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Reliability algorithm for reinforced concrete structures, an application

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This paper presents an algorithm of structural reliability evaluation based on design sensitivity analytical approach. To calculate the design derivatives of the field variables the direct differentiation (DDM) and the adjoint variable (AVM) methods are used. The DDM is applied to the structures with nonlinear behaviour. The presented algorithm is applied to RC plate – shell structures. The attention is focused on the analysis of an RC nuclear containment vessel.

Key words: *sensitivity, reliability, nuclear containment vessel, finite elements*

1. Introduction

The application of the reliability analysis and reliability based design is very wide. The analysis is particularly useful in the case of taking into account different types of material and geometric imperfections. An example of the sensitivity based approach, Refs. [1, 3, 9], is presented herein.

2. Reliability, general algorithm

Considering a stochastic process as a vector of random variables $\mathbf{U}$ which specific realization is the vector $\mathbf{x} = \{\mathbf{x}^{\mu}, \mathbf{x}^{\sigma}\}$, where $\mathbf{x}^{\mu}$ and $\mathbf{x}^{\sigma}$ are the vectors of the mean values and standard deviations of the probability distributions, the following failure function is introduced:

$$g(\mathbf{x}) = 0. \tag{2.1}$$

Transforming the vector $\mathbf{x}$ to a system of standard variables $\mathbf{u}$ using the Rosenblatt transformation $\mathbf{x} = T(\mathbf{u})$, the definition of the reliability index takes the form, cf. [3, 5],

$$\beta = ||\mathbf{u}^{\star}|| = \min ||\mathbf{u}|| \quad \text{with the condition} \quad g(\mathbf{u}) = 0. \tag{2.2}$$

The failure function g depends on the stochastic parameters vector $\mathbf{x}(\mathbf{u})$ and the performance measure Ψ, i.e. displacements $\mathbf{q}$ or the stress vector $\mathbf{S}$ depending also on $\mathbf{x}$

$$g\Big\{\mathbf{x}(\mathbf{u}),\ \Psi[\mathbf{x}(\mathbf{u})]\Big\} = 0. \tag{2.3}$$

The first derivative of the failure function w.r.t. the standard variables vector takes the form

$$\frac{\partial g}{\partial \mathbf{u}} = \frac{\partial g}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}} + \frac{\partial g}{\partial \Psi}\frac{\partial \Psi}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}. \tag{2.4}$$

The first derivative of the failure function is required during the optimization procedure, Eq. (2.1). The derivatives $\partial g/\partial \Psi$ and $\partial g/\partial \mathbf{u}$ are usually calculated explicitly. The derivatives $\partial \mathbf{x}/\partial \mathbf{u}$ are obtained exploiting the transformation $\mathbf{x} = T(\mathbf{u})$. The derivatives $\partial \Psi/\partial \mathbf{x}$ are the sensitivities (design derivatives) and are calculated using the methods described in [2].

3. Constraints functions

Let us consider a performance function depending on the state variables (stresses $\mathbf{S}$, nodal displacements $\mathbf{q}$) determined in the domain Ω and its boundary $\partial\Omega_\sigma$ fulfilling the displacement and stress boundary conditions. The function depends on the stresses and displacements determined at time $t + \Delta t$. The UL approach is used [11]. The stresses and the displacements are dependent implicitly on the design variable h. The function takes the form

$${}^{t+\Delta t}\Psi(\mathbf{S}, \mathbf{q}; h) = \int\limits_{\Omega^{\mathrm{t}}} G({}^{t+\Delta t}\mathbf{S}, {}^{t+\Delta t}\mathbf{q}; h)\, d\Omega^{\mathrm{t}} + \int\limits_{\partial\Omega^{\mathrm{t}}_{\sigma}} s({}^{t+\Delta t}\mathbf{q}; h)\, d(\partial\Omega^{t}_{\sigma}). \tag{3.1}$$

The total derivative of the function, Eq. 3.1 w.r.t. the design parameter h reads

$$\frac{d^{t+\Delta t}\Psi}{dh} = \int\limits_{\Omega^{\mathrm{t}}} \left[\frac{\partial G}{\partial \mathbf{S}}\frac{d^{t+\Delta t}\mathbf{S}}{dh} + \frac{\partial G}{\partial \mathbf{q}}\frac{\partial^{t+\Delta t}\mathbf{q}}{dh} + \frac{\partial G}{\partial h}\right] d\Omega^{\mathrm{t}} + \int\limits_{\partial\Omega^{\mathrm{t}}_{\sigma}} \left[\frac{\partial s}{\partial \mathbf{q}}\frac{d^{t+\Delta t}\mathbf{q}}{dh} + \frac{\partial s}{\partial h}\right] d(\partial\Omega^{t}_{\sigma}). \tag{3.2}$$

A particular case of the function, Eq. (3.1) may be considered

$$\Psi = |q| - q^a < 0 \tag{3.3}$$

where q is a choosen displacement and q^a is its allowable value.

