
Contributed Papers

Reliability-based optimization of spatial trusses

A. BADOWER¹⁾, S. JENDO²⁾, W.M. PACZKOWSKI¹⁾
and E. SILICKA¹⁾

*¹⁾Technical University of Szczecin
Faculty of Maritime Technology
Department of Theory of Structure
Al. Piastów 50, 70-311 Szczecin, Poland
optim@ps.pl*

*²⁾Institute of Fundamental Technological Research
Świętokrzyska 21, 00-049 Warszawa, Poland
sjendo@ippt.gov.pl*

In the paper a mathematical model of a discrete reliability-based polyoptimization problem is proposed. Spatial trusses made of steel tubes are analysed. The model takes into account random character of both the decision variables and the parameters. To each random variable an appropriate probability density function is attributed. The reliability level is measured as a probability of failure P_f and constitutes one of the criteria of evaluation. The presented method of reliability analysis is based on simulation methods, mainly on the Monte Carlo method. The crude Monte Carlo method has been modified to improve its efficiency.

Key words: *spatial truss, polyoptimization, reliability.*

1. Introduction

The development of numerical methods helps to design engineering objects in a better and more precise way. It also enables one to make the design process more controlled. For that purpose optimization methods become an indispensable tool. In the one- or multi-criteria optimization problems, an object is described by variables and parameters. The assumed variables are changed to obtain the best solution with regard to the criteria of evaluation. Such procedures might be very useful especially for some particular elements

or large-scale buildings [5] – they usually lead to a considerable reduction of costs of construction and maintenance.

To formulate the optimization problem properly it is necessary to define a vector of decision variables, a vector of objective functions and a vector of constraints. The decision variables describe the analyzed structure. They are changed during the process of optimization to satisfy the constraints and to achieve the optimum of the objective function in the case of a scalar optimization problem, or the Pareto set in the case of vector optimization problem. If more than one criterion of evaluation is assumed, a solution becomes usually more complex, but it better fits the engineering practice, where usually more than one feature of a structure is considered. One of the most frequently used criteria of evaluation is the mass of a structure or the volume of material. In case of large-scale objects, reliability becomes another important feature.

2. Random character of design variables

Reliability of engineering objects is usually measured as probability of failure. In the theory of reliability, failure is understood as a situation when some conditions defined by a designer are not satisfied [2, 7]. These conditions are called limit functions or safety margins $g_i(\mathbf{x})$. Reliability analysis enables one to take into account random character of quantities that describe both the object and the loads. Most of the variables have strongly stochastic character and are difficult to evaluate in the deterministic way. It concerns snow and wind loads especially. Such kinds of phenomena are recommended to be analyzed as time-varying stochastic processes. In engineering practice they are usually simplified and described by special probability density functions, e.g. Frechet or Gumbel distributions. Some random features are also typical for parameters describing the resistance of a structure. They may be defined as random variables with specified probability of occurrence [9, 1]. So it seems to be well-founded to treat both the load effect in the structure elements and the resistance of the structure as the functions of random variables, described respectively as

$$\begin{aligned} S &= S(x_{S1}, x_{S2}, \dots, x_{Si}), \\ R &= R(x_{R1}, x_{R2}, \dots, x_{Rk}). \end{aligned} \tag{2.1}$$

Then the limit function $g(\mathbf{x})$ is a combination of the resistance R and the load effect S . The function is defined in an i - D Cartesian space, where i is the number of assumed random variables. The limit function is graphically interpreted as the failure surface that divides the entire space of events Ω into

two subspaces called the safety region and the failure region. The realizations of random variables that satisfy the constraints $g(\mathbf{x}) = 0$ are situated on the failure surface (Fig. 1a). Then the probability of failure P_f means the probability of occurrence of realizations of the random variables that do not satisfy the constraints, for example the situation when the load effect S is greater than the resistance R [2, 7, 9]. If both S and R are described by probability density functions $f_S(x_S)$ and $f_R(x_R)$, respectively, the probability of failure may be defined as follows

$$P_f = P(R - S \leq 0) = \int_{-\infty}^{\infty} f_R(x_R) \left[\int_{-\infty}^{x_R} f_S(x_S) dx_S \right] dx_R = \int_{-\infty}^{\infty} f_R(x_R) F_S(x_R) dx_R \quad (2.2)$$

where $F(x)$ is the cumulative probability function. The Equation (2.2) is interpreted graphically in Fig. 1b.

