} = k_g \frac{dg}{g} + k_t \frac{dt}{t}$ and setting $k_g/k_t = \text{const.}$ according to the universality requirement, one finds two differential equations $k_g \equiv \frac{g}{f(g)} \frac{df(g)}{dg} = c_1$ and $k_t \equiv \frac{t}{h(t)} \frac{dh(t)}{dt} = c_2$ with solutions $f(g) = g^{c_1}$ and $h(t) = t^{c_2} = ((1 - w)e)^{c_2}$. Assume further that $g \propto cew$ where $c$ is the productivity of work. "*Presumably, people on the average work just enough so that the marginal value of wealth produced, or income earned, is equal to the marginal value of the time they lose when at work*" [26]. Consequently, people who work, possibly together with their families, optimize work and leisure time, i.e. their LQI. From $\frac{dL}{dw} = 0$ one determines $c_1 = c_2 \frac{w}{1-w}$ which together with $c_1 + c_2 = 1$ results in $L = g^w e^{1-w}$. For formal reasons we take the $1/(1 - w)$–th root and divide $g^q$ by $q = \frac{w}{1-w}$ which gives Eq. (3.4).

acceptance criterion for investments into projects for risk reduction:

$$\frac{dg}{de} = -\frac{\frac{\partial L}{\partial e}}{\frac{\partial L}{\partial g}} \geq -\frac{g}{e}\frac{1}{q} \tag{3.5}$$

or

$$\frac{dg}{g} + \frac{1}{q}\frac{de}{e} \geq 0. \tag{3.6}$$

Equality in Eq. (3.6) corresponds to "optimal" investments into life saving, ">" means that investments into life saving are inefficient and projects having "<" are not admissible. The latter projects would, in fact, be life-consuming and, thus, be in conflict with the constitutional right to live. Criterion (3.6) gives an indication of what is **necessary** and also **affordable** to a society for life saving undertakings. Equation (3.6) is easy to interpret. A relative negative change in $g$ (for a life saving investment, for example) must be compensated for by a relative positive change in $e$ multiplied by $\frac{1}{q}$. For example, a 1% increase in life expectancy requires yearly investments of about 5% of $g$. Whenever a given incremental increase in life expectancy by some life saving operation (positive $de$) is associated with larger than optimal incremental cost (negative $dg$) one should invest into alternatives of life saving. If the criterion Eq. (3.6) is applied to two alternatives of risk reduction the alternative which increases the LQI most should be selected. Equation (3.6), therefore, can be interpreted as efficiency criterion for life saving operations. From a practical point of view it is important that all quantities on the right-hand side of Eq. (3.6) are easily available and can be updated any time. The democratic equality principle dictates that average values for $g$, $e$ and $w$ have to be taken. Any deviations from average values for any specific group of people need to be justified carefully if Eq. (3.6) is applied to projects with involuntary risks. There is a certain dilemma arising from the actual unequal distribution of wealth and life expectancy in a society. A certain group in a society may benefit from safety interventions more than another. Then, it should be fair that the "gainers" compensate the "loosers" so that their LQI is at least maintained. For example, in projects where certain groups of people must take higher risks, voluntarily or involuntarily, it should be fair to provide compensation by higher incomes or more leisure time. One even may follow the requirement in [27] which states that the "gainers" should still have some left over. Much further discussion is provided in [22] and [26].

Practical application of Eq. (3.6) in a life saving operation is not always easy (see, however, the many examples in [26]). In general, the cost involved in some life saving operation can be estimated easily. The estimation of the effect of a life saving operation is more difficult. At first, we estimate the

cost of averting a fatality in terms of the gain in life expectancy $\Delta e$. The cost of the safety measure is expressed as a reduction $\Delta g$ of the GDP. This implied cost of averting a fatality (ICAF) can be obtained from the equality of Eq. (3.6) after separation and integration from $g$ to $g + \Delta g$ and $e$ to $e + \Delta e$, i.e. the cost $\Delta C = -\Delta g$ per year to extend a person's life by $\Delta e$ is:

$$\Delta C = -\Delta g = g\left[1 - \left(1 + \frac{\Delta e}{e}\right)^{-\frac{1}{q}}\right].$$

Because $\Delta C$ is a yearly cost and the (undiscounted) ICAF has to be spend for safety related investments into technical projects at the decision point $t = 0$, one should multiply by $e_r = \Delta e$ and

$$\text{ICAF}(e_r) = g\left[1 - \left(1 + \frac{e_r}{e}\right)^{-\frac{1}{q}}\right]e_r \tag{3.7}$$

follows. $\text{ICAF}(e_r)$ grows approximately linearly with $e_r$. In case of failure of a technical object $e_r$ is between $0.67e$ (for young groups with a triangular age distribution) and a little larger than $0.5e$ (for aging groups) on average. The societal equality principle prohibits to differentiate with respect to special ages within a group. Therefore,

$$\text{ICAF} = \int_0^{a_u} \text{ICAF}(e - a)\, h(a)\, da \tag{3.8}$$

where $h(a)$ is the density of the age distribution of the population. The density of the age distribution can be obtained from life tables. For the important case of a (stable) population with constant growth rate $n = b - m$ ($b$ = birth rate, $m$ = crude mortality) the age distribution is [21]:

$$h(a, n) = \frac{\exp[-na]\ell(a)}{\int_0^{a_u} \exp[-na]\ell(a)\, da}. \tag{3.9}$$

From Eq. (3.7) it is seen that ICAF is slightly smaller than the (undiscounted) earnings in the remaining life time. In countries with a fully developed social system ICAF is approximately the amount to support the (not working) relatives of the victims of an event by the social system. If no social system is present, it is useful to think of the amount an insurance should cover after an event. The premium for the insurance reduces the benefit of an economical activity. It is noted that the two systems are not quite identical as the consequences of a fatality are carried by the whole society in a social system essentially by redistribution and by the owner or operator of a technical facility in the second case via insurance given appropriate legal conditions.