4. Adjoint variable method

At first the AVM is introduced. The method is applied due to avoiding of the calculation of the displacement design derivatives directly. The adjoint method may be understood as the Lagrange multiplier method. Let us define the augmented Lagrange function in the form:

$$L(\mathbf{q}, \boldsymbol{\lambda}, \mathbf{h}) = \Phi - \boldsymbol{\lambda}^{\mathrm{T}}(\mathbf{K}\mathbf{q} - \mathbf{Q}), \tag{4.1}$$

where $\boldsymbol{\lambda}$ is the N-dimensional Lagrange multipliers vector (adjoint variables), $\mathbf{K}\mathbf{q} = \mathbf{Q}$ is the equilibrium equation. The stationary condition with respect to the displacements imposed on Eq. (4.1) is of the form:

$$\frac{\partial L}{\partial \mathbf{q}} = \frac{\partial \boldsymbol{\Phi}}{\partial \mathbf{q}} - \boldsymbol{\lambda}^{\mathrm{T}}\mathbf{K} = 0. \tag{4.2}$$

Considering the stationarity condition it is possible to calculate $\boldsymbol{\lambda}$. Differentiating Eq. (4.1) w.r.t. the design variables vector $\mathbf{h}$ we obtain:

$$\frac{dL}{d\mathbf{h}} = \frac{d\Phi}{d\mathbf{h}} - \boldsymbol{\lambda}^{\mathrm{T}}\frac{d}{d\mathbf{h}}(\mathbf{K}\mathbf{q} - \mathbf{Q}) - \left(\frac{d\boldsymbol{\lambda}}{d\mathbf{h}}\right)^{\mathrm{T}}(\mathbf{K}\mathbf{q} - \mathbf{Q}). \tag{4.3}$$

Considering that the equilibrium equation is fulfilled for the nominal and the perturbed value of the parameter $\mathbf{h}$, Eq. (4.3) takes the form:

$$\frac{dL}{d\mathbf{h}} = \frac{d\Phi}{d\mathbf{h}}. \tag{4.4}$$

Now, we may calculate the derivative of the augmented function, Eq. (4.1) instead of the derivative of the function, Eq. (3.1). The design derivative is expressed as follows:

$$\frac{dL}{d\mathbf{h}} = \frac{\partial L}{\partial \mathbf{h}} + \frac{\partial L}{\partial \mathbf{q}}\frac{d\mathbf{q}}{d\mathbf{h}} + \frac{\partial L}{\partial \boldsymbol{\lambda}}\frac{d\boldsymbol{\lambda}}{d\mathbf{h}}. \tag{4.5}$$

So, the total derivative of the function Φ is of the form:

$$\frac{d\Phi}{d\mathbf{h}} = \frac{\partial\Phi}{\partial \mathbf{h}} - \boldsymbol{\lambda}^{\mathrm{T}}\frac{d}{d\mathbf{h}}(\mathbf{K}\mathbf{q} - \mathbf{Q}) + \left(\frac{\partial\Phi}{\partial \mathbf{q}} - \boldsymbol{\lambda}^{\mathrm{T}}\mathbf{K}\right)\frac{d\mathbf{q}}{d\mathbf{h}}. \tag{4.6}$$

The last term in Eq. (4.6) disappears because of the stationarity condition, Eq. (4.2). It is clearly seen that it is possible to calculate the design derivative of the function (3.1) without calculation of the displacement design derivatives. However, it is necessary to introduce the additional linear adjoint equation:

$$\mathbf{K}\boldsymbol{\lambda} = \left(\frac{\partial \Phi}{\partial \mathbf{q}}\right)^{\mathrm{T}}. \tag{4.7}$$

The r.h.s of Eq. (4.7) is the explicit partial derivative of the function (3.1). Finally the expression for the design derivative of the function (3.1) takes the form:

$$\frac{d\Phi}{dh} = \frac{\partial \Phi}{\partial h} - \boldsymbol{\lambda}^{\mathrm{T}}\left(\frac{\partial \mathbf{K}}{\partial h}\mathbf{q} - \frac{\partial \mathbf{Q}}{\partial h}\right). \tag{4.8}$$

For the particular case of the function (3.1) in the form of Eq. (3.3), the r.h.s. of the adjoint equation is expressed as follows:

$$\frac{\partial \Phi}{\partial \mathbf{q}} = \operatorname{sign}(q)\left[0,\ldots,0,\frac{1}{q^a},0,\ldots,0\right]. \tag{4.9}$$

When applying the adjoint method it is necessary to solve as many adjoint equations as the design constraints. So, it is worthy to apply the method if the number of the constraints is lower than the number of the design variables. The direct differentiation method is useful in the opposite case.