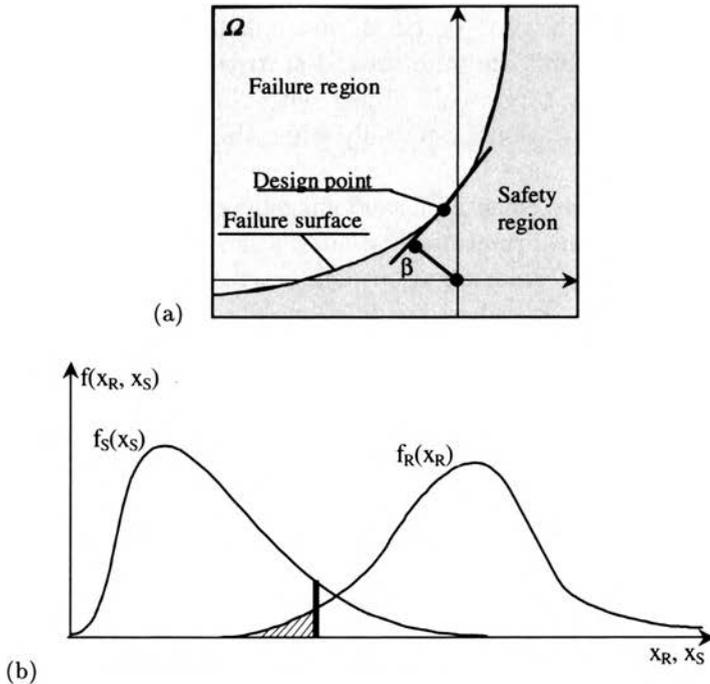


FIGURE 1. (a) Failure surface in FORM; (b) Interpretation of P_f .

Such a definition of probability of failure allows one to determine unambiguously reliability level of a structure. So it may be treated as the criterion of evaluation in polyoptimization problems.

3. Main methods of reliability analysis

There exist a few kinds of methods of reliability analysis. The most frequently used are:

- *First Order Reliability Methods (FORM)*,
- *Second Order Reliability Methods (SORM)*,
- *Stochastic Finite Element Method (SFEM)*,

and a group of simulation methods [2, 6, 7, 9].

In *FORM* reliability is expressed by the β index which is the distance between the failure surface and the origin of coordinate system. So it is required here to transform the space Ω to the normalized Gauss space and to expand the limit function into the Taylor series at the design point closest to the origin. In *SORM* similar idea is employed, but the expansion is more precise and better approximates the limit function. However, more calculations are required here.

SFEM is the modification of *FEM* that enables one to obtain some information about the first and the second statistic moments. It is possible to analyze a relatively large-scale object with this method, but it may be not precise in some situations, especially when the limit function is strongly nonlinear.

Simulation methods consist of random generation of a variable sample that after required transformations is treated as an empirical one. This group of methods includes information about character of the probability function that the variables are described by. It is usually employed when the limit function is given in an implicit way.

4. Formulation of the reliability-based polyoptimization problem

In the paper, a mathematical model of discrete reliability-based polyoptimization problem is proposed. Objects of the analysis are spatial trusses [4]. The model requires definition of a vector of decision variables \mathbf{x} , a vector of objective functions $\mathbf{f}(\mathbf{x})$, and vectors of constraints functions $\mathbf{g}(\mathbf{x})$ and $\mathbf{h}(\mathbf{x})$, similarly to the standard deterministic optimization task:

$$\mathbf{x} = \{x_1, x_2, \dots, x_N\}, \quad (4.1)$$

$$\mathbf{f}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_J(\mathbf{x})\}, \quad (4.2)$$

$$\mathbf{g}(\mathbf{x}) = \{g_1(\mathbf{x}) \leq 0, g_2(\mathbf{x}) \leq 0, \dots, g_K(\mathbf{x}) \leq 0\}, \quad (4.3)$$

$$\mathbf{h}(\mathbf{x}) = \{h_1(\mathbf{x}) = 0, h_2(\mathbf{x}) = 0, \dots, h_L(\mathbf{x}) = 0\}. \quad (4.4)$$