For example, if $g_{total} \approx 25\,000\,\text{PPP US\$}$ and thus, $g \approx 14\,500\,\text{PPP US\$}$, $e \approx 77$ years and $q \approx 0.19$, one calculates ICAF $\approx 500\,000\,\text{PPP US\$}$. As pointed out in [37] $g$ as well as $e$ grow with time and, thus, also the ICAF, i.e. the cost for society in an event. Therefore, this estimate needs to be updated from time to time.

There have been many attempts to estimate this quantity indirectly (among the rich literature for this subject see, for example, [51] and [25] and [46] for a collection of governmental stipulations), mostly by estimating the cost of some life saving operation like limiting highway speed [5], installing smoke detectors in homes or using seat belts in cars. Also, the compensation in risky jobs by higher wages has been used [52] as well as surveys with respect to hypothetical risky situations, so-called contingent valuation studies [33]. The values reported are between less than $1\,000\,000$ US\$ and more than $10\,000\,000$ US\$, i.e. more than 2 to 20 times as much as the (undiscounted) value of average lost earnings in case of a fatal accident at mid life. Most of those estimates have to be interpreted as "willingness-to-pay" (WTP) estimates or as "value of a statistical life" (VSL). Unfortunately, there is no commonly agreed definition of these notions. It appears as if the individual values her/his life rather high provided that the abstract public spends the money for its protection, ironically via anonymous taxes or costly but not individually felt or realised regulations. Here, we remark for the moment that our estimate is necessary, affordable and efficient. Other higher estimates are inefficient from the point of view of the life quality index and, strictly speaking, life-consuming as the extra investment could have been spent in other efficient life saving operations.

As mentioned, the direct quantification of $de/e$ is difficult but there is a good approximation if the life saving operation results in uniform changes of the age-dependent mortality rates. For a (small) proportional change $\delta = dm/m > 0$ in age dependent mortality $\mu(a)$, i.e. $\mu_\delta(a) = \mu(a)(1 + \delta)$, the change in $de/e$ is [21]:

$$\frac{de}{e} \approx \frac{\frac{d}{d\delta}\int_0^{a_u} \ell(a)^{1+\delta}\,da\,|_{\delta=0}}{\int_0^{a_u} \ell(a)da}\delta \approx \frac{\int_0^{a_u} \ln(\ell(a))\ell(a)da}{\int_0^{a_u} \ell(a)da}\delta = -C_\delta\delta \qquad (3.10)$$

where $C_\delta \approx 0.15$ (developed countries) to more than 0.5 (some developing countries) depending on the age structure and life expectancy of the group (see [38] for more details).

Recently, Pandey/Nathwani [31] improved the specific form of the LQI by considering the time preference of consumption (use of $g$) and by taking account of the age distribution of a population in line with general economic models. We shall briefly summarize their findings supporting and slightly

modifying the above considerations. Later on, they will offer the possibility to discuss societal discount rates in a broader perspective.

Denote by $c(\tau) > 0$ the consumption rate at age $\tau$ and by $u(c(\tau))$ the utility derived from consumption. Individuals tend to undervalue a prospect of future consumption as compared to that of present consumption. This is taken into account by some discounting. The life time utility for a person at age $a$ until she/he attains age $t > a$ then is:

$$U(a,t) = \int_a^t u\left[c(\tau)\right] \exp\left[-\int_a^\tau \rho(\theta)d\theta\right] d\tau$$

$$= \int_a^t u\left[c(\tau)\right] \exp\left[-\rho(\tau - a)\right] d\tau \quad (3.11)$$

for $\rho(\theta) = \rho$. Here it is assumed that consumption is not delayed, i.e incomes are not transformed into bequests. $\rho$ should be conceptually distinguished from a financial interest rate and is referred to as rate of time preference of consumption. A rate $\rho > 0$ has been interpreted as the effect of impatience, myopia, egoism, lack of telescopic faculty, etc. Another reason for this "subjective" rate of discount is the uncertainty about the future of a person. The economics literature also states that if no such "discounting" is applied more emphasis on the well being of future generations is placed rather than improving welfare of those alive at present assuming economic growth. A rate $\rho > 0$ is necessary for Eq. (3.11) to converge if future generations (and bequests) are inclued, i.e. if the utility integral must be extended to $t \to \infty$ [2]. $\rho$ is reported to be between 1 and 3% for health related investments, with tendency to lower values [50]. Empirical estimates reflecting pure consumption behavior vary considerably but are in part significantly larger [20].