5. Direct differentiation method

The design derivative of the incremental equation of equilibrium reads:

$$\int_{\Omega^t} \mathbf{B}_L^{\mathrm{T}} \frac{d\Delta \mathbf{S}}{dh}\, d\Omega^t = \frac{\partial \Delta \mathbf{Q}}{\partial h}. \tag{5.1}$$

Taking into account the explicitly integrated constitutive equation the design derivative of the stress increment $\Delta\mathbf{S}$ is calculated. The elasto-plastic matrix $\mathbf{C}^{(ep)}$ and the strains increment depend on the design derivative h. Simultaneously, the elasto-plastic matrix is a function of the total stresses $\mathbf{S}$ and internal variables vector $\boldsymbol{\gamma}$. The stress increment takes the form

$$\Delta \mathbf{S} = \mathbf{C}^{(ep)}\left({}^t\mathbf{S}, {}^t\boldsymbol{\gamma}, h\right)\Delta \mathbf{e}(h). \tag{5.2}$$

Differentiating Eq. (5.2) w.r.t. the design parameter the relation for the stress increment design derivative takes the form

$$\frac{d\Delta\mathbf{S}}{dh} = \left\{ \frac{\partial \mathbf{C}^{(ep)}}{\partial \mathbf{S}} \frac{d^t\mathbf{S}}{dh} + \frac{\partial \mathbf{C}^{(ep)}}{\partial \gamma} \frac{d^t\gamma}{dh} + \frac{\partial \mathbf{C}^{(ep)}}{\partial h} \right\} \Delta\mathbf{e} + \mathbf{C}^{(ep)} \left({}^t\mathbf{S}, {}^t\gamma, h\right) \frac{d\Delta\mathbf{e}}{dh}. \tag{5.3}$$

Substituting the design derivative stress increment in Eq. (5.1) to Eq. (5.3) and having in mind the strain-displacement relation written in FE form:

$$\Delta\mathbf{e} = \mathbf{B}_L^{\mathrm{T}} \Delta\mathbf{q}\,, \tag{5.4}$$

the expression for the displacement sensitivity increment is obtained as

$$\left(\int\limits_{\Omega^t} \mathbf{B}_L^{\mathrm{T}} \mathbf{C}^{(ep)} \mathbf{B}_L \, d\Omega^t \right) \frac{d\Delta\mathbf{q}}{dh} = \frac{\partial \Delta\mathbf{Q}}{\partial h} - \int\limits_{\Omega^t} \mathbf{B}_L^{\mathrm{T}} \left. \frac{d\Delta\mathbf{S}}{dh} \right|_{\Delta\mathbf{q}(h)=\mathrm{const}} d\Omega^t. \tag{5.5}$$

The first component of the above expression stands for the tangent stiffness and the last one is the design derivative of the nodal internal forces assuming that the displacement increment is independent of the design variable h. Solving Eq. (5.5) the displacement increments as well as the stress and internal variables increments and their design derivatives have to be accumulated in time.

6. A constitutive model for concrete: design derivatives

An outline of the DSA algorithm for a constitutive model applied to RC structures is presented herein [6]. The constitutive relations for plane stress state and their design derivatives acting in the principal directions for compression-compression state are presented below:

$$\begin{aligned} \sigma_i &= \frac{E_o \varepsilon_{iu}}{1 + \left(\dfrac{E_o}{E_s} - 2\right)\left(\dfrac{\varepsilon_{iu}}{\varepsilon_{ic}}\right) + \left(\dfrac{\varepsilon_{iu}}{\varepsilon_{ic}}\right)^2}, \\ E_i &= \frac{E_o \left[1 - \left(\dfrac{\varepsilon_{iu}}{\varepsilon_{ic}}\right)^2\right]}{\left[1 + \left(\dfrac{E_o}{E_i} - 2\right)\dfrac{\varepsilon_{iu}}{\varepsilon_{ic}} + \left(\dfrac{\varepsilon_{iu}}{\varepsilon_{ic}}\right)^2\right]^2}, \end{aligned} \tag{6.1}$$

where E_o is the initial tangent modulus, E_s is the secant modulus, ε_{iu} are the equivalent uniaxial strains, ε_{ic} are the maximum equivalent uniaxial strains and σ_{ic} are the maximum equivalent uniaxial stresses in plane stress state. The relations for σ_{ic} and for ε_{ic} are determined by the Kupfer-Gerstle curve,

$$\begin{gathered}\sigma_{2c} = \frac{1+3.65\beta}{(1+\beta)^2} f_c\,, \qquad \sigma_{ic} = \beta\sigma_{2c}\,, \qquad \varepsilon_{ic} = \varepsilon_c \left(3\frac{\sigma_{2c}}{f_c} - 2\right), \\ \varepsilon_{2c} = \varepsilon_c \left[-1.6\left(\frac{\sigma_{1c}}{f_c}\right)^3 + 2.25\left(\frac{\sigma_{1c}}{f_c}\right)^2 + 0.35\left(\frac{\sigma_{1c}}{f_c}\right)\right].\end{gathered} \tag{6.2}$$

where β is the principal stresses ratio σ_1/σ_2, f_c and ε_c are the maximum strength and maximum strains taken from the uniaxial compression test.