The random character of both the decision variables and the parameters are considered. It needs to be defined which entries are to be described as the random variables. Reduction of the number of random variables improves greatly the efficiency of the computational process. The assumed random variables are put into the vector $\mathbf{x}_{RV} = \{x_{RV1}, x_{RV2}, \dots, x_{RVt}\}$. An appropriate marginal probability distribution is attributed to each variable. It may be also assumed that the variables are stochastically independent. Then t -dimensional joint probability distribution $P_0(\mathbf{x}_{RV})$ is easily established. The function $P_0(\mathbf{x}_{RV})$ is discretized and then each realization is described by the vector \mathbf{x}_{RV} and the probability of occurrence. In the case of large-scale truss systems, limit functions are often given in a non-analytical or implicit way. They usually concern the resistance or critical load effect in a single bar (local functions), or maximum displacement or stability (global functions). The reliability of each analyzed variant of the structure, measured as P_f , is then used as one of the criteria of evaluation.

The problem solution is a set of nondominated solutions X_{ND} and nondominated valuations Y_{ND} . In discrete problems, Y_{ND} is defined as

$$\begin{aligned} Y_{ND} &= \mathbf{f}(x_{ND}) = \text{Opt}_{\mathbf{x} \in X} \mathbf{f}(\mathbf{x}) \\ &= \{y_{ND} \in Y : \neg \exists y_i \in Y : y_i \neq y_{ND} \wedge y_{ND} \in y_i + \Lambda\}, \quad (4.5) \end{aligned}$$

where y_i is an optional realization vector of evaluations, y_{ND} - a nondominated evaluation, Λ - a cone of domination. The preferred evaluation is selected from the set Y_{ND} , and it determines the preferred solution by the inverse transformation $\mathbf{f}^{-1}(\mathbf{x})$. Note that this stage of the model is the same as in the deterministic polyoptimization problem [5, 8].

The presented method of reliability analysis is based on the group of simulation methods [2, 7, 9]. The crude Monte Carlo method has been modified to improve its efficiency. The sample is not taken randomly, but is controlled during the process of integration. The control system is based mainly on the fact that the failure region is a convex domain. The integration proceeds in one specified direction until a realization that fulfils the condition is found. Then another direction is examined. Such a modification improves greatly the efficiency without the decrease of precision.

The polyoptimization problem is realized as a two-level procedure. The proposed algorithm of solution is shown in Fig. 2. The external process is