The expected remaining present value life time utility at age $a$ (conditional on having survived until $a$) then is (see [2, 44, 40, 14, 19, 4]):

$$L(a) = E\left[U(a)\right] = \int_a^{a_u} \frac{f(t)}{\ell(a)} U(a,t)dt$$

$$= \frac{1}{\ell(a)} \int_a^{a_u} f(t) \int_a^t u\left[c(\tau)\right] \exp\left[-\rho(\tau - a)\right] d\tau dt \quad (3.12)$$

$$= \frac{1}{\ell(a)} \int_a^{a_u} u\left[c(t)\right] \exp\left[-\rho(t - a)\right] \ell(t)dt = u\left[c\right] e_d(a)$$

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[2] Exponential population growth with rate $n$ can be considered by replacing $\rho$ by $\rho - n$ giving larger weight to the utility of consumption at a later date but considering that families are by a factor $\exp[nt]$ larger. The correction $\rho > n$ appears always necessary simply because future generations are expected to be larger and wealthier.

where $f(t)dt = \left(-\frac{d}{dt}\ell(t)\right)dt = \left(\mu(\tau)\exp\left[-\int_0^t \mu(\tau)d\tau\right]\right)dt$ is the probability of dying between age $t$ and $t+dt$ computed from life tables and $\mu(\tau)$ is the age-dependent mortality rate. The first expression in the second line is obtained upon integration by parts. Also, a constant consumption rate $c$ independent of $t$ has been introduced. Note that $L(a)$ is finite throughout due to $a_u < \infty$. The "discounted" life expectancy $e_d(a)$ at age $a$ can be computed from

$$e_d(a) = \frac{\exp(\rho a)}{\ell(a)} \int_a^{a_u} \exp\left[-\int_0^t (\mu(\tau) + \rho)d\tau\right] dt. \qquad (3.13)$$

"Discounting" affects $e_d(a)$ primarily when $\mu(\tau)$ is small (i.e. at young age) while it has little effect for larger $\mu(\tau)$ at higher ages. Curves $e_d(a)$ become more compact with increasing $\rho$. It is important to recognize that "discounting" by $\rho$ is initially with respect to $u[c(\tau)]$ but is formally packed into the life expectancy term. Clearly, there is $e_d(0) \le e$ for $\rho > 0$. Depending on the value of $\rho$ life expectancy $e_d(a)$ is substantially reduced with increasing $\rho$.

For $u[c]$ we select a power function

$$u[c] = \frac{c^q - 1}{q} \qquad (3.14)$$

used frequently in economics with $0 \le q \le 1$. There is a minimum consumption necessary for food, clothing and housing without running the risk to die (in welfare states about half of mean consumption). Even then, it is assumed that the utility function can be approximated by Eq. (3.14). The form of Eq. (3.14) reflects the reasonable assumption that marginal utility $\frac{du[c]}{dc} = c^{q-1}$ decays with consumption $c$. $u[c] = \frac{c^q-1}{q}$ is a concave function since $\frac{du[c]}{dc} > 0$ for $q \ge 0$ and $\frac{d^2u[c]}{dc^2} < 0$ for $q < 1$. $q \ge 0$ implies that a person prefers more consumption rather than less at any period in life. $q < 1$ implies that a person is financially risk averse. $q = 1$ means indifference against risks. $q > 1$ indicates risk proneness which is considered as irrational in our context. Equation (3.14) with constant $q$ ($0 \le q \le 1$) implies constant relative risk aversion according to Arrow-Pratt constant elasticity of substitution or constant elasticity of marginal utility [3]. It is noteworthy that the power function form of Eq. (3.14) is also the result of the derivations in note 1 after some

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[3] Elasticity is defined as the relative derivative of a function $f(x)$: $\epsilon = \frac{\frac{df(x)}{dx}}{\frac{f(x)}{x}} = x\frac{f'(x)}{f(x)}$. The elasticity of marginal utility, i.e. the change in the slope of the utility function of consumption is: $\epsilon = -\frac{c\frac{d^2u(c)}{dc^2}}{\frac{du(c)}{dc}}$. $\epsilon > 0$ defines risk aversion, $\epsilon = 0$ risk neutrality and $\epsilon < 0$ risk proneness. For the utility function in Eq. (3.14) we have constant $\epsilon = 1 - q$. Therefore, for $q \approx 0.2$ it is $\epsilon \approx 0.8$. This value is also used in [44].

manipulation. Much further discussion can be found in [31]. The appropriate value of $q$ is in turn found by applying the work-leisure optimization principle in note 1.

Assuming consumption $c$ proportional to the (available) GDP $g \gg 1$ one can define a modified life quality index $L_d = \frac{g^q}{q} e_d(0)$. Pandey/Nathwani [31] went one important step further. They defined a **Societal Life Quality Index (SLQI)** by averaging $L(a)$ over the (undiscounted) age distribution, $h(a, n)$, in a stable population and obtained:

$$L_{\bar{E}} = \frac{g^q}{q} \int_0^{a_u} e_d(a) h(a, n) da = \frac{g^q}{q} \bar{E}, \qquad (3.15)$$

which is formally the same as Eq. (3.4) with $e$ replaced by $\bar{E}$. For $\rho = 0$ the averaged "discounted" life expectancy $\bar{E}$ is a quantity which is about 60% of $e$ and considerably less than that for larger $\rho$. Taking $\bar{E}$, now being an artificial parameter as opposed to $e$, is considered as an advantage because averaging over the age distribution better reflects the composition of the population and, consequently, the exposition of the population to hazards in or around technical objects. In good approximation, it is possible to define a new coefficient relating uniform changes in mortality ($\delta = \frac{dm}{m} > 0$) to changes in averaged "discounted" life expectancies, similar to Eq. (3.10):

$$\frac{d\bar{E}}{\bar{E}} \approx -C_{\delta\bar{E}}(\rho) \frac{dm}{m} = -C_{\delta\bar{E}}(\rho)\,\delta. \qquad (3.16)$$

The coefficient $C_{\delta\bar{E}}(\rho)$ for averaged "discounted" life expectancies turns out to be somewhat larger than that computed by Eq. (3.10) with "undiscounted" and not averaged life expectancies. This is in part due to the different age structure of the populations. $C_{\delta\bar{E}}(\rho)$ decreases with increasing $\rho$. This is shown in Table 1 for some countries for uniform proportional changes in mortality. The population growth rates $n$ have been taken into account [12].