The r.h.s. of Eq. (5.5) appropriate for reinforced concrete acting in plane stress state is of the form

$$\begin{aligned}\left.\frac{d\Delta\mathbf{F}}{dh}\right|_{\Delta\mathbf{q}(h)=\mathrm{const}} = &\left(\int_{\Omega^t} \mathbf{B}_L^{\mathrm{T}} \frac{d\mathbf{C}(E_i,\nu)}{dh}\, d\Omega^t\right)\Delta\mathbf{q} \\ &+ \left(\int_{\Omega^t} \mathbf{B}_L^{\mathrm{T}} \frac{d\mathbf{C}^{(ep)}}{dh}\, d\Omega^t\right)\Delta\mathbf{q}\,.\end{aligned} \tag{6.3}$$

The matrix $\mathbf{C}$ is the constitutive matrix of the orthotropic material depending on the actual tangent moduli in the pricipal directions i. The matrix $\mathbf{C}^{(ep)}$ is the elasto-plastic matrix for the reinforcing steel acting in the uniaxial direction.

The stress increment in the direction i depending on the state of the material may be expressed as follows

$$\Delta\sigma_i = E_i\Big(\varepsilon_{iu}, \varepsilon_{ic}(\beta,h), \sigma_{ic}(\beta,h)\Big)\Delta\varepsilon_{iu}. \tag{6.4}$$

The design derivative of the stress increment, Eq. (6.4) takes the form

$$\frac{d\Delta\sigma_i}{dh} = E_i\frac{d\Delta\varepsilon_{iu}}{dh} + \frac{dE_i}{dh}\Delta\varepsilon_{iu}. \tag{6.5}$$

The design derivative of the tangent moduli is of the form

$$\begin{aligned}\frac{dE_i}{dh} = \frac{\partial E_i}{\partial\varepsilon_{iu}}\frac{d\varepsilon_{iu}}{dh} &+ \frac{\partial E_i}{\partial\varepsilon_{ic}}\left(\frac{\partial\varepsilon_{ic}}{\partial\beta} + \frac{\partial\varepsilon_{ic}}{\partial h}\right) \\ &+ \frac{\partial E_i}{\partial\sigma_{ic}}\left(\frac{\partial\sigma_{ic}}{\partial\beta}\frac{d\beta}{dh} + \frac{\partial\sigma_{ic}}{\partial h}\right) + \frac{\partial E_i}{\partial h}\,.\end{aligned} \tag{6.6}$$

The partial derivatives in Eq. (6.6) can be obtained differentiating the expressions (6.1), (6.2), the ordinary derivatives are accumulated in time.

7. Computer program, general description, features

The computer program consists of two main modules: the reliability module solving the problem defined by Eq. (2.2) and the module solving the static equilibrium problem formulated by Eq. (5.1) with the implemented DSA. The DSA is necessary for the calculation of the design derivatives w.r.t. the standardized variables given by Eq. (2.4). The general DSA formulation is expressed by Eqs. (3.1) and (3.2). The reliability module is the program COMREL-TI [4]. The nonlinear static equilibrium problem, Eq. (5.1) and the DSA, Eq. (5.5) or Eq. (4.7) are solved by a significantly extended version of the program NASHL [6]. The AVM is used for the linear problems and the DDM for the nonlinear ones. The programs are implemented on UNIX workstations and a mainframe.

8. Numerical example, nuclear containment vessel

The nuclear containment vessel structure consists of a cylinder (radius 20 m) and a dome, height of the structure is 64 m, Fig. 1. The vessel is discretized with 640 shell elements, Fig. 2. The structure is prestressed with a system of tendons and the action of the active reinforcement is replaced by the action of the external pressure distributed approximately in compliance with the zones division. The number of nodes is 2600 and number of d.o.f. is 12800. More detailed description is given in [10].

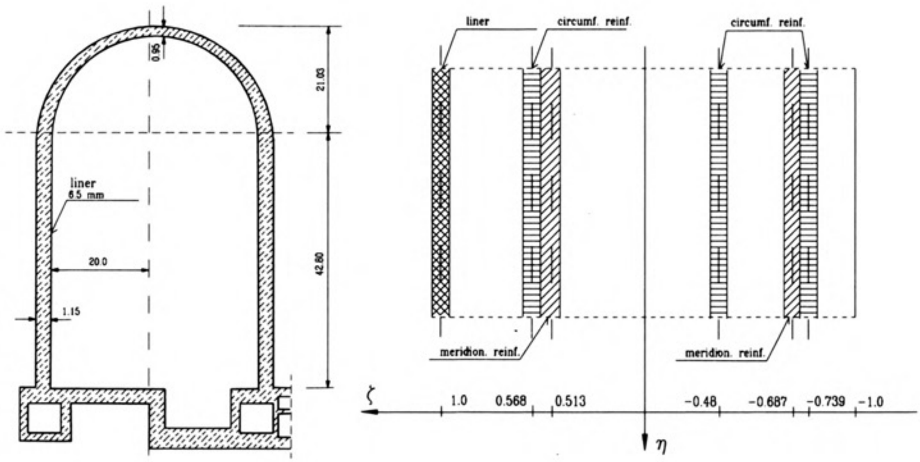

FIGURE 1. Containment vessel, vertical cross-section (left) and a typical cross-section of the wall (right).