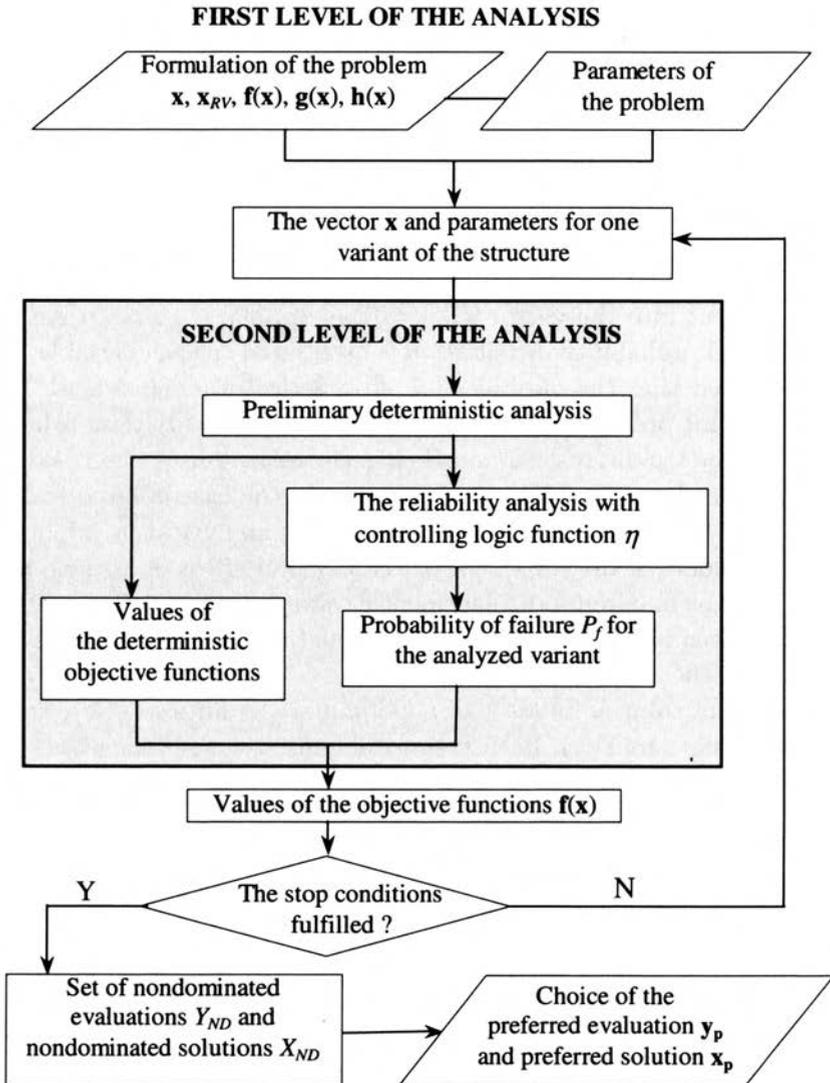


FIGURE 2. The algorithm of solution of the problem.

the optimization loop. Here the initial data concerning loads and material properties are established. Next, scalar optimization problem is solved for the assumed variant of the structure. To that purpose *Optytruss* system is used. It enables one to obtain axial force and to select the best possible profile for each element of the truss. Profiles are matched on the basis of a discrete catalogue of commercially available cross-sections [2]. For the assumed variant of a

structure appropriate evaluations are obtained in a deterministic way. In parallel, the probability of failure is defined. For that purpose the second internal loop is used. Such a formulation of the algorithm makes the solution clear and easy to interpret.

5. Numerical example

To test the proposed model, a one-layer truss dome has been analyzed. The span of the structure is 30 m and the rise – 9.25 m (Fig. 3).

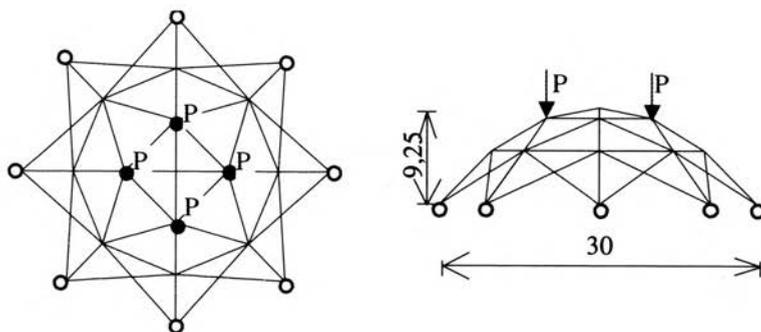


FIGURE 3. The analyzed truss dome.

The truss consists of 52 elements and 21 nodes. It is loaded symmetrically by four concentrated forces $P = 1000$ kN at the joints (Fig. 3). Supports of the truss are realized at eight nodes as perfectly hinged. It is also assumed that bars of the structure are made of elements with the same cross-section. To solve the problem, the one-element vector of decision variables has been assumed

$$\mathbf{x} = \{x_1\} \quad (5.1)$$

where x_1 is the cross-section of the truss bars. The cross-sections are selected from the catalogue of commercially available steel tube elements [3]. The x_1 takes finite number of values contained in the discrete space of the solution A . Three criteria of evaluation have been chosen – minimum volume of steel, minimum of the biggest node displacement, and minimum probability of failure. The criteria are uniquely defined in the vector of objective function, respectively

$$\mathbf{f}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})\}. \quad (5.2)$$