Similar $C_{\delta\bar{E}}(\rho)$ can also be computed if a uniform change in mortality concerns only specific age ranges [31, 38]. The reasoning leading to the special form of the utility of consumption lets one speculate that developing countries might prefer a higher value of $\rho$ while developed countries prefer lower ones, thus diminishing the economic and demographic differences among countries. Nevertheless, it is remarkable that a change in mortality by some life saving operation results in changes in "discounted" averaged life expectancy which are close to those calculated by Eq. (3.10) for pure demographic life expectancies [38]. They are considered sufficiently close in view of the significant uncertainties when quantifying the effect of a life saving operation in terms of changes in mortalities.

TABLE 1. Dependence of $C_{\delta\bar{E}}$ on rate $\rho$ of time preference of consumption.

| | | Germany | Poland | Sweden[b] | Japan[b] | Canada[a] | USA[b] |
|---|---|---|---|---|---|---|---|
| $e$ | | 77 | 73 | 79 | 80 | 79 | 77 |
| $n$ | | 0.27% | -0.03% | 0.02% | 0.17% | 0.99% | 0.90% |
| $\rho$ | 0% | 0.27 | 0.31 | 0.26 | 0.26 | 0.29 | 0.27 |
| | 1% | 0.24 | 0.28 | 0.23 | 0.23 | 0.24 | 0.25 |
| | 2% | 0.22 | 0.25 | 0.21 | 0.20 | 0.21 | 0.21 |
| | 3% | 0.20 | 0.24 | 0.19 | 0.19 | 0.19 | 0.19 |
| | 4% | 0.19 | 0.23 | 0.18 | 0.18 | 0.17 | 0.17 |

[a] [24], [b] [9], others from complete life tables from national statistical offices.

The assumption of a constant $c$ over age is debatable. One would presume that consumption is below (life time) average at young ages, is above average between age 25 and 60, say, and then decays again below average. The details of the variations are difficult to assess (however, see [45, 44, 19]). However, in [44] it is shown that a constant consumption rate corresponds to the optimal rate under perfect market conditions. Constancy of $c$ is crucial for arriving at the simple product form Eq. (3.16) of the SLQI[4].

Application of the work-leisure optimization principle in [26] leads to the same form as in Eq. (3.6) with $e$ replaced by $\bar{E}$:

$$\frac{dg}{g} + \frac{1}{q}\frac{d\bar{E}}{\bar{E}} \approx \frac{dg}{g} - \frac{1}{q}C_{\delta\bar{E}}(\rho)\frac{dm}{m} \geq 0. \qquad (3.17)$$

---

[4] At the expense of some more numerical effort and some loss of transperancy it is possible to consider non-constant consumption functions $c(\tau)$ which may be optimal determined by the calculus of variations or by dynamic optimization given certain wealth and/or solvency constraints and known earning functions or can even be suboptimal. Then, the utility function is given by the left-hand expression in the third line of Eq. (3.12). In order to determine the SLQI we additionally need to average over the age distribution as in Eq. (3.15). Then, SLQI can no more be decomposed into two factors.

Giving up the intuitively appealing interpretation of the life quality index being the product of a term involving consumption and a term involving life expectancy, it is convenient to start from Eq. (3.3) in the form:

$$\frac{dc}{dm} = -\frac{\frac{\partial L_{\bar{E}}}{\partial m}}{\frac{\partial L_{\bar{E}}}{\partial c}}$$

where $dm$ is a small change in mortality and $dc$ is a small change in consumption. Remember that changes in life expectancy are related to changes in mortality and changes in the GDP are related to changes in consumption. Shepard/Zeckhauser [44] and others call $dc/dm$ "willingness to pay" (WTP). As before age-dependent mortality changes ac-

These interesting considerations add to a clear interpretation and understanding of the LQI. It appears as if demographic aspects play a less important role for SLQI than for LQI. On the other hand, estimation of the rate $\rho$ is difficult and still under debate in health related and other long-term economics. In most applications clear support for decisions can, however, be reached by using either Eq. (3.6) or (3.18). Are similar adjustments also necessary for the ICAF? The author is inclined to negate it because the compensation cost calculated approximately by the ICAF in Eq. (3.8) become real in an adverse event and have to be carried by the social system or insurance or both.

## 4. Application to technical facilities

We are now ready to apply these findings to safety regulations for structures or other technical facilities. It can reasonably be assumed that the life risk in and from such facilities is uniformly distributed over the age and sexes of those affected. Also, it is assumed that everybody uses such facilities and, therefore, is exposed to possible fatal accidents. The total cost of a safety related regulation per member of the group and year is

$$dg = -dC_Y(\mathbf{p}) = -\frac{1}{N}\sum_{i=1}^{n} dC_{Y,i}(\mathbf{p}) \tag{4.1}$$

where $n$ is the total number of objects under discussion, each with incremental cost $dC_{Y,i}$ and $N$ is the group size. Inserting into Eq. (3.17) gives:

$$\frac{-dC_Y(\mathbf{p})}{g} + \frac{1}{q}\left(-C_{\bar{E}\delta}\frac{dm}{m}\right) \geq 0.$$

Let $dm$ be proportional to the failure rate $dh(\mathbf{p})$ $(t \to \infty)$. Then, one finds

$$\frac{dC_Y(\mathbf{p})}{dh(\mathbf{p})} \geq -k\frac{C_{\bar{E}\delta}}{m}g\frac{1}{q} = -k\, G_F \tag{4.2}$$