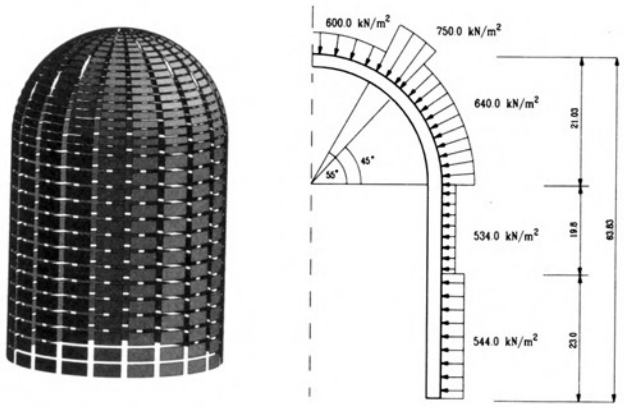

FIGURE 2. FE discretization, shrink plot (left), reinforcement zones (right).

Young modulus of the reinforcement steel of the liner and the passive reinforcement is $2.1{\cdot}10^8\,\mathrm{kN/m^2}$ and the prestressed one is $1.025{\cdot}10^8\,\mathrm{kN/m^2}$. The yield limits of the steel used for the liner, passive and prestressed reinforcement are $1.68{\cdot}10^5\,\mathrm{kN/m^2}$, $4.2{\cdot}10^5\,\mathrm{kN/m^2}$ and $3.2{\cdot}10^5\,\mathrm{kN/m^2}$, respectively. The hardening modulus for all types of steel is $3.0{\cdot}10^7\,\mathrm{kN/m^2}$. The initial Young modulus for concrete is $3.0{\cdot}10^7\,\mathrm{kN/m^2}$, yield limit in compression is $3.2{\cdot}10^4\,\mathrm{kN/m^2}$, strength in tension is $3.2{\cdot}10^3\,\mathrm{kN/m^2}$.

The behaviour of concrete is described by the constitutive model presented above and steel is modelled using elasto-plastic model with isotropic hardening. Aspects of the equilibrium analysis, DSA and crude Monte-Carlo method have been described in [7, 8].

8.1. Linear case: AVM

The adjoint variable method is very suitable to solve linear problems with high number of design variables [1]. Consequently, the method is very efficient when attempting to solve this type of reliability problems.

An example of such a stochastic system is created. The stochastic parameters are the thicknesses of the reinforcement layers in particular elements and load multiplier. The normal distributions are assumed for the steel layers with the standard deviations as follows: 0.15, 0.15, 0.015, 0.15, 0.15, 0.16, counting from the liner to the most external layer. For the load multiplier the log-normal distribution is assumed and the standard deviation is 0.20. The whole stochastic model consists of 3841 variables. The structure is loaded with the external equivalent pressure and the reliability index is evaluated for the internal pressure load multiplier 6.5. The failure function is described

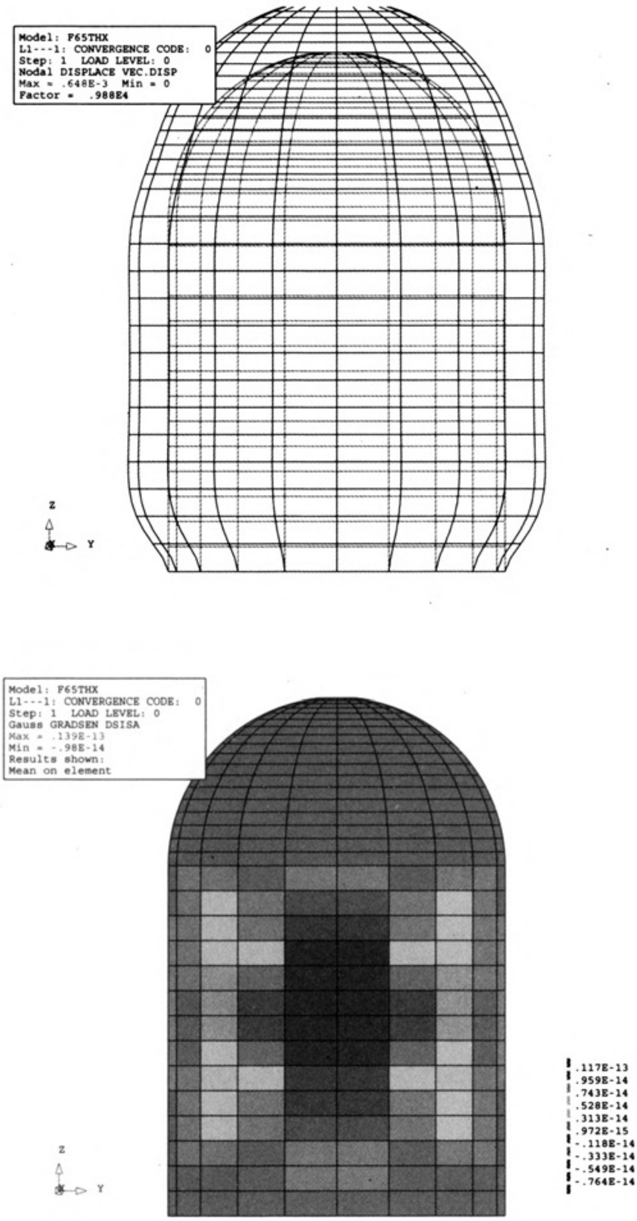

FIGURE 3. Shape of the structure for the considered load level (upper), displacement sensitivity w.r.t. the thicknesses of the liner in particular elements, deterministic solution (lower).