The constraints have been imposed on the decision variable. They restrict upper and lower values of the diameter and the wall thickness of the tube. The

constraints apply also to maximum displacement and maximum probability of failure

$$177.8 \text{ mm} \leq D \leq 355.6 \text{ mm}, \quad (5.3)$$

$$8 \text{ mm} \leq t \leq 20 \text{ mm}, \quad (5.4)$$

$$f_{\max} \leq 2.5 \text{ cm}, \quad (5.5)$$

$$P_f \leq 10^{-4}. \quad (5.6)$$

The discrete constraints are imposed by the catalogue [3]. In the analyzed example, random characters of diameters and wall thickness of tubes and the yield stress of steel have been taken into account. These quantities are put into the vector of random variables, respectively,

$$\mathbf{x}_{RV} = \{x_{RV1}, x_{RV2}, x_{RV3}\}. \quad (5.7)$$

The random variables are described by the probability density functions and parameters $(\mu; \nu)$, where μ is the mean and ν – the standard deviation;

x_{RV1} – normal distribution ($D_0; 12,5\%$), D_0 – nominal value;

x_{RV2} – normal distribution ($t_0; 6\%$), t_0 – nominal value;

x_{RV3} – Gumbel distribution for minimum ($f_y; 8\%$), $f_y = 287 \text{ MPa}$.

The assumed distributions have been constrained and discretized to a finite number of intervals. At the beginning, three limit functions were considered as constraints for reliability analysis. The first one concerns the probability of resistance loss of the truss bar, the second one – the probability of not satisfying the displacement constraints, and the third one – the probability of global stability loss. Each function was evaluated before the analysis. The results showed that the second and the third functions were not significant for further analysis. Thus, only the first one has been taken into account. The limit function is defined as follows:

$$g(\mathbf{x}_{RV}) = \frac{S}{R(\mathbf{x}_{RV})} \leq 1, \quad (5.8)$$

where R is the resistance of the weakest bar of the structure and S – the load effect (axial force) in the bar.

During the analysis, 30 different profiles of the tubes have been taken into account. The results from the procedures in the internal reliability loop showed that a few variants do not satisfy constraints on the allowable probability of the failure. That fact is very significant, because all these variants fulfilled the conditions contained in design codes [10].

After the analysis, it was found that all the solutions from the feasible domain were nondominated. The results are shown in Fig. 4. The preferred evaluation

$$\mathbf{y}_p = \{4.466e - 3 \text{ m}^3/\text{m}^2; 15.75 \cdot 10^{-3} \text{ m}; 1.0344 \cdot 10^{-9}\} \quad (5.9)$$

was selected with the use of distance function method with the norm $\|p\| = 2$. To make the choice more objective all valuations were normalized. Next, the preferred solution was found by the transformation

$$\mathbf{x}_p = \mathbf{f}^{-1}(\mathbf{y}_p) = \{298.5/10\}. \quad (5.10)$$

It means that the tube with the diameter 298,5 mm and wall thickness 10 mm fulfills at best the assumed criteria.

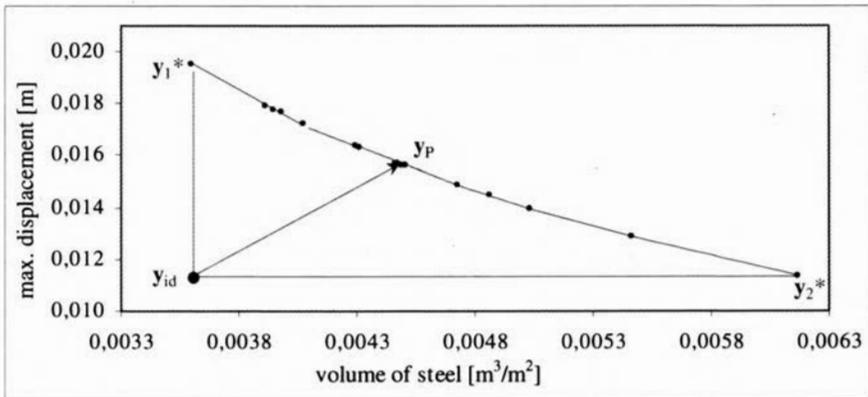


FIGURE 4. Set of nondominated evaluations.

6. Concluding remarks

- It is possible to evaluate in an efficient way the reliability of a structure measured as the probability of failure.
- It seems to be justified to treat the probability of failure as a criterion in polyoptimization problems.
- Taking into account random character of design variables may significantly influence the results of optimization of structure.

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