---

cording to $\mu_\delta(a) = \mu(a)(1+\delta) = \mu(a)(1+dm/m)$ and, similarly, consumption changes according to $c_\Delta(a) = c(a)(1+\Delta) = c(a)(1+dc/c)$ such that, for example, the following budget constraint $\int_0^{a_u} c_\Delta(a)da = c\, a_u \geq 0$ is fulfilled, thus avoiding Fréchet differentiation. Taking derivatives of $L_{\bar{E}}$ with respect to $\delta$ and $\Delta$, respectively, leads after some simple manipulations to a criterion similar to Eq. (3.17) of the form:

$$\frac{dc}{dm} = -\frac{1}{q}\frac{C_{m\bar{E}}}{m}c \geq 0$$

with $c = g$. It is believed that the case of constant consumption is more relevant if public concerns are of interest.

where $dm = k\ dh(\mathbf{p})$, $0 < k \le 1$, the proportionality constant $k$ relating the changes in mortality to changes in the failure rate and $G_F = \frac{1}{q}\frac{C_{\bar{E}\delta}}{m}g$. The constant $k$ may be interpreted as a person's probability of actually being killed in case of failure. Note that for any reasonable intervention there is necessarily $dh(\mathbf{p}) < 0$. For the same data as used for ICAF above and $m \approx 0.01$, $\rho \approx 0.01$ and therefore $C_{\bar{E}\delta} \approx 0.25$ the constant $G_F$ is $1\,900\,000$ PPP US\$. If $C_{\bar{E}\delta}$, $m$ and $q$ remain essentially constant over time the right hand side of Eq. (4.2) grows as $g$ (see [37] for further investigations of time aspects). It is important to recognize that discounting is only with respect to the rate of pure time preference via the demographic constant $C_{\bar{E}\delta}$.

It remains to answer the question whether a criterion like Eq. (4.2) derived for safety-related regulations for a larger group in a society or the entire society can also be applied to individual technical projects. The constant $G_F$ and, similarly, the ICAF are derived from general considerations of changes in mortality by changes in safety-related but costly measures implemented in a regulation, code or standard by the public. $G_F$ as well as ICAF were related to one anonymous person. For a specific project it makes sense to apply criterion (4.2) to the whole group exposed. One may think of a number of technical projects each with $N_F$ potential fatalities in an exposed group of size $N$. Therefore, the "life saving cost" of a technical project with $N_F$ potential fatalities is:

$$H_F = \text{ICAF } k\ N_F. \tag{4.3}$$

Criterion (4.2) changes into:

$$\frac{dC_Y(\mathbf{p})}{dh(\mathbf{p})} \ge -k\,G_F\,N_F = K_F. \tag{4.4}$$

All quantities in Eq. (4.2) are related to one year. They apply to a safety related regulation under the assumption that there is steady-state building or production activity. For a particular technical project all cost, denoted by $dC(\mathbf{p})$ must be raised at the decision point $t = 0$. Any discounting, therefore, applies to $dC_Y(\mathbf{p})$ and $g$ as well, i.e. one obtains the same discount factor on both sides of Eq. (4.2). However, the yearly cost must be replaced by the erection cost at $t = 0$ on the left hand side of Eq. (4.2). The method of discounting is the same as for discharging an annuity. For infinite discounting time $(t \to \infty)$ consistent with the strategy of systematic reconstruction there is $dC_Y(\mathbf{p}) = dC(\mathbf{p})\gamma$. $dC_Y(\mathbf{p})$ may be interpreted as cost of societal financing of $dC(\mathbf{p})$. The interest rate to be used must, of course, be the societal interest rate to be discussed below. The LQI-criterion for individual technical projects then is:

$$\frac{dC(\mathbf{p})}{dh(\mathbf{p})} \ge -\frac{\exp[\gamma t] - 1}{\gamma \exp[\gamma t]}K_F \xrightarrow[t \to \infty]{} -\frac{K_F}{\gamma}. \tag{4.5}$$

Note that discounting in $G_F$, i.e. in the constant $C_{\bar{E}\delta}$, by the rate $\rho$ approximately cancels with the time preference part contained in $\gamma$. $N_F$ as well as $k$ must be estimated taking account of the average number of persons endangered by the event, the severity and suddenness of failure, possibly availability and functionality of rescue systems, etc. $N_F$ and $k$ also depend on the cause of failure and, frequently, are dependent on the safety measures themselves. The estimation of realistic values of $N_F$ and $k$ might belong to the most difficult tasks in actual practical applications. It is typically the subject of risk analysis or, more precisely, failure consequence analysis.

## 5. Optimization for technical components

For the special task in Eq. (2.1) with (2.2) we have

**Maximize:**

$$Z(\mathbf{p}) = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F)\frac{\lambda P_f(\mathbf{p})}{\gamma + \lambda P_f(\mathbf{p})}$$

**Subject to:** $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (5.1)

$$f_k(\mathbf{p}) \le 0, \quad k = 1, \ldots, q,$$

$$\nabla_p C(\mathbf{p}) + \frac{K_F}{\gamma} \nabla_p \big(\lambda P_f(\mathbf{p})\big) \ge 0,$$

where the first condition represent some restrictions on the vector $\mathbf{p}$ of optimization variables. The second condition represents the LQI – acceptability criterion written out for vectorial parameter $\mathbf{p}$ and $t \to \infty$. The failure consequences are now decomposed into direct cost $H_M$ (including indirect failure cost such as loss of business, service, etc.) and life saving cost $H_F$.