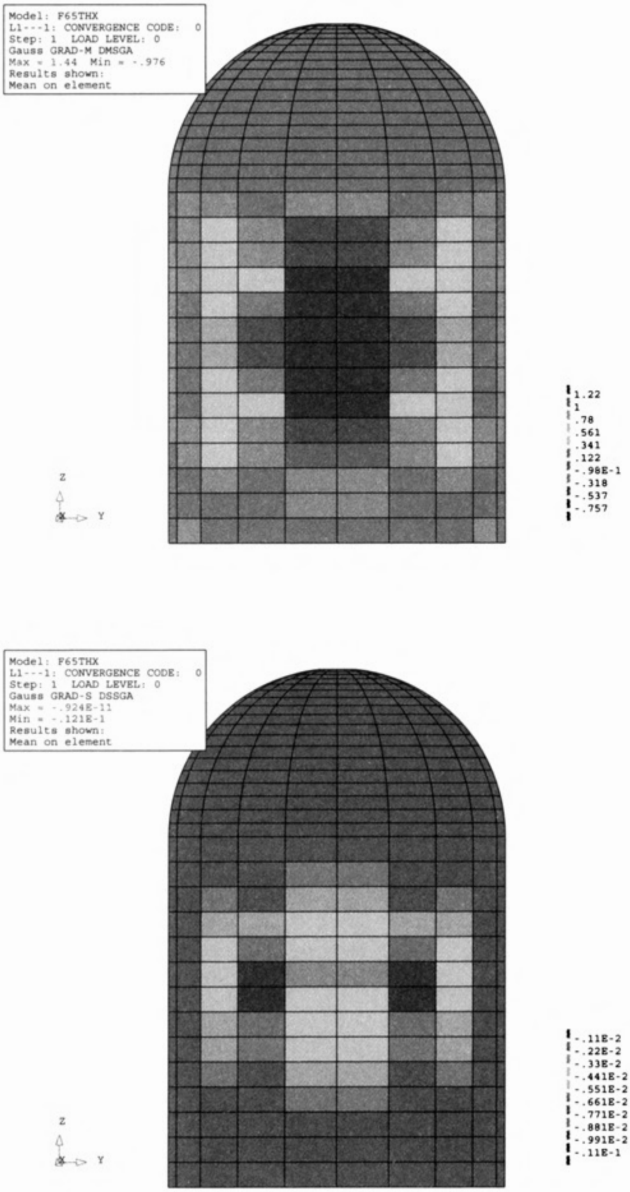

FIGURE 4. Reliability index sensitivity w.r.t. mean values (upper) and standard deviations (lower) of the thicknesses of the liner in particular elements.

by the excessive horizontal displacement of the point (651) in the midspan of the cylinder, $g(x) = q^{\star} - q^{h}_{651}$. Its allowable value is $0.15{\cdot}10^{-2}$ m.

The deterministic solution is presented in Fig. 3. The shape of the structure and the design sensitivity field of the choosen horizontal displacement are shown. The design variables are the thicknesses of the liner in particular elements. The deterministic displacement is $0.4824{\cdot}10^{-3}$ and the highest absolute sensitivity gradients are in the neighbourhood of the constraint and on the sides (left and right) of the cylinder.

The estimated reliability index is 6.08 and corresponds to the probability of violation of the allowable displacements – $6.169{\cdot}10^{-10}$. The beta index sensitivities with respect to the mean values and standard deviations are presented in Fig. 4. The distribution of the beta index sensitivities (Fig. 4, upper) is qualitatively similar to the design sensitivities of the chosen displacement calculated during the deterministic solution. The distribution of the beta index derivatives w.r.t the standard deviation is qualitatively different from the design sensitivities, the lowest beta index sensitivities corresponds to the the highest design sensitivities in the deterministic solution. A more extended analysis of the vessel in the linear range is given in [10].

The run of the program for this example takes about 5 hours CPU and the reliability solution is reachead after 17 iterations. This example was calculated on SUN HPC Enterprise 10000 platform.

8.2. Nonlinear case: DDM

The design sensitivity analysis of the vessel is carried out for the range of load starting from the prestressing phase (external pressure) passing to the increase of uniform internal pressure up to failure. The design parameter is the thickness of the internal circumferential reinforcement layer in a chosen element (141).