The formulation Eq. (5.1) includes the SLQI-criterion Eq. (4.5). Assume that the conditions $f_k(\mathbf{p}) \le 0$ are not active in the solution point. At the optimum there must be $\nabla_p Z(\mathbf{p}) = 0$, i.e. for $\mathbf{p} = \mathbf{p}^*$:

$$\nabla_p C(\mathbf{p}) + \left[ (C(\mathbf{p}) + H_M + H_F)\nabla_p \left( \frac{\lambda P_f(\mathbf{p})}{\gamma + \lambda P_f(\mathbf{p})} \right) \right] = 0, \qquad (5.2)$$

which is to be compared with the equality of Eq. (4.5) written out for vectorial parameter $\mathbf{p}$ and $t \to \infty$:

$$\nabla_p C(\mathbf{p}) + \frac{K_F}{\gamma} \nabla_p \big(\lambda P_f(\mathbf{p})\big) = 0. \qquad (5.3)$$

If there is $(C(\mathbf{p}) + H_M + H_F)/\gamma \ge K_F/\gamma$ the optimal solution for Eq. (5.2) will automatically fulfill the SLQI-criterion Eq. (4.5). It can be shown that

this will frequently be the case under conditions of interest, especially $\lambda P_f(\mathbf{p})$ $\ll \gamma$. Optimal structures are almost always safer than the SLQI-criterion would require.

## 6. Societal discount rates

The cost for saving life years in Eq. (4.5) also enters into the objective function (2.1) and with it the question of discounting those cost also arises. At first sight this is not in agreement with our moral value system. However, a number of studies summarized in [32] and [23] express a rather clear opinion based on ethical and economical arguments. The cost for saving life years must be discounted at the same rate as other investments, especially in view of the fact that our present value system is expected to be maintained for future generations, a goal which is supported by empirical studies on human preferences quoted in [23]. Otherwise serious inconsistencies cannot be avoided.

What should then the societal interest rate be? It is clear that it is different from the interest rates on the financial market nor can it be taken as the rate prescribed for governmental purposes in some countries without careful investigation of its long-term implications. In view of the time horizon of some 20 to more than 100 years (i.e. several generations) it should be a long-term average, i.e. $\gamma = E[\gamma(t)] = \frac{1}{t_u} \int_0^{t_u \gg t_s} \gamma(\tau) d\tau$ where $t_s$ is some anticipated service time. It should be net of inflation and taxes[5]. Weinstein/Stason [54] require that interest rates for life saving investments should be the same as for cost and thus equal to the real market interest rate, simply for consistency reasons. This appears to be an extreme point of view. The long time horizon suggests to prefer smaller rates. For example, for $\gamma = 0.075$ 1$ benefit (or loss) in 100 years is presently worth less than 0.1 cent, which appears unacceptable if human lives (in present and future generations) are concerned. But $\gamma = 0.015$ gives 0.23$, which let us feel a little more comfortable, yet still unsatisfied. The other extreme of not discounting intergenerationally at all is expressed in [43], [10] and [13], based primarily on ethical grounds in the context of $CO_2$-induced global warming, nuclear waste disposals, depletion of natural resources, etc. In this case the rationale of our basic optimization model Eq. (2.1) together with Eq. (4.5) and part of the considerations in section 3 break down. Further, it is beyond the author's grasp to imagine an eco-

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[5] Let $i$ be the monetary nominal interest rate and $\pi$ the inflation rate determined from the prices of representative market goods. Then, the real interest rate is $\gamma = i - \pi$ with $\pi = \frac{dp(t)/dt}{p(t)}$ and $p(t)$ is the price level. Real interest rates usually are between 2 and a little more than 5% in most countries.

nomic world without discounting. Presumably, there is something in between which can be founded rationally. There have been ongoing but somewhat inconclusive discussions when discounting public investments into health care (see, for example, [50]). Those discussions have been revived recently in the context of sustainable development, long term public investments in general and intergenerational justice – aspects which appear particularly relevant in our context. Our choices of discount rates for technical objects should at least be consistent with those for a sustainable economic development and should equally fulfill the requirement of intergenerational equity. Therefore, in the following the main stream of arguments is reviewed and an attempt is made to give some guidelines for practical applications.

Due to the requirement $\beta > \gamma_{\max}$ stated just below Eq. (2.4), the interest rate is strongly related to the benefit a society earns from its various activities, i.e. its real economic growth. The growth rate measures the success of all activities of a society – among them also activities for saving lives. It is sometimes called "natural interest rate" and measures technological progress. In most developed industrial countries the growth rate was about 2% over the last 50 years. The United Nations Human Development Report 2001 [49] gives values between 1.2 and 1.9% for industrialized countries during 1975-1998. If one considers the last 120 years and the data in [30] for some selected countries one determines a growth rate $\zeta = \frac{\ln(g_{1992}/g_{1870})}{1992-1870}100$ for exponential growth of about 1.8% (see Table 2) which is also the growth

TABLE 2. Data for some selected countries.

| Country | 1870 $g$ [a] | 1992 $g$ [a] | $\zeta$ [%] | $\epsilon$ [b] | $n$ [%] [c] | $\rho$ [%] | $\beta$ [%] [d] |
|---|---|---|---|---|---|---|---|
| United Kingdom | 3263 | 15738 | 1.3 | 0.81 | 0.23 | 0.4 | 1.3 |
| USA | 2457 | 21558 | 1.8 | 0.78 | 0.90 | 1.2 | 2.4 |
| France | 1858 | 17959 | 1.9 | 0.83 | 0.37 | 0.7 | 1.9 |
| Netherlands | 2640 | 16898 | 1.5 | 0.86 | 0.55 | 0.8 | 1.8 |
| Sweden | 1664 | 16927 | 1.9 | 0.82 | 0.02 | 0.3 | 1.6 |
| Germany | 1913 | 19351 | 1.9 | 0.83 | 0.27 | 0.6 | 1.8 |
| Australia | 3801 | 16237 | 1.2 | 0.79 | 0.99 | 1.2 | 2.0 |
| Japan | 741 | 19425 | 2.7 | 0.79 | 0.17 | 0.7 | 2.3 |
| Mean | 2292 | 13060 | 1.8 | – | – | 0.8 | 1.9 |

[a] in PPP US$, 1990,  [b] [38],  [c] [12],  [d] lower upper bound in Eq. (6.3).

rate for Western Europe, the socalled Western Offshots, USA, Canada and
Australia, and Japan. In [30] it is shown that growth was, in fact, nearly
exponential in the long run. For Southern Europe, Latin America and Asia
one finds from the same data similar growth rates $\bar{\zeta} \approx 1.7\%$, for Eastern
Europe still $\bar{\zeta} \approx 1.4\%$ but for Africa only $\bar{\zeta} \approx 0.9\%$. But convergence to a
universal growth rate can be observed. The high value for Japan (as well as
for a few other East Asian countries) appears to be a special (time limited)
case.

Some more insight can be gained from modern economic growth theory.
Nordhaus [29] and others follow the classical Ramseyan approach (see [39, 47]
and [6]) for optimal economic growth:

$$\gamma = \rho + \epsilon\zeta > 0 \tag{6.1}$$

where $\gamma$ is the real market interest rate, $\rho$ the rate of pure time preference
of consumption, $\epsilon > 0$ the elasticity of marginal consumption (income) and
$\zeta$ the consumption (income) growth rate[6].

Here, a perfect market with stable growth is assumed. In such a market
$\gamma$ equals the real growth rate of the total output of goods and services. With
$\rho \approx 0.03$ and $\zeta \approx 0.02$ as well as $\epsilon = 1$ Nordhaus [29] obtains $\gamma \approx 0.05$.
Arrow [1] estimates $\gamma \approx 0.03$ assuming $\rho \approx 0.01$, $\zeta \approx 0.012$ and $\epsilon = 1.5$ (!),

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[6] According to Solow [47] optimal economic growth will be achieved if the utility integral,
appropriately discounted by the rate of time preference of consumption, $\rho$, is maximum:

$$W = \int_0^\infty \exp[-(\rho - n)t]u[c(t)]dt \text{ subject to } \frac{dk(t)}{dt} = y(t) - c(t)$$

$n$ is the population growth rate, $y(t)$ denotes income and $k(t)$ is investment net of depre-
ciation, all per capita. Solow shows by dynamic optimization that on the optimal path the
following condition must hold for every $t$:

$$\frac{\frac{d}{dt}\frac{d}{dc}u[c(t)]}{\frac{d}{dc}u[c(t)]} = -(\gamma^*(t) - \rho)$$

where $\gamma^*(t)$ is marginal productivity and there is a steady state. Differentiation in the
steady state yields

$$\frac{\frac{d^2 u[c(t)]}{dc^2}\frac{dc(t)}{dt}}{\frac{d}{dc}u[c(t)]} = \frac{c\frac{d^2}{dc^2}u[c]}{\frac{d}{dc}u[c]}\frac{1}{c}\frac{dc}{dt} = -\epsilon\frac{\frac{dc}{dt}}{c} = -(\gamma^* - \rho).$$

Hence, on the optimal path

$$\gamma^* = \rho + \epsilon\zeta$$

where $\zeta = \frac{\frac{dc}{dt}}{c}$ is the growth rate of the economic output and $\epsilon$ is assumed to be constant
asymptotically (see [6] for mathematical details). Remember that $\epsilon$ is in fact constant for
the utility function Eq. (3.14) (see footnote 3).

however with tendency to larger values. In many other studies for sustainable development discount rates cluster around 5%. All those values are close to the real market rates or only a little smaller.

Solow [47], who presumes $\rho \approx 0.01$ to $0.02$, adds a convergence condition for the (infinite) utility integral

$$\rho + \epsilon \zeta > n + \zeta \tag{6.2}$$

to Eq. (6.1). However, there are many authors in economics as well as philosophical and political sciences including Ramsey who refuse convincingly to accept a rate $\rho > 0$ in intergenerational contexts on ethical grounds ([43, 10, 13, 34, 18, 7]) while it is considered fully acceptable for intragenerational discounting. Also, positive rates $\rho > 0$ are shown to be not mandatory for investments into health care (see, for example, [3]). Intergenerational and intragenerational rates of pure time preference, if greater than zero, should be the same if strongly counterintuitive results are to be avoided [13]. On the other hand, intergenerational equity arguments in [1], while fully accepting zero time preference rates from a moral point of view, indicate that there should be $\rho > 0$ in order to remove an "... *incredible and unacceptable strain on the present generation*". Rabl [34], who sets $\rho = 0$, argues that there must be $0 < \gamma < \epsilon \zeta$ in the framework of long-term public investments. However, Rabl neglects demographic aspects. As noted in note 2 we must have $\rho > n$ and, therefore, with $\rho \approx n$ at least $0 < \gamma < n + \epsilon \zeta$. On the basis of the Solow condition (6.2) one can, in fact, justify a rate $\rho$ even slightly larger than $n$. One derives:

$$n + \zeta(1 - \epsilon) < \rho < \gamma \leq \gamma_{\max} < \beta = n + \epsilon \zeta \quad \text{or} \quad \beta = n + \zeta \tag{6.3}$$