The results of the design sensitivity analysis are presented in Figs. 5 and 6. The shape of the structure in the prestressing phase is presented in Fig. 5 (upper) and the corresponding design sensitivity field is given in Fig. 6 (upper). The shape of the structure in the failure phase is given in Fig. 5 (lower) and the displacement sensitivity field is shown in Fig. 6 (lower). The corresponding internal pressure is $10.64\,\mathrm{kN/m^2}$ and the horizontal displacement of a point in the midspan of the cylinder is 0.174 m. A qualitative difference is manifested when comparing the design sensitivity fields for both phases. During the prestressing phase the behaviour of the structure is almost linear, the highest values of the design sensitivity gradients are concentrated mostly close to the investigated element whereas in the failure phase when

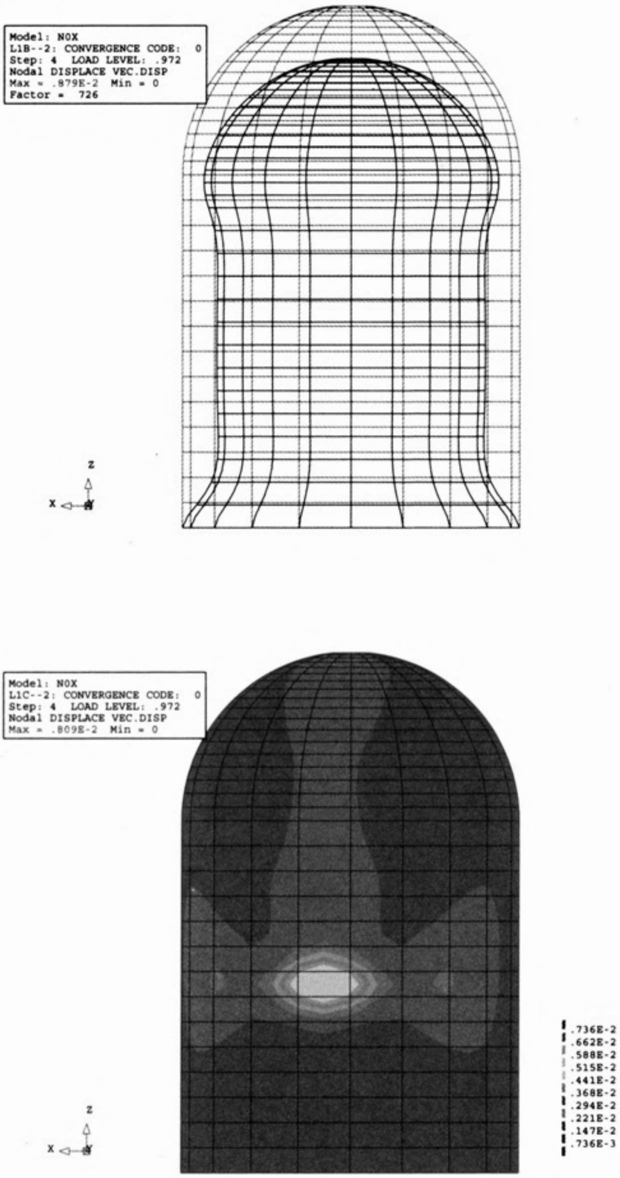

FIGURE 5. Shape of the structure, prestressing phase (upper), displacement sensitivity field (lower), design parameter – thickness of reinforcement layer, internal, circumferential, element 141.