with preference for the smaller upper bound resulting from $\rho = n$. The larger upper bound is obtained by using $\rho = n + \zeta(1 - \epsilon)$ in Eq. (6.1). Values for $\rho$ and $\beta$ are also presented in Table 2 ranging from 0.003 to 0.012 and from 0.015 to 0.029, respectively, for the larger upper bound and from 0.013 to 0.024, respectively, for the smaller upper bound. The high value for Japan is due to its large recent economic growth rate (and exceptionally high saving rates), the high values for USA are partly due to a relatively high population growth rate (including immigration). It appears that $\rho$ is small enough to be acceptable in view of the controversy about the rate of time preference of consumption. Also the values for $\beta$ appear reasonable and acceptable. It is then possible to compute $\gamma_{\max} < \beta$ from Eq. (2.4). $\gamma_{\max}$ usually is only insignificantly (1 to 20%) smaller than $\beta$ depending on the specific case at hand, i.e. the particular sensitivities of $C(\mathbf{p})$ and $h(\mathbf{p})$ with respect to $\mathbf{p}$. The interest rates $\gamma_{\max}$ implied by the value of $\beta$ are considerably lower, around 1.9%, than the usual real market interest rates.

The above considerations based on a simple, ideal, steady state Ramseyan growth model in a closed economy can at least define the range of benefit and interest rates as well as reasonable rates of pure time preference to be used in long-term investments into life saving operations. It is believed that the steady state assumption of the Ramsey model is not too far from reality in developed countries. Also, the assumption of an infinite time horizon is consistent with our general setting. Historical long-term population and economic growth rates cannot be questioned. The value of $\epsilon$ varies very little, say between 0.75 and 0.85. Only the pure time preference rate $\rho$ to be used in Eq. (3.11) and possibly in Eq. (4.5) can be subject to discussion. It is suggested to take the lowest possible value which is $\rho = n + \zeta(1 - \epsilon) > 0$. Of course, our considerations do not exclude larger rates for the time preference of consumption in special projects if there are no potential intergenerational conflicts. Table 3 contains some additional data for the same countries as in Table 1.

TABLE 3. Economic and demographic data for countries in Table 1.

|  | Germany | Poland | Sweden | Japan | Canada | USA |
|---|---|---|---|---|---|---|
| GDP [a] | 25010 | 9030 | 23770 | 26460 | 27330 | 34260 |
| $g$ [b] | 14660 | 5630 | 12620 | 15960 | 16040 | 22030 |
| $\gamma$ [%] [c] | 1.9 | 1.6 | 1.9 | 2.7 | 2.0 | 1.8 |
| $m$ [%] [d] | 1.042 | 0.998 | 1.061 | 0.834 | 0.730 | 0.870 |
| $w$ [e] | 0.14 | 0.16 | 0.15 | 0.17 | 0.17 | 0.18 |
| $\rho$ [%] | 0.61 | 0.26 | 0.35 | 0.70 | 1.40 | 1.30 |
| $C_{\delta E}$ | 0.25 | 0.30 | 0.25 | 0.23 | 1.23 | 0.27 |
| $\beta$ [%] | 1.8 | 1.3 | 1.6 | 2.3 | 2.6 | 2.3 |
| ICAF $\times 10^5$ | 5.6 | 1.9 | 4.7 | 5.9 | 6.8 | 8.6 |
| $G_F \times 10^6$ | 2.1 | 0.9 | 1.7 | 2.1 | 2.5 | 3.1 |

[a] [56],  [b] [49],  [c] [30],  [d] [12],  [e] [38].

In the literature the adequacy of the Ramseyan model is sometimes questioned. For example, so-called overlapping generation models or generation adjusted discounting models are advocated instead (see [8] for theoretical considerations and [7] and [15] for applications). But it is not expected that those refinements change our results significantly. Other extensions and/or modifications have been proposed but they apply particularly to investments into sustainable development.

Some precautionary remarks are in order. The main body of environmental and economics literature on sustainable development agrees that economic growth will not persist, at least not at the long-term historical level. In many industrialized countries a very small or negative population growth rate is expected for the future accompanied by larger life expectancies and a significant change in the age structure (at the same time world population will grow by 1.2% each year, mainly in developing countries!). Natural resources will be depleted and arable land will become scarce. Many people raise serious doubts whether those demographic changes and the increasing scarcity of natural resources can be fully compensated by technological progress. Optimists, on the other hand, are confident that technology will provide solutions. It is hard to predict what will happen. But there is an important mathematical result which may guide our choice. Weitzman [53] showed that the far-distant future should be discounted at the lowest possible rate if there are different possible scenarios each with a given probability of being true. It is obvious that the results about the right public interest rate for long-term investments are not yet fully conclusive and still controversial. More research and discussion is necessary.

Since public benefit and interest rates are close together so that $Z_S(\mathbf{p}^*)$ (index "S" stands for optimization in the society's interest, "O" for optimization in the owner's interest) is positive but close to zero we postulate that the acceptable risk for any undertaking is the risk associated with $\mathbf{p}^*$ as the result of optimization in the public interest. Even if the (long-term) growth rate of the GDP is negative but $\beta$ small and $\gamma$ still positive, an optimization of Eq. (2.1) is still possible but the requirement of $Z_S(\mathbf{p}^*)$ being positive can no more be maintained. One has to be satisfied with the owner's solution of $Z_O(\mathbf{p})$ being maximum and positive. However, the public must require that Eqs. (3.6) or (3.17) and their derivates Eqs. (4.2) or (4.5) are always satisfied.

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