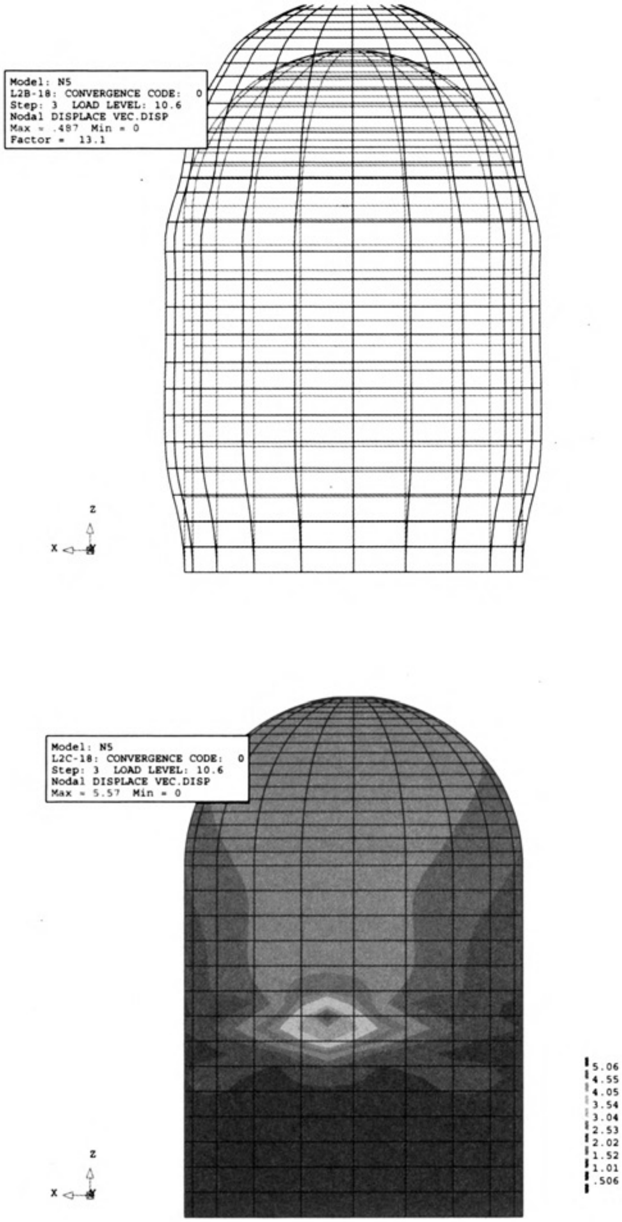

FIGURE 6. Increase of the internal pressure, shape of the structure (upper), displacement sensitivity field (lower), design parameter – thickness of reinforcement layer, internal, circumferential, element 141.

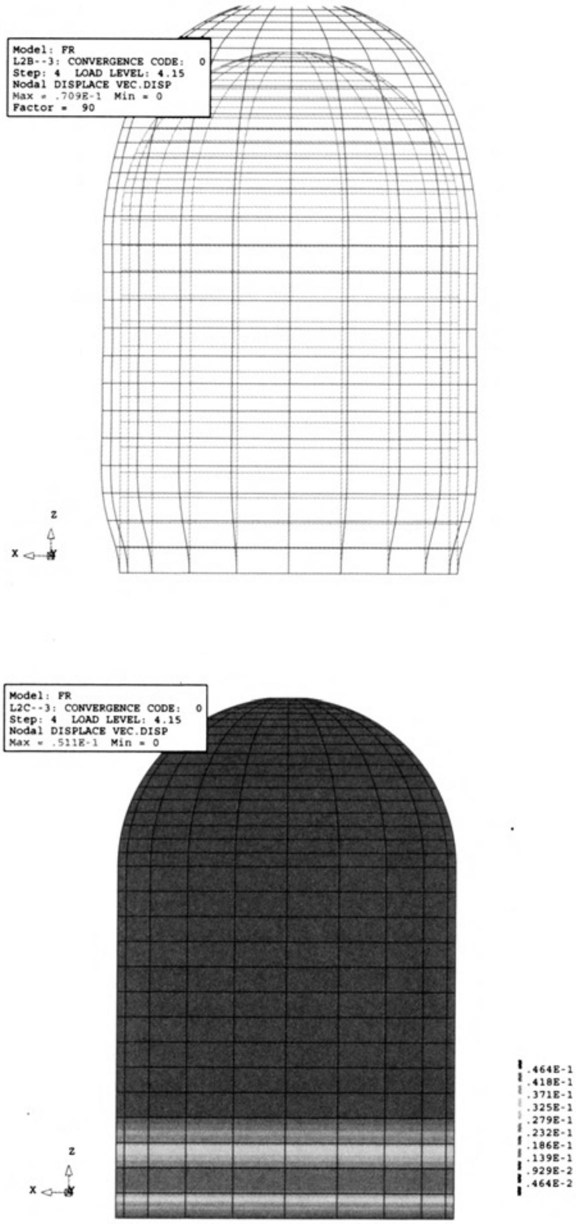

FIGURE 7. Increase of internal pressure, shape of the structure (upper) and the displacement sensitivity field (lower), design parameter – distance of the liner from the midsurface.

the nonlinear behaviour of the material is very significant the high values of the gradients are distributed over the whole structure.

In the next step the reliability analysis of the system is performed. The stochastic parameters are the distances of whole reinforcement laters from the midsurface. The stochastic system consists of 6 parameters connected with the reinforcement and load multiplier.

Normal distributions are assumed for the distances from the midsurface of the shell with the following standard deviations: 0.008, 0.004, 0.005, 0.005, 0.001, 0.005. The log-normal distribution with standard deviation 0.20 is assumed for the load multiplier.

The displacement failure function is considered. As in the linear case the design constraint is set on the horizontal displacement of the point 651 in the midspan of the cylinder. The horizontal displacement should not exceed 0.035 m. The shape of the structure is given in Fig. 7 (upper) and the design sensitivity field from the deterministic solution is presented in Fig. 7 (lower). The reliability analysis is carried out for the load level 415 kPa. The deterministic displacement for this load level is $0.186 \cdot 10^{-3}$. The obtained reliability index is 6.561.

Full run of the equilibrium and sensitivity analyses up to failure phase takes about 20 hours CPU (DEC Alpha, 677 MHz, 4 CPU). One iteration of the reliability module for the investigated failure function with the assumed constraint above and the considered pressure level takes 12 hours CPU. The reliability index was reached in 49 iterations. Total CPU time of the reliability module stands for almost negligible per-cent of the total CPU needed.

9. Final remarks

A short description of an algorithm concerning the reliability index evaluation is described. The sensitivity coefficients are calculated employing the DDM and AVM along with the analytical approach. The effectiveness of the algorithm is proved by presenting the numerical example. The computational effort is still very high, however, it may be decreased by further improvement of the efficiency of the equilibrium and the design sensitivity analyses using the vectorized and parallel solvers.

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